Optical Properties and Thermal Modeling of Dust Grains in Circumstellar Environments

Justin Bracks
Consortium for Undergraduate Research
Los Angeles Community College
Los Angeles, USA
Justinbracks@gmail.com

***Abstract*— Python modules are developed to calculate the effective optical properties of theoretical composite circumstellar grains. Additionally, a separate suite of modules was developed to use the effective emissivities of those composite mixtures to create thermal models of circumstellar debris belts around a variety of star types.**

***Index Terms*—Circumstellar Matter, Effective Medium Theory, Thermal Modeling.** *(key words)*

I. Introduction *(Heading 1)*

In recent decades astronomers have had an ongoing conversation regarding the composition of the majority of circumstellar debris in mature star systems spurred by the discovery of hundreds of planetary debris disks. By the 1990's the community generally assumed an asteroidal composition for these debris systems. Meaning the debris would be rockier in composition. However, in more recent times the assumption has shifted to that of icier, more cometary compositions.

Because these debris belts are made of many smaller particles, their overall surface area is far greater than that of any exoplanet in their system. This means that they absorb and reemit the light from their host star far more readily, making them distinctly easier to see than exoplanets.

More developed instruments and techniques have allowed for a deeper and more refined probe of these star systems.

For instance, Herschel images of many of these star systems show features resembling a two-belt configuration with an outer and inner belt analogous to our own asteroid and Kuiper belts. The characteristics of these exo-astroid belts and exo-kuiper belts may offer significant insight into the stellar neighborhoods in which they have developed. This in turn can tell us a great deal about the planets potentially developing there. Moreover, through modeling and further observation we can hope to gain a deeper understanding of the typical stages of the evolution of star systems. Currently scientists are attempting to associate specific disk traits to corresponding characteristic evolutionary phases in star system development. This could aid immensely in humanity's effort to characterize the current flood of data regarding exoplanets.

II. Optical Properties

In this work we have progressed with the assumption that the grains of dust in an exo-debris belt are comprised of some mixture of astronomical silicates (astrosil) akin to feldspar and dirty water ice which itself is composed by pure water ice with inclusions of carbon. Because the optical properties of a grain of such a composition are unclear, an effective emissivity must be calculated for any mixture the team sought to create a thermal model for.

To this end effective medium theory (EMT) is applied using the Maxwell-Garnett matrix/inclusion method. This EMT method assumes a mixture with two constituent materials, in this case dirty ice and astrosil. One of the two components serves as the matrix, and the other is then theoretically embedded into the matrix as the inclusion. Without derivation for brevity, the equation used for effective emissivity of a compound material is given as:

$$\epsilon_{eff} = \left(1 + \frac{3f\left(\frac{\epsilon_i - \epsilon_m}{\epsilon_i + 2\epsilon_m}\right)}{1 - f\left(\frac{\epsilon_i - \epsilon_m}{\epsilon_i + 2\epsilon_m}\right)} \right) \tag{1}$$

Here ϵ_{eff} is the effective emissivity of the composite material. f is the filling factor of the inclusion. ϵ_i is the emissivity of the inclusion and ϵ_m is the emissivity of the matrix. For the purposes of this work we've assumed spherical grains.

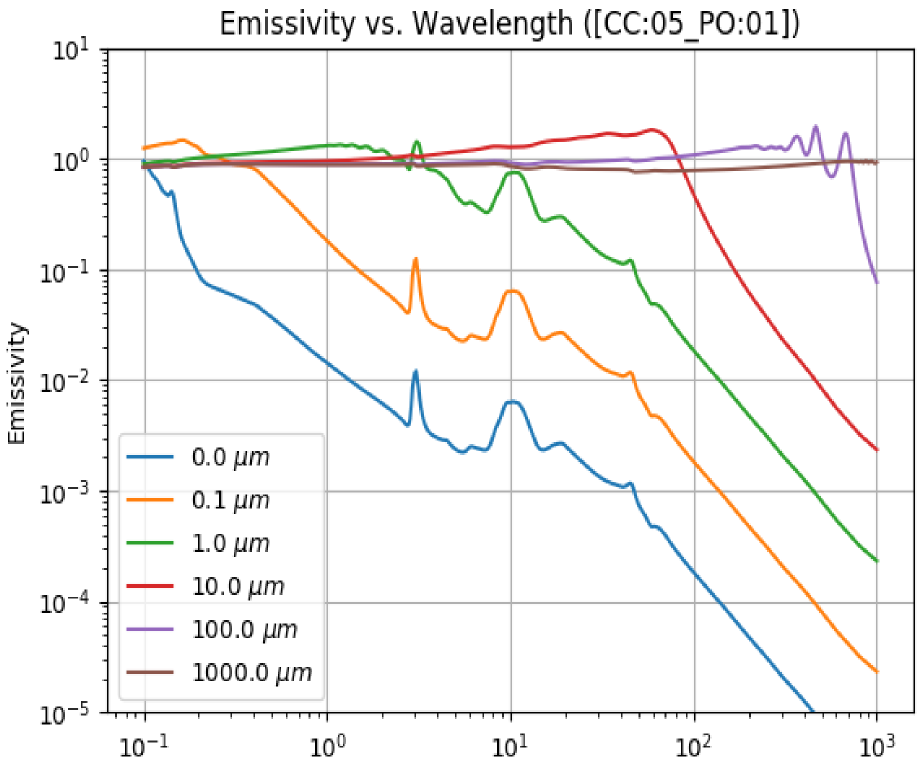
III. Particle Properties

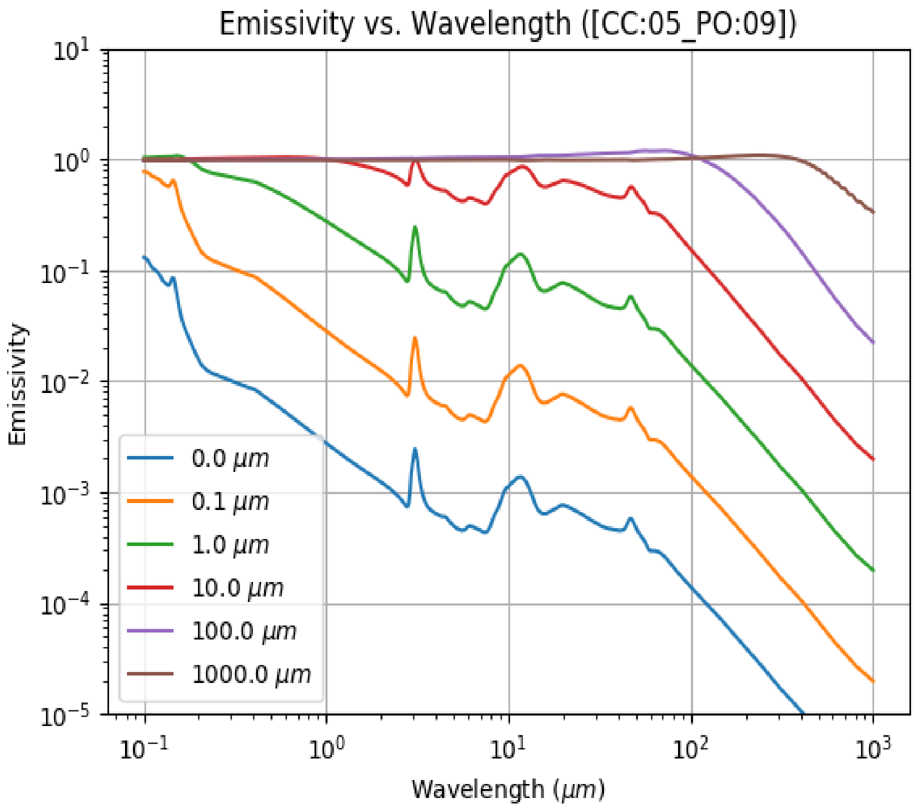
Two main characteristics were considered before calculating the effective emissivity of each particle type. First the filling factor of the inclusions. As an example, if we assume the dirty ice is the matrix and the astrosil the inclusion, a filling factor of 0.5 would indicate a 50% filing factor of rocky silicate inclusions by volume per grain. $f = 1$ would indicate an entirely silicate grain, f = 0 an entirely icy grain.

The choice of matrix verses inclusion is an important distinction; even with a 50-50 mixture, the resulting ϵ_{eff} changes slightly based on which material was chosen as the matrix. To illustrate this more intuitively, imagine a sphere of ice with stone marbles embedded in it. Even if the marbles constitute 50% of the sphere's overall volume, they can only contribute a small fraction of the sphere's surface area where light is incident. This intuitively weights the refractive index of the grain toward that contribution of the matrix. This in turn effects how the grain absorbs energy and ultimately what energy it can then re-emit.

The second characteristic considered was porosity. We must take into consideration that particles formed by the processes of collision might be somewhat consisted of porous pockets mixed into the compound material. The effect of porosity on a grain's emissivity was an important concern. Intuitively, a porosity of 0 represents a completely solid composite material, a porosity of 1 represents empty void; void ought not emit. Consistent with intuition, plotting emissivity verses wavelength incident for several porosity levels shows that as porosity rises, emissivity lowers.

In the title of these graphs, CC refers to the filing factor of rock. 50% was used throughout the summer as a test composition. PO refers to the porosity factor. Here we see a significant drop in emissivity from a porosity of 10% in the top graph to the following graph which depicts a porosity of 90%. Each colored line represents a specific grain size.



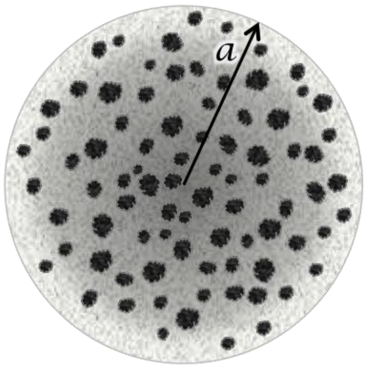


Note: for these graphs: the blue line is not truly a grain with radius 0.0, matplot simply rounded 0.01.

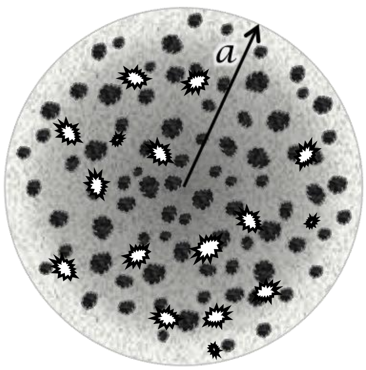
By scaling these two factors we are afforded a variety of grain compositions to model. For example a grain of porosity 0 and filling factor 1 would essentially be solid astrosil.

Solid Astrosil

A grain with a porosity factor of 0 filling factor of 0.3 would be a solid ice matrix with tightly held astrosil inclusions.



Water Ice Matrix with Astrosil Inclusions



Water Ice Matrix with Astrosil Inclusions and Porosity

As a final example, by maintaining the filling factor of 0.3 but increasing the porosity to 0.2 we have an ice matrix with astrosil inclusions, and inclusions of void.

IV. Thermal Balance

Once the effective emissivity of a composition can be determined per grainsize, a temperature can be calculated by finding the thermal balancing point between the energy flux coming into the grain from the host star and the energy being emitted by the grain itself. We employ the Planck function to calculate both energy in and energy out, given as:

$$B_{\nu,T} = \frac{2h\nu^3}{c^2}\frac{1}{e^{\left(\frac{h\nu}{k_B T}\right)}-1} \tag{2}$$

λ

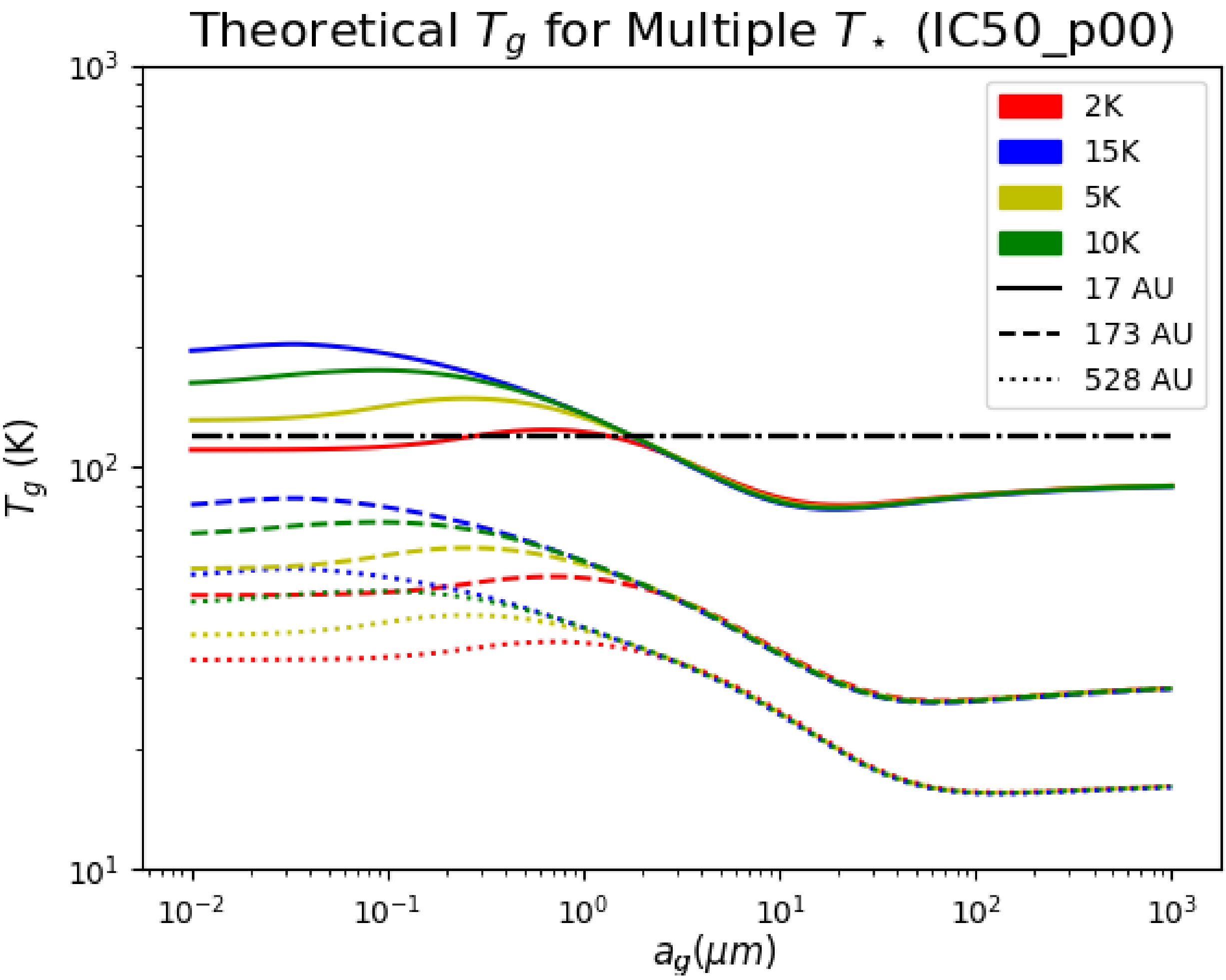
Where k_B is the Boltzman constant, h is the Planck constant, c is the speed of light in the medium, ν is the frequency of incident light and T is absolute temperature of the star. The emissivity of the composition must be considered in flux absorbed and emitted by the material. Finally, the cross-sectional area of the grain must also factor in, giving:

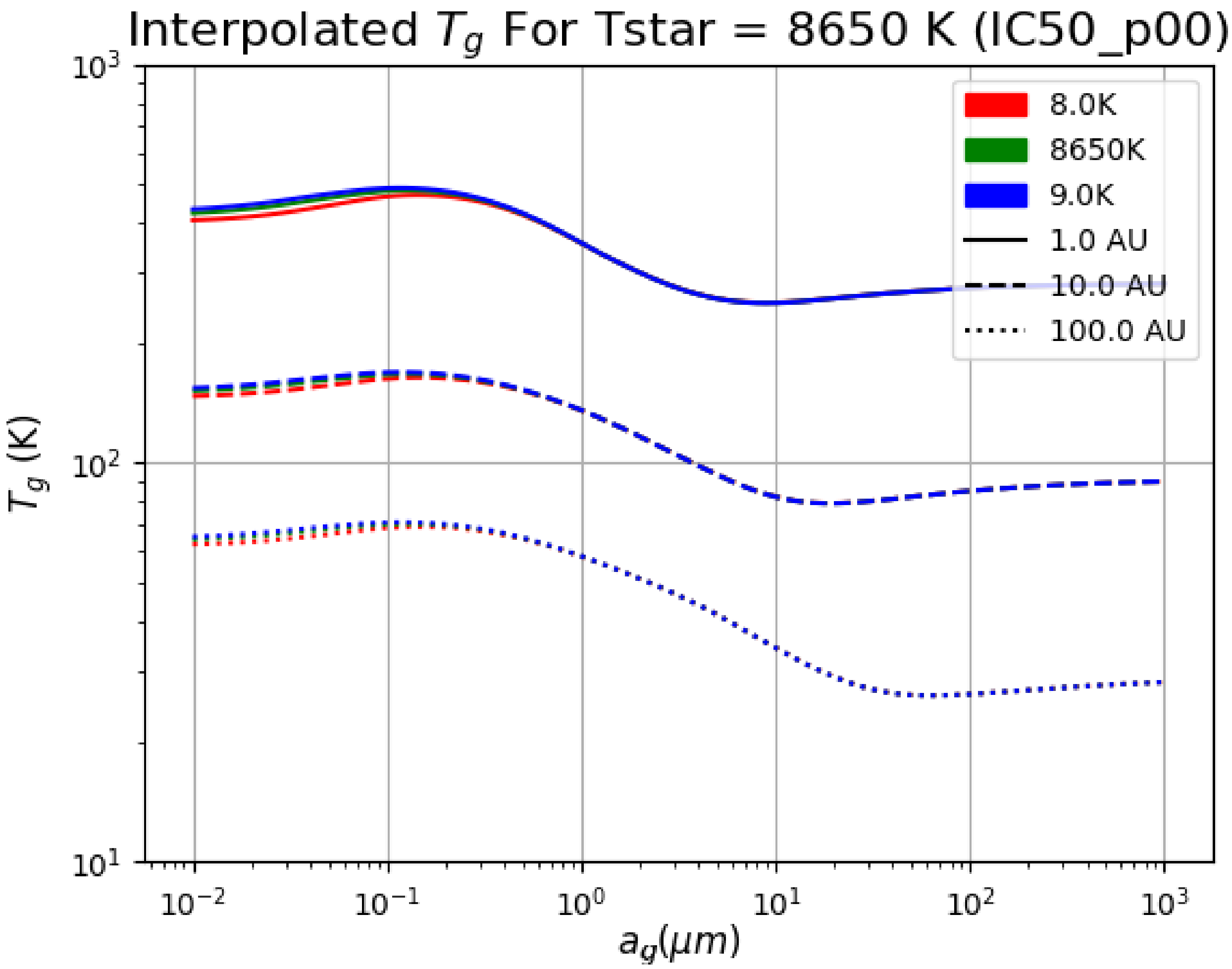
$$E_{out} = 4\pi a2 \cdot \pi \int \epsilon_{(a,\lambda)} \cdot B_{(\lambda,T_{dust})}\, d\lambda \tag{3}$$

$$E_{in} = \frac{\pi a^2 \cdot 4\pi R^2}{4\pi r^2} \cdot \pi \int \epsilon_{(a,\lambda)} \cdot B_{(\lambda,T_{dust})} d\lambda = \frac{\pi a2 \cdot L \cdot (1 - A)}{4\pi r^2} \tag{4}$$

Here r is the radial distance from the star to the grain, a represents the radius of the grain, λ is wavelength, and R is the radius of the star. By interpolating between E_{out} and E_{in} we find a thermal balancing point. However, the Planck function can only calculate flux for one frequency of light at a time, so all reasonable frequencies are calculated and a total flux is accounted for by integrating over the spectral energy distribution (SED).

Multiple parameters must be considered for the thermal modeling of grains in realistic star systems. The code written in this summer internship accounts for a range of radial locations from 1 AU to 1,000 AU, grainsizes of $0.01\mu m$ to $1{,}000\mu m$, and star temperatures ranging from L dwarves (not technically stars) to A types. The program builds a three-dimensional temperature table for every composition, making a separate table of $T_{(a,r)}$ for each of 14 intervals of object temperatures from 2,000 to 15,000 K. Plotting this many variables in a meaningful and readable manner was particularly difficult. These catalogues can then be used to interpolate grain temperatures for more specific star temperatures. The graph below depicts the temperatures for theoretical icy grains around four stars of different absolute surface temperatures, at 3 different radial locations. The graph on the following page shows three different star temperatures. Two were drawn from the catalogues previously created, and the third is an arbitrary temperature whose thermal profile was interpolated based on the catalogued profiles of stars immediately hotter and colder than it.





V. Conclusion

While the program will eventually need several more bells and whistles, the fundamental skeleton has been set down and each and every project objective was meet with time and effort to spare for a few peripheral tasks. The student was able to design and code additional modules for the project to use in the future. One such module was a peripheral add-on that can determine the percent by mass of each element in a grain by the percent volume.