

Chapter 1 – Performing Operations and Evaluating Expressions
 Section 3 – Operations with Fractions and Proportions; Converting Units (1 day)

Before Lesson: Read Section 1.3 Teaching Tips

Materials Needed: Possibly use flash-based pizza fraction on www.mathsisfun.com/fractions

Objectives

1. Describe the meaning of a fraction.
2. Explain why division by zero is undefined.
3. Describe the rules for $a \cdot 1$, $a/1$, a/a .
4. Perform operations with fractions.
5. Find the prime factorization of a number.
6. Simplify fractions.
7. Find proportions.
8. Convert units of quantities.

Vocabulary:

1. equivalent expressions
2. prime factorization
3. simplify a fraction
4. reciprocal
5. least common multiple
6. least common denominator
7. proportion (statistics definition)

Opening/Purpose

Take attendance during this exercise.

Prime Factorization (Objective 5)

Make sure you and everyone at your table knows how to find the prime factorization of an integer. Find the prime factorization of 4, 6, 8, 10, 12, 14, 16, 18, 21, 24, 35, 36, 45, 49.

Lesson/Activity

OBJECTIVE 1 – Describe the meaning of a fraction.

1. Shade a portion of a circle to illustrate the meaning of $5/8$ of a pizza.
2. Use two pizzas with 4 slices each to show that $8/4 = 8 \div 4 = 2$.

The fraction a/b means $a \div b$.

OBJECTIVE 2 – Explain why division by zero is undefined.

Use the concept that division is repeated subtraction to show why division by 0 is undefined (see the top of page 24 of the textbook).

Division by Zero

The fraction a/b is undefined if $b = 0$. Division by 0 is undefined.

OBJECTIVE 3 – Describe the rules for $a \cdot 1$, $a/1$, a/a .

$a \cdot 1 = a$, $a/1 = a$, $a/a = 1$, where a is nonzero

When we write statements such as $a \cdot 1 = a$, we mean that if we evaluate $a \cdot 1$ and a for any value of a in both expressions, the results will be equal.

We say the expressions $a \cdot 1$ and a are **equivalent expressions**.

OBJECTIVE 4 – Perform operations with fractions.

Multiplying Fractions

If b and d are nonzero, then $a/b \cdot c/d = ac/bd$.

3. $3/5 \cdot 7/2$

4. $4/9 \cdot 5/7$

OBJECTIVE 5 – (Opening) Find the prime factorization of a number.

5. Prime factor 24.

6. Prime factor 36.

OBJECTIVE 6 – Simplify fractions.

Use a drawing of a pizza to show that $6/8 = 3/4$.

Simplifying a Fraction

To simplify a fraction.

1. Find the prime factorizations of the numerator and the denominator.

2. Find an equal fraction in which the numerator and the denominator do not have common positive factors other than 1 by using the property $ab/ac = a/a \cdot b/c = 1 \cdot b/c = b/c$ where a and c are nonzero.

7. Simplify $6/8$.

8. Simplify $24/18$.

9. Simplify $7/49$.

OBJECTIVE 4 – (revisited) Perform operations with fractions.

10. $4/25 \cdot 35/8$

11. $14/8 \cdot 10/21$

Dividing Fractions

If b , c , and d are nonzero, then $a/b \div c/d = a/b \cdot d/c$.

12. $9/2 \div 3/4$

13. $45/16 \div 35/8$

(When you use a calculator to check work with fractions, enclose each fraction in parentheses.)

14. Use a drawing of a pizza to show that $3/8 + 4/8 = 7/8$.

Adding Fractions with the Same Denominator

If b is nonzero, then $a/b + c/b = (a + c)/b$.

Subtracting Fractions with the Same Denominator

If b is nonzero, then $a/b - c/b = (a - c)/b$.

Adding (or Subtracting) Fractions with Different Denominators

To add (or subtract) two fractions with different denominators, use the fact that $a/a = 1$, where a is nonzero, to write an equal sum (or difference) of fractions for which each denominator is the LCD. Throughout the course, if a result is a fraction, it must be simplified.

15. $7/12 + 2/12$

16. $5/6 + 7/2$

17. $4/5 + 2/3$

18. $5/6 + 3/8$

Use a drawing of a pizza to show that $7/9 - 2/9 = 5/9$.

19. $5/6 - 1/6$

20. $7/6 - 2/3$

21. $5/8 - 3/10$

OBJECTIVE 7 – Find proportions.

Definition: Proportion

In statistics, a proportion is a fraction of the whole. A proportion can also be written as a decimal number. A proportion is always between 0 and 1, inclusive.

Sum of the Proportions Equals 1

If an object is made up of two or more parts, then the sum of their proportions equals 1.

When working with data values, proportions are usually written as decimal numbers.

We will use capital letters to emphasize the mathematical meanings of "AND," "OR," and "NOT."

22. About $\frac{3}{20}$ of 1.2 million people surveyed think that Five Guys has the best burger out of all fast food chains (Source: YouGov). What proportion of those surveyed do NOT think Five Guys has the best burger out of all fast food chains?

Proportion of the Rest

Let a/b be the proportion of the whole that has a certain characteristic.

Then the proportion of the whole that does NOT have that characteristic is $1 - a/b$.

23. Of all bribes made in the multinational industry, $\frac{1}{10}$ of the bribes are made in extraction (of oil, coal, and so on) and $\frac{3}{20}$ of the bribes are made in construction (Source: OECD). What proportion of the bribes are made in sectors other than extraction and construction?

24. A total of 1012 Americans were asked whether God plays a role in determining which team wins a sporting event. Their responses are summarized in the following table.

	God Plays a Role in Sporting Events					
	Completely Agree	Mostly Agree	Mostly Disagree	Completely Disagree	Don't Know/Refused	Total
# of People	101	162	203	516	30	1012

Source: Public Religion Research Institute

Find the proportion (rounded to the third decimal) of those surveyed who

- completely agreed.
- did NOT completely agree.
- mostly disagreed OR completely disagreed.

OBJECTIVE 8 – Convert units of quantities.

Converting Units

To convert the units of a quantity,

- Write the quantity in the original units.
- Multiply by fractions equal to 1 so that the units you want to eliminate appear in one numerator and one denominator.

Make the indicated unit conversions. Round approximate results to two decimal places.

25. The Sears Tower in Chicago is 1450 feet tall. What is its height in yards?

26. A jug contains 5 gallons of water. How many cups of water does it contain?

27. A car travels at a speed of 65 miles per hour. What is the car's speed in feet per second?

Closure/Review

Give Homework assignment.

Homework/Assessment

1, 3, 21, 25, 33, 41, 49, 67, 73, 81, 87, 93, 95, 99, 103, 107