Meteoroid Stream Modeling in the Inner Solar System

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-Abstract-

The statement that some small bodies in the Solar System - asteroids, comets, meteors (of cometary origin) - travel in co-orbital streams, would be accepted by planetary scientists without argument. After all, streams have been observed of fragments of at least one comet (Scotti and Melosh, 1993; Weaver et al., 1993), asteroids (Drummond, 1991; Rabinowitz et al., 1993; Binzel and Xu, 1993) and meteoroids of asteroidal origin, like Innisfree (Halliday et al., 1990; cf. Drummond, 1991). Whether members of a stream can be recognized from compositional studies of meteorites recovered on Earth and linked to a common source is more controversial since such linkage would imply variations in the Earth’s sampling of extraterrestrial material that persist for tens of Myr.

To begin to address these open issues, we simulate the trajectories of several near-Earth meteoroid streams---some with orbital elements corresponding to suspected streams, others randomly chosen. To integrate the trajectories as accurately as possible, we use an error-optimized modified 13th order Störmer integration scheme, capable of handling close planet/meteoroid approaches (Grazier et al., 1998). We use Drummond’s (1998) $D'$ criteria to determine stream membership and coherency as a function of time. We find that stream coherency beyond 100 Ky, certainly beyond 1 My, is rare.

-Introduction-

The dates of fall of H chondrites show that many – including Clusters of May, 1855-1895, September, 1812-1831 and Sept.-Oct., 1843-1992 - apparently derive from specific meteoroids (Lipschutz et al., 1997). Contents of highly volatile elements in these 3 Clusters (selected by one criterion, fall circumstances), when analyzed using multivariate statistical techniques demonstrate that members of each Cluster (i.e. stream) share a totally different characteristic: a thermal history distinguishable from random H chondrite falls (cf. Lipschutz et al., 1997, for specific references). Antarctic H chondrites with terrestrial ages >50 Myr (Michlovich et al., 1995) also show this. Metallographic and thermoluminescence data for these H chondrites which also reflect their thermal histories, and support the existence of such meteoroid streams (Sears et al., 1991; Benoit and Sears, 1993), but cosmogenic noble gas contents do not (Loeken et al., 1993; Schultz and Weber, 1996). Important unanswered orbital dynamic questions are how long a meteoroid stream should be recognizable and what dynamic conditions are implied by Clusters, whose members have cosmic ray exposure ages of some Myr.

-Project Methods and Techniques-

Since stream coherency is the primary focus of this endeavor, Jack D. Drummond’s $D'$ criterion was chosen to differentiate between significant streams and individually coincident meteoroid orbits. The following equation combines various orbital elements of the particles being considered and weights the values into a linear discriminant that contains all directionless components, with each orbital element contributing equally to $D'$:

$$D'^2 = \frac{(e_1 - e_2)^2}{e_1 + e_2} + \frac{(q_1 - q_2)^2}{q_1 + q_2} + \left(\frac{l}{180^\circ}\right)^2 + \frac{e_1 + e_2}{2} \left(\frac{\theta}{180^\circ}\right)$$

where $l = \arccos[\cos{(i_1)}\cos{(i_2)} + \sin{(i_1)}\sin{(i_2)}\cos(\Omega_1 - \Omega_2)]$

and $\theta = \arccos[\sin{\beta_1}\sin{\beta_2} + \cos{\beta_1}\cos{\beta_2}\cos(\lambda' - \lambda_2)]$

where $\beta'$ (ecliptic latitude of perihelion point) = $\arcsin(\sin{(i_1)}\sin{(i_2)})$

and $\lambda'$ (ecliptic longitude of perihelion point) = $\Omega + \arctan(\cos{\tan{\lambda}})$
The value of $D'$ therefore ranges from 0, which is a perfectly coherent orbit, to 1, which is completely unrelated.

Since there is an obvious lack of coherent meteoroid streams in the present-day Solar System, fictitious particles were utilized in addition to the orbital elements from comet and meteoroid streams present in the current Solar System in order to create enough data for this simulation. A 13\textsuperscript{th}-order backwards-difference Störmer integrator (Grazier et al., 1998) was chosen to integrate the positions and velocities of these particles primarily because of its error optimization and accuracy. The equations for computing kinematics of this nature are used as infinite series with finite time steps, and thus only a certain number of terms are chosen that contribute significantly to the solution. This integrator is of the 13\textsuperscript{th} order to ensure that all computations are performed at machine precision, stretching computational accuracy to the limits of the digital computers available. In order to maintain such high computational reliability, however, the integrator must additionally manage both truncation error and roundoff error appropriately.

To combat truncation error, this particular Störmer method employs a backwards-difference implementation which is analytically equivalent but computationally superior to other forms of the Störmer method. It allows the integrator to update information about a particle using previous positions and accelerations, independently of the order of the infinite series used. The most common source of roundoff error in the large amounts of computations used is evidenced when values with significantly different orders of magnitude are calculated. To reduce this error, "significance-ordered computation" was used, where all operations are grouped with similar orders of magnitude. These error-optimization routines combined insure that error growth remains linear and not systematic.

In order to test the accuracy of the integrator, the planets were integrated forward in time for several Ky. The final positions and velocities were then used as starting points, and the Solar System was integrated backwards for the same time period. This process was repeated, and at the end of each backwards integration, the relative errors in energy and angular position of the entire system were found to be minimal. This provided superior, method-independent confirmation of the integrator’s accuracy.

When integrating the outer solar system, as was done in previous studies (Grazier et al., 1998) it is possible to assume the masses of the inner planets are added into the sun to simplify the calculations with a negligible effect on the results. However, since the inner solar system is now being considered, this is not feasible. The first challenge which arises when considering the inner planets is Mercury’s orbital precession, which can only be accurately replicated with the addition of general relativity. In order to achieve this, a segment of code was added to the original integrator, which calculated the contributions to acceleration from relativity of a central body and any number of orbiting bodies.

-\textbf{Adjustments to Address This Issue-}\n
Mercury's orbital precession posed some interesting challenges. In Newtonian physics, under standard assumptions for the two-body problem, a lone object orbiting a spherical mass would trace out an ellipse with the spherical mass at one of the foci. Perihelion, in the case of our solar system, specifically the point of closest approach, is fixed. Orbital precession can be the effect of a number of factors, such as the gravitational pulls of the other planets in the solar system or the oblateness of the Sun, but none of these would cause such a significant change in Mercury's orbit. In 1859, Urbain le Verrier first recognized this as a problem with celestial mechanics when his calculated value from Newton’s equations disagreed with the observations from 1697 to 1848.

-\textbf{Current Status-}\n
The project is currently ready to begin the testing phase. In order to reach this point, however, the integrity of the integrator needed to be verified. After adding the relativistic changes to account for Mercury's precession, the first test integrated Mercury in two ways. Initial conditions were used without any other particles in the system for both. Mercury was integrated with and without the relativistic corrections for 1 Myr. Since Mercury precesses at the rate of 43 arcseconds per century, the two ending positions differed by precisely the expected amount, in this case 119.44°. Now that the integrity of the integrator is assured, it is ready to begin computing the close encounters for the particles to examine the existence of meteoroid streams in the inner solar system (or lack thereof.)
This numerical investigation utilizes the most accurate numerical techniques ever employed on these issues, as pioneered by (Grazier et all, 1998). This statistical methodology has been applied to a different area of interest with the same precision and reliability as used in the Outer Solar System. Some streams such as the Lagrange points and perhaps the Hilda group remained stable for large portions of the Solar System’s existence in the Jupiter/Saturn zone, and although the outcomes of these previous studies have determined that planetesimal niches are rare, insight into the evolutionary process of the inner Solar System may give us new understandings of our primordial origins.

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-References-