Chapter 1 – Performing Operations and Evaluating Expressions
Section 7 – Exponents, Square Roots, Order of Operations, and Scientific Notation

Objectives
1. Describe the meaning of exponent.
2. Describe the meaning of the exponent zero.
3. Describe the meaning of a negative-integer exponent.
4. Describe the meaning of square root and principal square root.
5. Approximate a principal square root.
6. Use the rules for order of operations to perform computations and evaluate expressions.
7. Use the rules for order of operations to make predictions.
8. Find the area of part of an object.
9. Use scientific notation.

Vocabulary
1. base
2. exponent/power
3. exponentiation
4. nth
5. square/perfect square
6. square root/principal square root
7. radical/radical sign
8. radicand
9. scientific notation

Opening/Purpose

Lesson/Activity

OBJECTIVE 1 − Describe the meaning of exponent.

$2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16$ $4^3 = 4 \cdot 4 \cdot 4 = 64$

Definition: Exponent

For any counting number n, $b^n = b \cdot b \cdot b \cdot \ldots \cdot b$ (n factors of b)

We refer to b^n as the **power**, the **nth power of b**, or **b raised to the nth power**.

We call b the **base** and n the **exponent**.

1. 3^4 2. $(-2)^5$ 3. $(-5)^2$ 4. -5^2

For an expression of the of the form $-b^n$, we calculate b^n before taking the opposite.

OBJECTIVE 2 − Describe the meaning of the exponent zero.

$2^4 =$
$2^3 =$
$2^2 =$
$2^1 =$
$2^0 =$

Definition: Zero exponent

For nonzero b, $b^0 = 1$

5. 269.72^0

6. $(-9)^0$

OBJECTIVE 3 − Describe the meaning of a negative-integer exponent.

2^2 =
2^1 =
2^0 =
2^{-1} =
2^{-2} =
2^{-3} =

Definition: Negative-integer exponent

If n is a counting number and b is nonzero, then $b^{-n} = \frac{1}{b^n}$.

7. 4^{-2}

8. 3^{-4}

OBJECTIVE 4 − Describe the meaning of square root and principal square root.

Definition: Principal square root

If a is a nonnegative number, then $\sqrt{a}$ is the nonnegative number we square to get *a*.
We call $\sqrt{a}$ the **principal square root** of *a*.

The symbol $\sqrt{\ }$ is called a **radical sign**.
An expression under a radical sign is called a **radicand**.
A radical sign together with a radicand is called a **radical**.
An expression that contains a radical is called a **radical expression**.
A square root of a negative number is not a real number.

Find the square root.

9. $\sqrt{25}$

10. $\sqrt{-25}$

11. $-\sqrt{25}$

12. $-\sqrt{-25}$

OBJECTIVE 5 − Approximate a principal square root.

State whether the square root is rational or irrational. If the square root is rational, find the (exact) value. If the square root is irrational, estimate its value by rounding to the second decimal place.

13. $\sqrt{144}$

14. $\sqrt{24}$

OBJECTIVE 6 − Use the rules for order of operations to perform computations and evaluate expressions.

We do operations that lie within grouping symbols before we perform other operations.
The order of operations does matter:

$(4 + 2) \cdot 5$	$4 + (2 \cdot 5)$
$6 \cdot 5$	$4 + 10$
30	14

For a fraction, such as $\frac{2+5}{9-4}$, the following use of parentheses is assumed:

$$\frac{2+5}{9-4} = \frac{(2+5)}{(9-4)} = \frac{7}{5}$$

For a radical expression, such as $\sqrt{16 + 9}$, the following use of parentheses are assumed:

$$\sqrt{16 + 9} = \sqrt{(16 + 9)} = \sqrt{25} = 5$$

Warning: $\sqrt{16 + 9}$ does *not* equal $\sqrt{16} + \sqrt{9}$.

Order of Operations

We perform operations in the following order:

Perform operations within parentheses or other grouping symbols, starting with the innermost group.

1. Then perform exponentiations.
2. Next, perform divisions and multiplications, going from left to right.
3. Last, perform subtractions and additions, going from left to right.

15. (4 − 8)(9 − 2)

16. $\frac{2-8}{3-7}$

17. $3 - 8 \div 4$

18. $4 + (-2)^3$

19. $2(5 - 8) - (4 - 9)$

20. $3 - [6 - 2(3 + 4)]$

21. $3^3 - 2(3 - 5)^2 \div (-4)$

22. $\frac{9}{10} - \frac{3}{5} \div \frac{2}{7}$

23. $\frac{9+4+7+1+9}{5}$

24. $\frac{4-(-2)^3}{2+4^2}$

Operation	Computation with 10s
Exponentiation	10^{10} = 10,000,000,000
Multiplication	10 · 10 = 100
Addition 10 + 10 = 20	

Order of Operations and the Strengths of Operations

After we have performed operations in parentheses, the order of operations goes from the most powerful operation, exponentiation, to the next-most-powerful operations, multiplication and division, to the weakest operations, addition and subtraction.

Use a calculator to perform the indicated operations.

25. $\frac{(1-3)^2 + (2-3)^2 + (6-3)^2}{3-1}$

26. $(44 - 35) - 4\sqrt{\frac{4^2}{2} + \frac{7^2}{49}}$

27. $\sqrt{\frac{(2.3-5.4)^2+(5.6-5.4)^2+(8.3-5.4)^2}{3-1}}$

28. Evaluate $\frac{a-b}{c-d}$ for a = −3, b = 5, c = 4, and d = −6.

29. Evaluate $\bar{x} - t\frac{s}{\sqrt{n}}$ for $\bar{x}$= 12, t = 3, s = 10, and n = 25.

30. Let **x** be a number. Translate the sentence "Subtract 5 from the product of the number and −3." Then evaluate the expression for x = −2.

OBJECTIVE 7 − Use the rules for order of operations to make predictions.

31. The number of knee replacement surgeries among Medicare participants was about 244 thousand in 2010 and has increased by approximately 8 thousand surgeries per year until 2014 (Source: Journal of the American Medical Association). Complete the following table to help find an expression that stands for the number of such surgeries at t years since 2010. Show the arithmetic to help see a pattern.

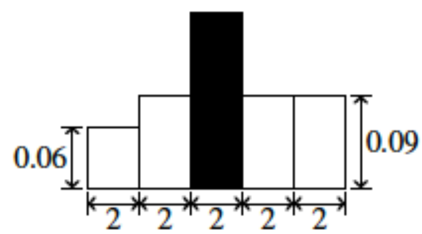
		Number of Knee Replacement Surgeries
Years Since 2010	Show Arithmetic	Among Medicare Participants (in thousands)
0		
1		
2		
3		
t		

Evaluate your result for t = 4 and discuss what the result means in this situation.

32. The number of age-related retirements of U.S. pilots was 226 retirements in 2010, and it increased by 20.8% in 2011 (Source: KitDarby.com Aviation Consulting). What was the number of age-related retirements in 2011?

OBJECTIVE 8 − Find the area of part of an object.

33. Use the fact that the total area of the following object is 1 to find the area of the shaded bar. The object has not been drawn to scale.



OBJECTIVE 9 − Use scientific notation.

Definition: Scientific notation

A number is written in scientific notation if it has the form N X 10^k, where k is an integer and the absolute value of N is between 1 and 10 or is equal to 1.

Examples: 6.74 X 10^7, 9.2 X 10^{-6}

Write the number in standard decimal notation.

34. 5.36 X 10^3 35. 8.192 X 10^6 36. 4.6 X 10^{-3} 37. 2.99 X 10^{-5}

Converting from Scientific Notation to Standard Decimal Notation

To write the scientific notation $N \times 10^k$ in standard decimal notation,
we move the decimal point of the number N as follows:

- If k is positive, we multiply N by 10 k times and hence move the decimal point k places to the right.
- If k is negative, we divide N by 10 k times and hence move the decimal point k places to the left.

Write the number in scientific notation.

38. The first evidence of life on Earth dates back to 3,600,000,000 years ago.

39. The wavelength of violet light is about 0.00000047 meter.

Converting from Standard Decimal Notation to Scientific Notation

To write a number in scientific notation, count the number k of places that the decimal point must be moved so that the absolute value of the new number N is between 1 and 10 or is equal to 1.

If the decimal point is moved to the left, then the scientific notation is written as $N \times 10^k$.

If the decimal point is moved to the right, then the scientific notation is written as $N \times 10^{-k}$.

- Some calculators represent 7.29×10^{26} as 7.29 E 26.
- Some calculators represent 5.96×10^{-23} as 5.96 E −23.

Homework/Assessment

1, 3, 5, 19, 25, 39, 45, 53, 67, 71, 73, 79, 81, 85, 87, 91, 97, 103, 105, 109, 117, 123, 129, 131