Supercomputer Numerical Simulation of Astrophysical Processes

Tannaz Oskui
astrophysicschick@gmail.com
Astrophysics- Pasadena City College
Astrophysics and Space Sciences Section
NASA-Jet Propulsion Laboratory
California Institute of Technology
Two Pure Research Topics

• Pure Mathematics-Monte Carlo Analysis
• Stellar Formation
• Follow up work for Hubble, Spitzer
• Prelude for James Webb Space Telescope
Modeling Photonic, Atomic or Nuclear Processes Requires Doing Numerical Integration - *Curse of Dimensionality*

- 7 dimensions typically required to specify just one single particle state - non excited
- 3 x, y, z position, 3 $v_x$, $v_y$, $v_z$ velocity and time
- Integrals 7 dimensional or higher $\iiint d x_1 \ldots d x_7$
- Error proportional to the seventh root! $\frac{1}{\sqrt[7]{n}}$
- *Two decimal point improvement* ↓ $10^{14}$ more computations! (About 200,000 years or ½ of a management meeting)
Ulam, Feynman, von Neumann to the rescue!!!
Monte Carlo Method

\[ \frac{1}{s\sqrt{n}} \text{ vs } \frac{1}{2\sqrt{n}} \] by the Central Limit Theorem

• Major advance and problems then become dimension free
• Drawback: Solutions are Probabilistic NOT Deterministic
• Pseudo-random number generation difficult mathematical topic
The Monte Carlo Method
Volume of an Arbitrary Shape

\[
\begin{bmatrix}
1.0 \\
0.5 \\
0.0 \\
0.5 \\
1.0 \\
\end{bmatrix}
\]
Can do even better converge at $\frac{1}{\sqrt{N}}$ vs $\frac{1}{N}$

Random sequence on left less uniform than Halton sequence on the right
1-D Convergence with just Random and 1-D Halton = Van der Corput.

\[ \int_{I_1} e^y \, dy \]

Error improved by an order of magnitude!

![Convergence Rates](chart.png)
1-D Koskma Inequality

- Let $f$ be of bounded variation $V(f)$ on $[0,1]$ then for any real numbers $P=\{x_1, x_2, ..., x_N\}$ with star Discrepancy, $D^* (x_1, x_2, ..., x_N)$ then

\[ \left| \int_{I_1} f(x) dx - \frac{1}{N} \sum_{k=1}^{N} f(x_k) \right| \leq V(f) D^*(P) \]

- $V(f)$ constant so only $P=\{x_1, x_2, ..., x_N\}$ determines the error

- Recall \[ \left| \int_{I_1} f(x) dx - \frac{1}{N} \sum_{k=1}^{N} f(x_k) \right| = O(\sigma \cdot N^{-\frac{1}{2}}) \]

- Make correspondences $V(f) \rightarrow \sigma$ and $D^*(P) \rightarrow N^{-\frac{1}{2}}$

- $\Rightarrow$ Error $= O \left( \left( \frac{\log N}{N} \right)^{\frac{1}{2}} \right) $
Weyl Criterion for Uniformity Mod(1)

• Let \( P = \{x_k\}, \ k=1,2,3... \) be a sequence of points in \( I^S \), then \( \{x_k\} \), is uniformly distributed Mod 1 iff for all complex valued functions \( f(x) \) on \( I^S \)

\[
\lim_{N \to \infty} \frac{1}{N} \sum_{k=1}^{N} f(x_k) = \int_{I^S} f(x) dx
\]

• (Weyl Criterion) \( P = \{x_k\} \) uniformly distributed Mod 1 iff, for all \( h \in Z^S, h \neq 0 \)

\[
\lim_{N \to \infty} \frac{1}{N} \sum_{k=1}^{N} e^{2\pi i (h \cdot x_k)} = 0
\]
Future Work to be done on the Pleiades SuperComputer

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