

Chapter 4 – Summarizing Data Numerically  
Section 2 – Measures of Spread

Objectives

1. Compute the range of some data.
2. Compute the standard deviation of some data.
3. Compare the means and standard deviations of two groups of data.
4. Apply the Empirical Rule to some data.
5. Determine whether an observation is unusual.
6. Compare the range and the standard deviation.
7. Compute the variance of some data.

Vocabulary

1. range
2. standard deviation
3. empirical rule
4. variance

Lesson/Activity

OBJECTIVE 1 – Compute the range of some data.

**Definition: Range**

The **range** of some data values is given by

$$R = \text{largest number} - \text{smallest number}$$

1. The prices of 8 randomly selected calculators are shown in the following table.

132.99	110.63	132.00	60.00
49.99	91.50	137.81	138.82

Source: Amazon.com

Find the range in prices and explain what it means in this situation.

OBJECTIVE 2 – Compute the standard deviation of some data.

**Definition: Standard deviation**

The **standard deviation** of  $n$  data values  $x_1, x_2, x_3, \dots, x_n$  is given by

$$s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n - 1}}$$

**The standard deviation measures the spread. The greater the spread, the greater the standard deviation will be.**

2. The highway gas mileages of different versions of the Audi R8 Spyder are 23, 20, 22, and 19, all in miles per gallon. Find the standard deviation of the gas mileages.

OBJECTIVE 3 – Compare the means and standard deviations of two groups of data.

3. A household suffers from food insecurity if at some point in the year the household eats less, goes hungry, or eats less nutritional meals because there is not enough money for food. The percentages of households suffering from food insecurities are shown in the following table for the West North Central and Pacific states.

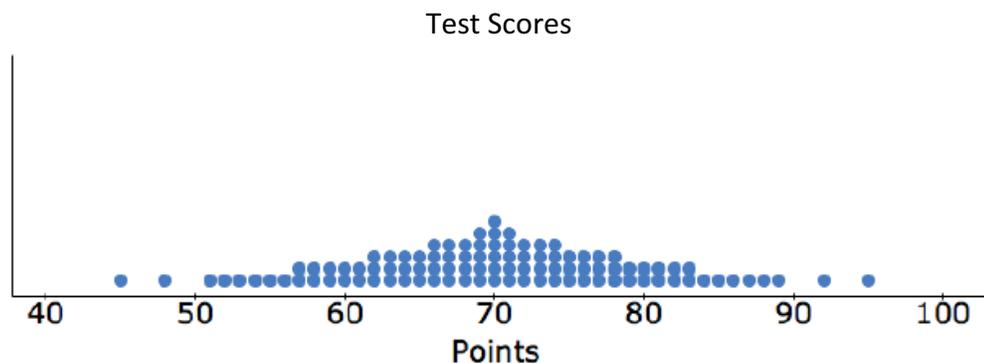
West North Central State	Food Insecurity (% of households)	Pacific State	Food Insecurity (% of households)
Iowa	12.7	Alaska	14.0
Kansas	14.8	California	16.2
Minnesota	10.7	Hawaii	14.2
Missouri	17.1	Oregon	16.7
Nebraska	13.4	Washington	15.0
North Dakota	7.7		
South Dakota	12.3		

Source: Feeding America; U.S. Department of Agriculture

- Compare the mean food insecurities for the West North Central states and for the Pacific states. What does your comparison mean in this situation?
- Compare the standard deviations of the food insecurities for the West North Central states and for the Pacific states. What does your comparison mean in this situation?
- Even though the mean food insecurity for the West North Central states is less than the mean food insecurity for the Pacific states, one of the West North Central states (Missouri) has a larger percentage of households with food insecurity than every one of the Pacific states. Explain why this is not surprising by referring to the standard deviation of food insecurities for the West North Central states and the Pacific states.

OBJECTIVE 4 – Apply the Empirical Rule to some data.

4. Suppose that 100 statistics students' scores (in points) on a test are described by the following dotplot. The mean is 70 points and the standard deviation is approximately 10 points.



- Describe the distribution.
- Find the percentage of the scores that are within one standard deviation of the mean.
- Find the percentage of the scores that are within two standard deviations of the mean.
- Find the percentage of the scores that are within three standard deviations of the mean.

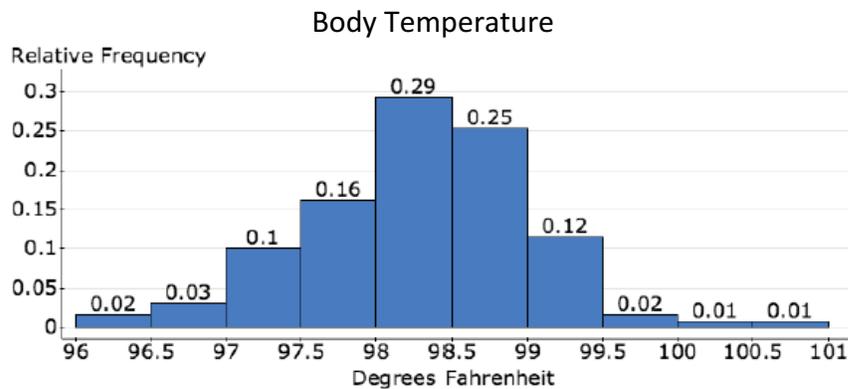
### Empirical Rule

If a distribution is unimodal and symmetric, then the following statements are true.

- Approximately 68% of the observations lie within one standard deviation of the mean.  
So, approximately 68% of the observations lie between  $\bar{x} - s$  and  $\bar{x} + s$ .
- Approximately 95% of the observations lie within two standard deviations of the mean.  
So, approximately 95% of the observations lie between  $\bar{x} - 2s$  and  $\bar{x} + 2s$ .
- Approximately 99.7% of the observations lie within three standard deviations of the mean.  
So, approximately 99.7% of the observations lie between  $\bar{x} - 3s$  and  $\bar{x} + 3s$ .

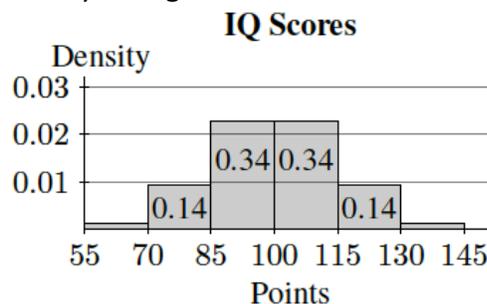
**Before applying the Empirical Rule to a particular distribution, we must make sure the distribution is unimodal and symmetric. Other distributions can have quite different percentages.**

5. The body temperatures of 130 adults are described by the following relative frequency histogram. The mean is 98.25°F and the standard deviation is 0.73°F (Source: American Statistical Association).



Source: American Statistical Association

- Explain why the Empirical Rule can be applied.
  - Apply the Empirical Rule.
  - Use the Empirical Rule to estimate the number of the adults who have body temperatures between 97.52°F and 98.98°F. Verify your result by referring to the histogram.
  - Use the Empirical Rule to estimate the percentage of the adults who have body temperatures less than 96.79°F OR greater than 99.71°F?
  - Use the Empirical Rule to estimate the number of the adults who have body temperatures greater than 99.71°F.
6. The Wechsler IQ test measures a person's intelligence. The IQs (in points) of some people are described by the following density histogram. Some of the bars have been left blank on purpose.



- Estimate  $M$ .
- Estimate  $\bar{x}$ .
- Estimate  $s$ .

OBJECTIVE 5 – Determine whether an observation is unusual.

**If an observation is more than two standard deviations away from the mean, we refer to the observation as unusual.**

7. In Problem 5, the 130 adults have a mean body temperature of 98.25°F with standard deviation 0.73°F. Is a body temperature of 96.3°F unusual?

OBJECTIVE 6 – Compare the range and the standard deviation.

### Summary of Measuring Center and Spread

- If a distribution is unimodal and symmetric, then we usually use the mean to measure the center and the standard deviation to measure the spread.
- If a distribution is skewed, then we usually use the median to measure the center and the range to measure the spread.

OBJECTIVE 7 – Compute the variance of some data.

### Definition: Variance

The **variance** is the square of the standard deviation.

8. In Problem 2, we found that the standard deviation of the gas mileages of different versions of the Audi R8 Spyder is 1.8257 miles per gallon. Find the variance.

The meanings and symbols of the measures we have discussed in Sections 4.1 and 4.2 are shown in the following table.

Measure	What It Measures	Symbol
mean	center	$\bar{x}$
median	center	M
mode	center	(no standard symbol)
range	spread	R
standard deviation	spread	s
variance	spread	$s^2$

### Homework/Assessment

1, 3, 7, 9, 13, 19, 21, 23, 27, 29, 31, 33, 35, 37, 43, 45, 47, 55