Supercomputer Numerical Simulation of Astrophysical Processes

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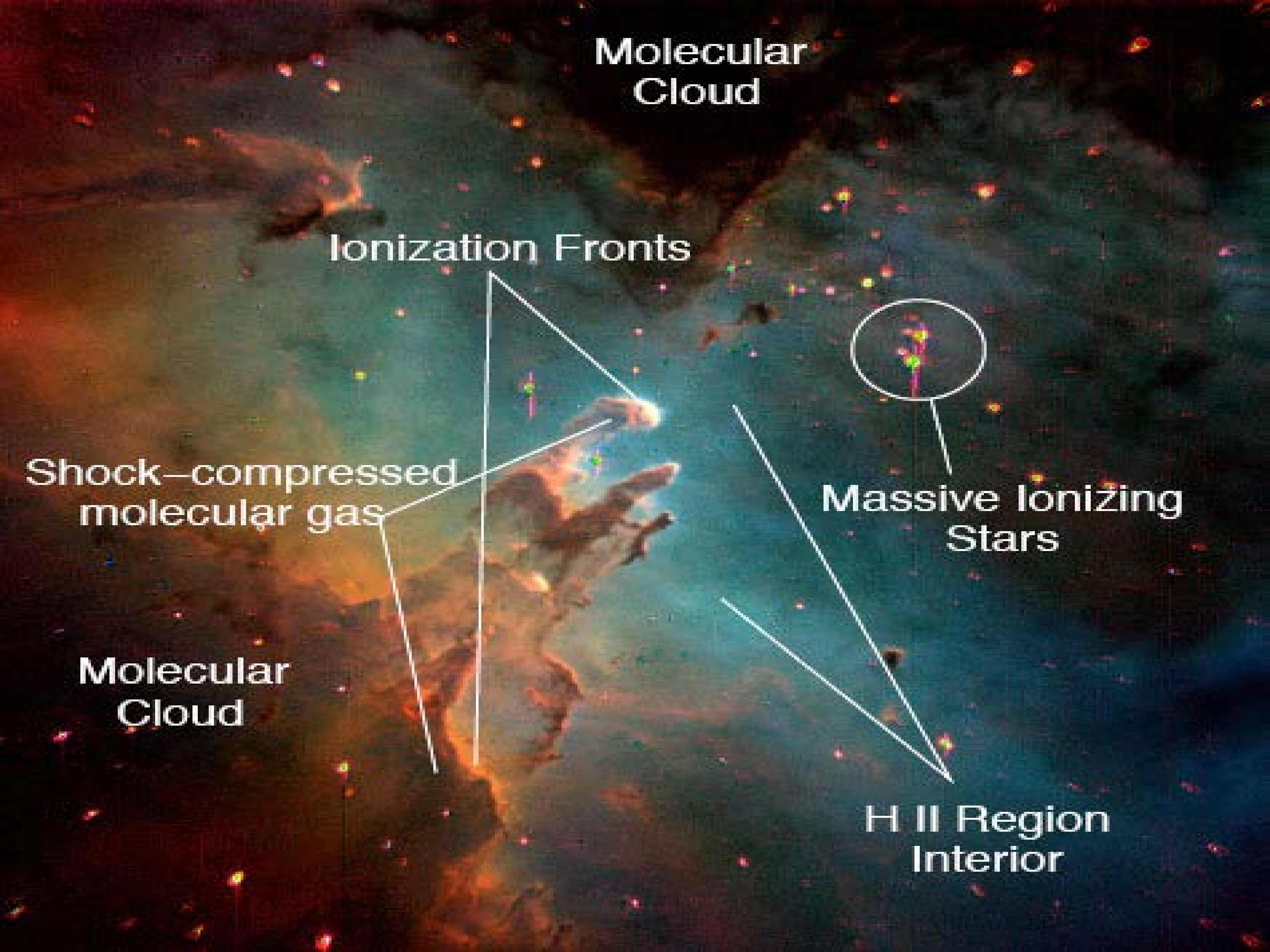

Molecular
Cloud
Ionization Fronts
Massive Ionizing
Stars
Shock–compressed
molecular gas
Molecular
Cloud
H II Region
Interior

Two Pure Research Topics

- Pure Mathematics-Monte Carlo Analysis
- Stellar Formation
- Follow up work for Hubble, Spitzer
- Prelude for James Webb Space Telescope

Modeling Photonic, Atomic or Nuclear Processes Requires Doing Numerical Integration-*Curse of Dimensionality*

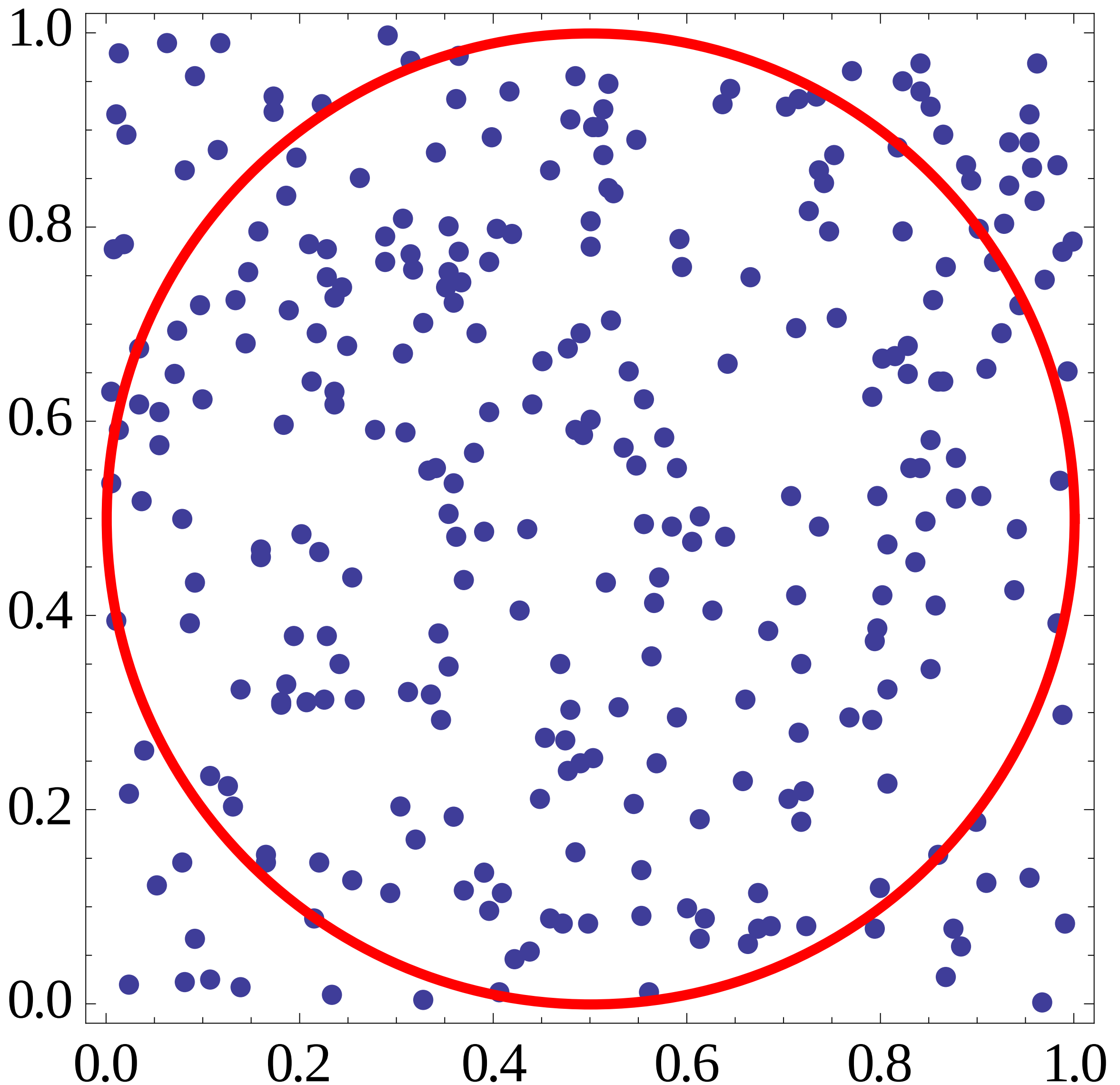
- 7 dimensions typically required to specify just one single particle state-non excited
- 3 x,y,z positon, 3 v_x, v_y, v_z velocity and time
- Integrals 7 dimensional or higher $\iiint d\,x_1 \ldots dx_7$
- Error proportional to the seventh root! $\frac{1}{\sqrt[7]{n}}$
- *Two decimal point improvement*↓10^{14} *more computations! (About 200,000 years or ½ of a management meeting)*

Ulam, Feynman, von Neumann to the rescue!!!

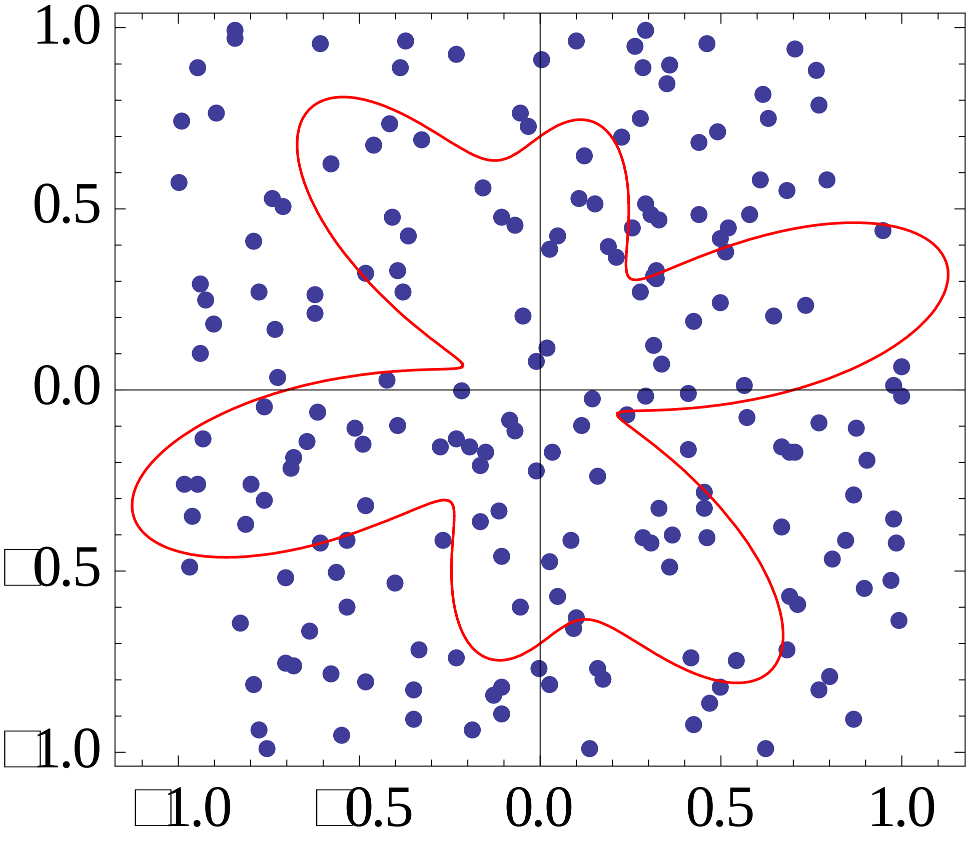
Monte Carlo Method

- $\frac{1}{\sqrt[s]{n}}$ vs $\frac{1}{\sqrt[2]{n}}$ by the Central Limit Theorem
- Major advance and problems then become dimension free
- Drawback: Solutions are Probabilistic NOT Deterministic
- Pseudo-random number generation difficult mathematical topic

The Monte Carlo Method



Volume of an Arbitrary Shape



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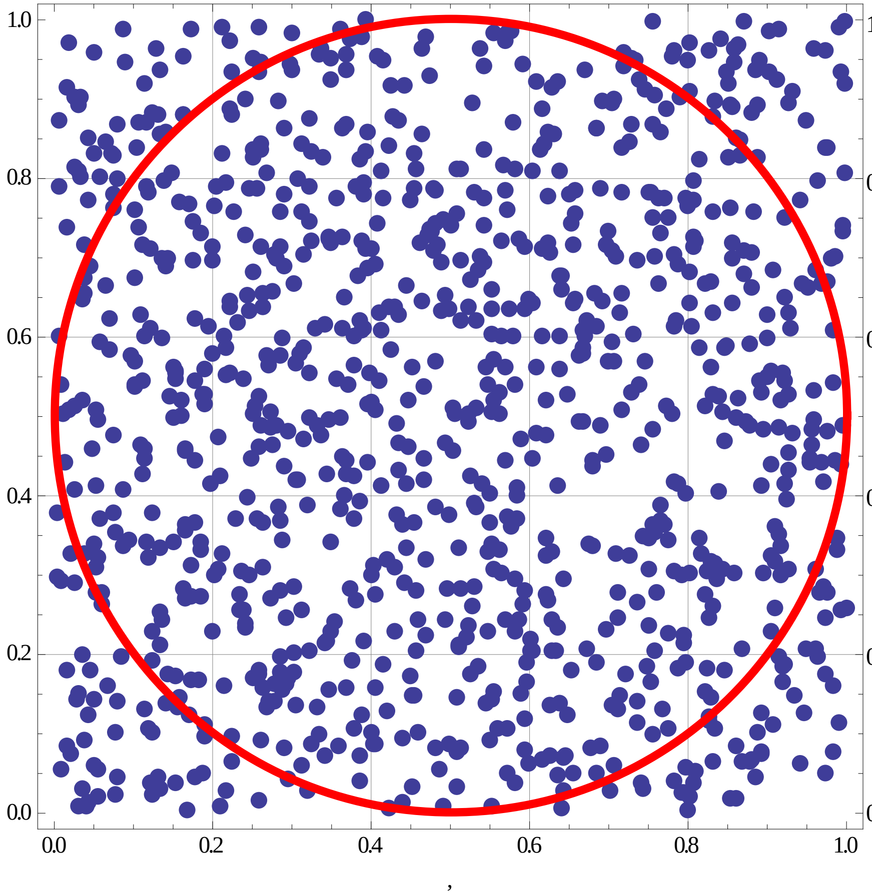
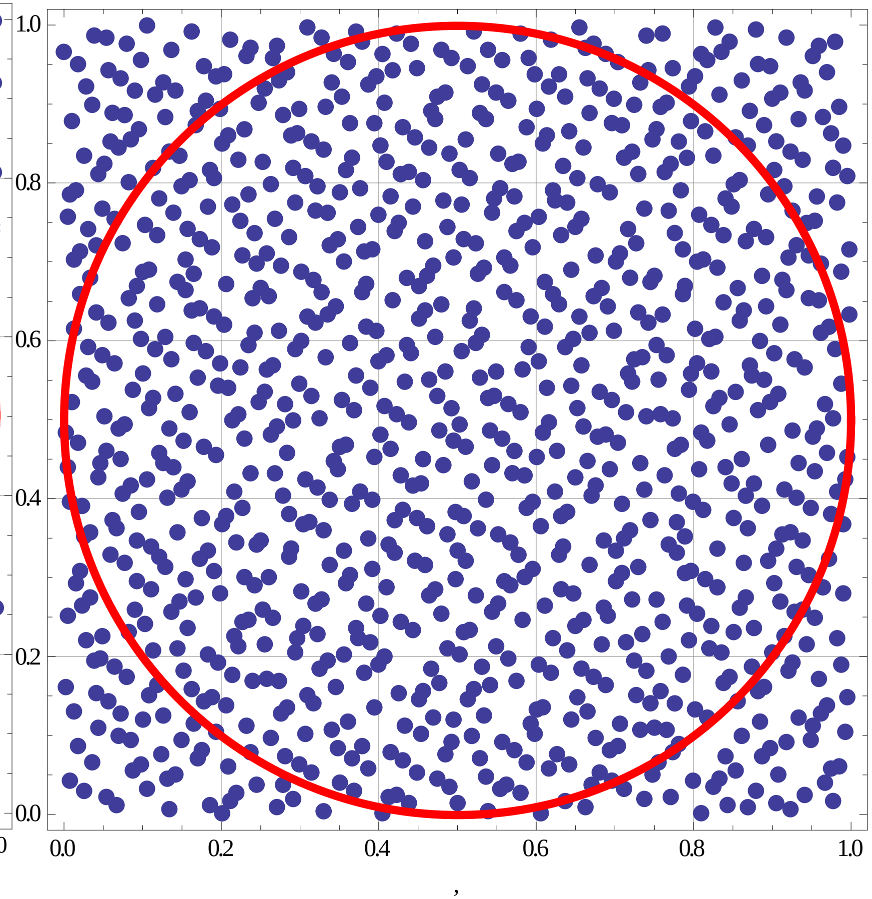
Can do even better converge at $\frac{1}{\sqrt{N}}$ vs $\frac{1}{N}$

Random sequence on left less uniform than Halton sequence on the right

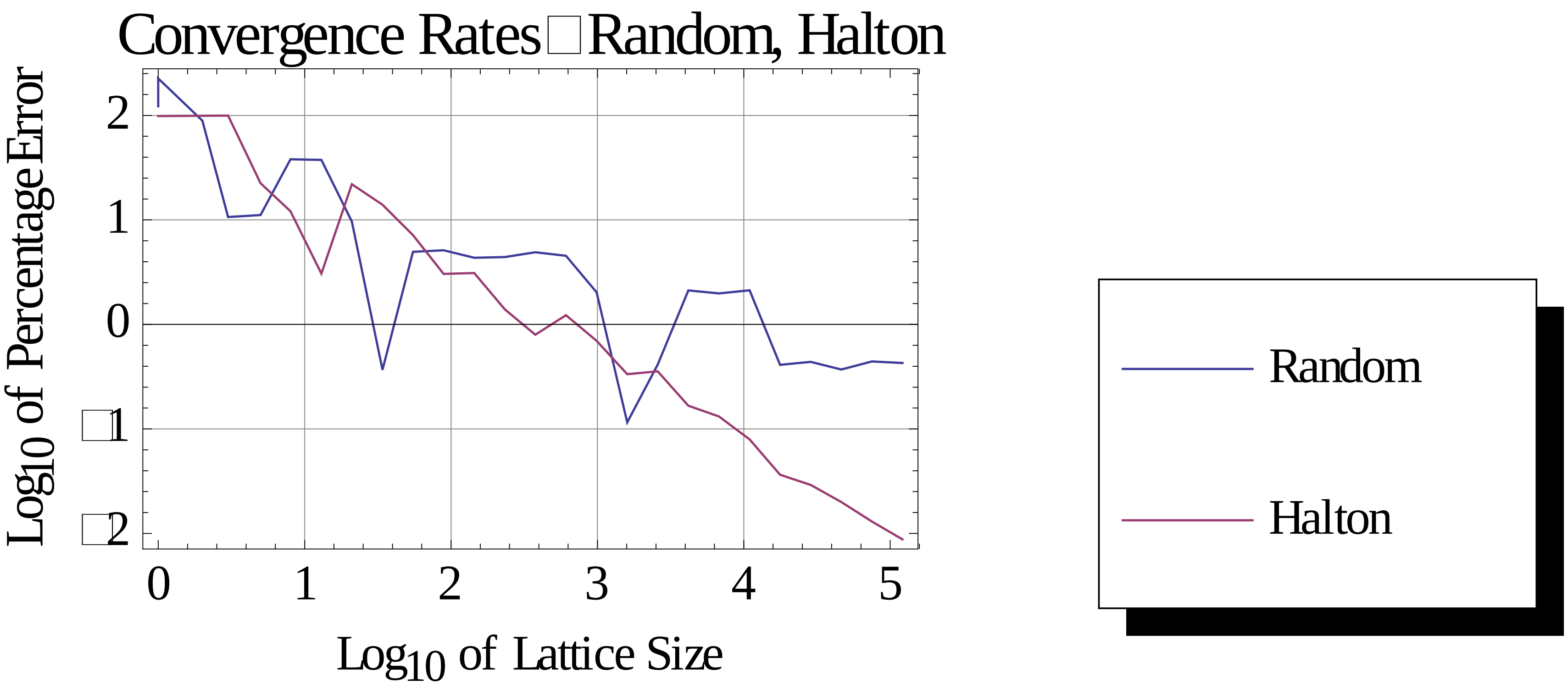




1-D Convergence with just Random and 1-D Halton =Van der Corput.

$$\int_{I^1} e^y \, dy$$

Error improved by an order of magnitude!



1-D Koskma Inequality

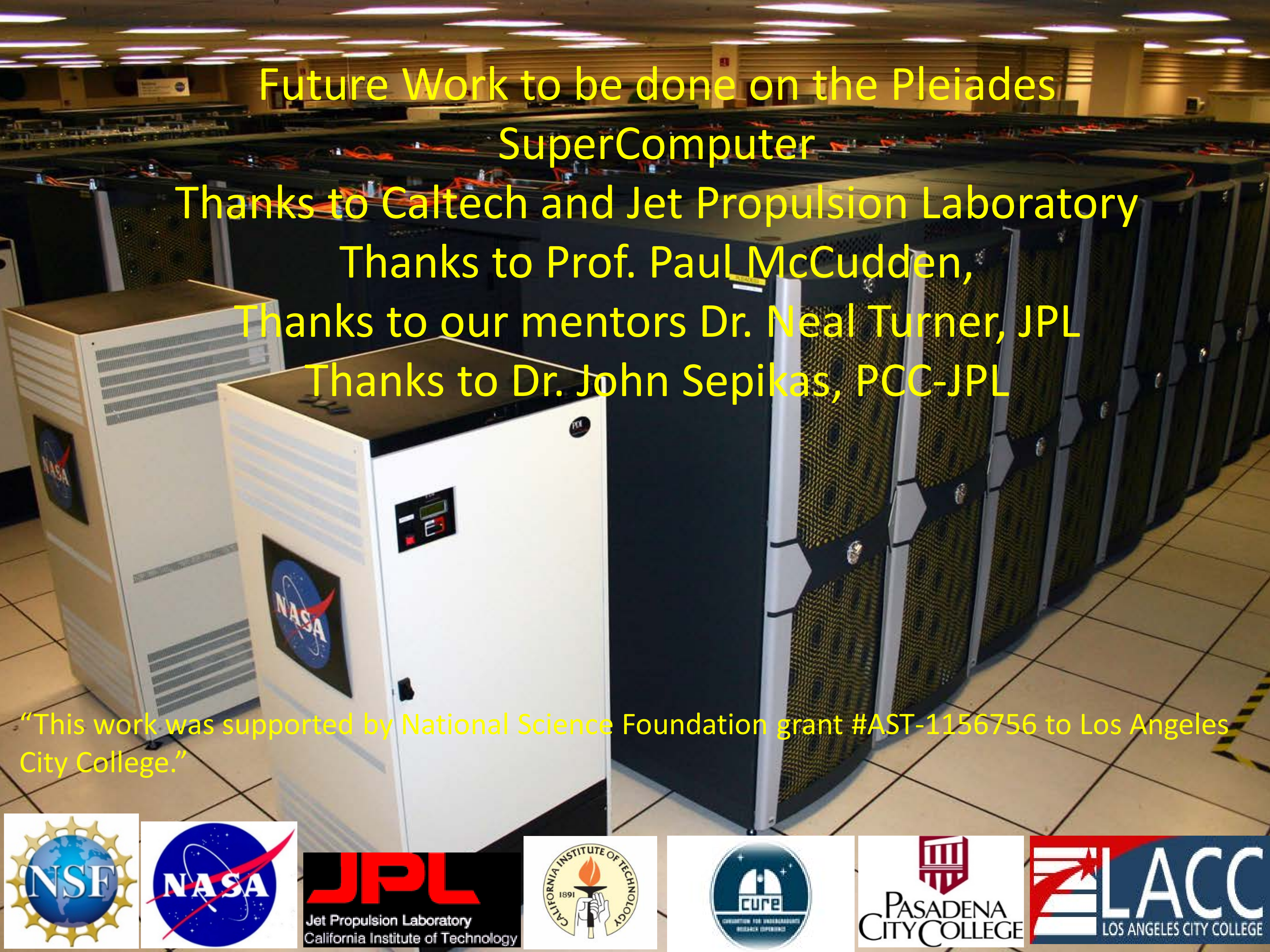
- Let f be of bounded variation V(f) on [0,1] then for any real numbers P= $\{x_1, x_2, \dots, x_N\}$ with star Discrepancy, $D^*(x_1, x_2, \dots, x_N)$ then
- $\left|\int_{I^1} f(x)dx - \frac{1}{N}\sum_{k=1}^{N} f(x_k)\right| \leq$ V(f) D^*(P)
- V(f) constant so only P= $\{x_1, x_2, \dots, x_N\}$ determines the error
- Recall $\left|\int_{I^s} f(x)dx - \frac{1}{N}\sum_{k=1}^{N} f(x_k)\right| = O(\sigma \cdot N^{-\frac{1}{2}})$
- Make correspondences V(f)→ σ *and* D^*(P) → $N^{-\frac{1}{2}}$
- => Error= $O\left(\frac{(Log\ N)^1}{N^1}\right)$

Weyl Criterion for Uniformity Mod(1)

- Let P={$\boldsymbol{x}_k$}, k=1,2,3…be a sequence of points in I^s, then {$\boldsymbol{x}_k$}, is uniformly distributed Mod 1 iff for all complex valued functions $f(x)$ on I^s

$$\lim_{N\to\infty} \frac{1}{N} \sum_{k=1}^{N} f(\boldsymbol{x}_k) = \int_{I^s} f(\boldsymbol{x}) d\boldsymbol{x}$$

- (Weyl Criterion) P={$\boldsymbol{x}_k$} uniformly distributed Mod 1 iff, for all $\boldsymbol{h} \in \boldsymbol{Z}^s, \boldsymbol{h} \neq \boldsymbol{0}$
- $\lim_{N\to\infty} \frac{1}{N} \sum_{k=1}^{N} e^{2\pi i(\boldsymbol{h}\cdot\boldsymbol{x}_k)} = 0$


Future Work to be done on the Pleiades SuperComputer
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