

Chapter 1 – Performing Operations and Evaluating Expressions
Section 6 – Ratios, Percents, and Multiplying and Dividing Real Numbers

Objectives

1. Find the ratio of two quantities.
2. Describe the meaning of percent.
3. Convert percentages to and from decimal numbers.
4. Use proportions and percentages to describe authentic situations.
5. Find the percentage of a quantity.
6. Multiply and divide real numbers.
7. Describe which fractions with negative signs are equal to each other.

Vocabulary

1. unit ratio
2. percent

Opening/PurposeLesson/Activity

OBJECTIVE 1 – Find the ratio of two quantities.

Suppose that there are 8 women and 4 men on a softball team. The ratio of women to men is

$$\frac{8 \text{ women}}{4 \text{ men}} = \frac{2 \text{ women}}{1 \text{ man}}$$

This means that there are 2 women to 1 man. This ratio is called a **unit ratio**.

A **unit ratio** is a ratio written as

$$\frac{a}{b} \text{ or } a:b \text{ with } b = 1.$$

1. The blink of an eye takes 400 milliseconds. One beat of a hummingbird's wings takes 20 milliseconds. Find the unit ratio of the time it takes for the blink of an eye to the time it takes for one beat of a hummingbird's wings. What does your result mean in this situation?
2. The median sales prices of existing homes and the median household incomes in 2011 are shown in the following table for four regions of the United States.

Region	Median Sales Price of Existing Homes (dollars)	Median Household Income (dollars)
Northeast	237,500	51,862
Midwest	135,400	49,549
South	135,200	42,590
West	201,300	53,367

Sources: U.S. Census; Federal Reserve Bank of St. Louis

- a. Find the unit ratio of the median sales price of existing homes to the median household income in the Northeast. What does the result mean?
- b. For each of the four regions, find the unit ratio of the median sales price of existing homes to the median household income. Taking into account the median household income of each region, list the regions in order of affordability of existing homes, from greatest to least.
- c. A person believes that existing homes in the South are more affordable than in the Midwest, because the median price of existing homes is lower in the South than in the Midwest. What would you tell that person?

OBJECTIVE 2 – Describe the meaning of percent.

- If 57 of 100 songs are hip hop, then 57% of the songs are hip hop.
- Percent means “for each hundred”: $a\% = \frac{a}{100}$
- A percent is a ratio.

OBJECTIVE 3 – Convert percentages to and from decimal numbers.

Converting Percentages to and from Decimal Numbers

- To write a percentage as a decimal number, remove the percent symbol and divide the number by 100 (move the decimal point two places to the left).
- To write a decimal number as a percentage, multiply the number by 100 (move the decimal point two places to the right) and insert a percent symbol.

3. Write 8% as a decimal number.

4. Write 0.145 as a percent.

OBJECTIVE 4 – Use proportions and percentages to describe authentic situations.

5. About 63% of Americans feel the distribution of money and wealth in the United States should be more evenly distributed (Source: CBS News Poll). Use a proportion to describe this situation.
6. The proportion of Yahoo employees who are Asian is 0.39 (Source: Yahoo). Use a percentage to describe this situation.
7. Of 905 surveyed drivers, 163 drivers said it takes them at least three tries to parallel park (Source: Hankook Tire Fall Gauge Index survey). Use a percentage to describe the situation.

OBJECTIVE 5 – Find the percentage of a quantity.

Finding the Percentage of a Quantity

To find the percentage of a quantity,
multiply the decimal form of the percentage and the quantity.

8. Of 4672 surveyed students in grades 3–5, 85% knew how to use a keyboard to type answers to questions (Source: Idaho’s Smarter Balanced Field Test survey). How many of the students knew how to use a keyboard?
9. A person buys some take-out food for \$8 (not including state meals tax) in Boston, Massachusetts, which has a state meals tax of 6.25%. How much money is the state meals tax?

One Hundred Percent of a Quantity

One hundred percent of a quantity is all of the quantity.

OBJECTIVE 6 – Multiply and divide real numbers.

Use repeated addition to show that $4(-3) = -12$.

Multiplying Two Numbers with Different Signs

The product of two numbers that have different signs is negative.

10. a. Find the products $3(-4)$, $2(-4)$, $1(-4)$, $0(-4)$.
b. Explain why the results of Part (a) suggest that $-1(-4) = 4$, $-2(-4) = 8$, and $-3(-4) = 12$.
c. Form a theory about the sign of the product of two numbers that have the same sign.

Multiplying Two Numbers with the Same Sign

The product of two numbers that have the same sign is positive.

11. $3(-8)$ 12. $-5(-9)$ 13. $-0.2(-0.1)$ 14. $-\frac{4}{9} \cdot \frac{3}{2}$

The rules for dividing numbers are similar to those for multiplying numbers, because to divide by a number, we multiply by the reciprocal of the number.

Multiplying or Dividing Real Numbers

The product or quotient of two numbers that have different signs is negative.

The product or quotient of two numbers that have the same sign is positive.

15. $-12 \div (-2)$ 16. $-35 \div 7$ 17. $475 \div -25$ 18. $-\frac{5}{49} \div \frac{25}{21}$
19. A person has credit card balances of -3580 dollars on a Discover account and -1590 dollars on a MasterCard account.
- a. Find the unit ratio of the Discover balance to the MasterCard balance.
- b. If the person wishes to gradually pay off both accounts in the same amount of time, describe how the result in Part (a) can help guide the person in making his next payment.

20. $-4 - (-7)$ 21. $8 \div (-2)$ 22. $-15 + 11$
23. $(-7)(-10)$ 24. $-5.8 + 2.9$ 25. $-0.2(-0.3)$
26. $-\frac{27}{20} \div \frac{9}{8}$ 27. $-\frac{7}{4} - \frac{3}{2}$

OBJECTIVE 7 – Describe which fractions with negative signs are equal to each other.

Compare the results of $\frac{-6}{3}$, $\frac{6}{-3}$, $-\frac{6}{3}$ to motivate the following property.

Equal Fractions with Negative Signs

If $b \neq 0$, then $\frac{-a}{b}$, $\frac{a}{-b}$, $-\frac{a}{b}$.

28. $\frac{-32}{20}$ 29. $\frac{7}{9} + \frac{-4}{9}$ 30. $\frac{7}{6} - \frac{5}{-4}$

Homework/Assessment

1, 3, 9, 11, 15, 19, 27, 29, 35, 45, 55, 65, 69, 77, 83, 87, 89, 97