Chapter 6 – Describing Associations of Two Variables Graphically
Section 3 – Modeling Linear Associations

Objectives
1. Use a line to model an association between two numerical variables.
2. Use a linear model to make estimates and predictions.
3. Describe the meaning of input and output.
4. Find errors in estimations.
5. Describe the meaning of interpolate, extrapolate, and model breakdown.
6. Find intercepts of a line.
7. Find intercepts of a linear model.
8. Modify a model.

Vocabulary
1. linear model
2. input/output
3. error
4. interpolation/extrapolation
5. model breakdown
6. x/y intercept

Lesson/Activity

OBJECTIVE 1 − Use a line to model an association between two numerical variables.

1. Let **n** be the number of bank robberies per 100 branches at **t** years since 1995.

Year	Robberies per 100 Branches	Year	Robberies per 100 Branches
1997	10.6	2007	6.0
1999	8.8	2009	6.0
2001	10.3	2011	5.3
2003	8.7	2012	4.5
2005	7.7		

Source: FBI

   a. Construct a scatterplot.
   b. Is there a linear association, a nonlinear association, or no association?
   c. Compute r. On the basis of r and the scatterplot, determine the strength of the association.
   d. Draw a line that comes close to the points of the scatterplot.

Definition: Model

A **model** is a mathematical description of an authentic situation.
We say that the description models the situation.

Definition: Linear model

A **linear model** is a nonvertical line that describes the association between two quantities in an authentic situation.

OBJECTIVE 2 − Use a linear model to make estimates and predictions.

2. Use the robbery model to estimate the number of bank robberies per 100 branches in 2004.
3. Use the model to estimate when there were 5.6 bank robberies per 100 branches.

- We construct a scatterplot of data to determine whether there is a linear association. If there is a linear association, we draw a line that comes close to the data points and use the line to make estimates and predictions.

- It is a common error to try to find a line that contains the greatest number of points. However, our goal is to find a line that comes close to all the data points.

OBJECTIVE 3 − Describe the meaning of input and output.

Definition: Input, output

An **input** is a permitted value of the explanatory variable that leads to at least one **output**, which is a permitted value of the response variable.

4. In Problem 2, identify the input and the output.

OBJECTIVE 4 − Find errors in estimations.

The **error** in an estimate is the amount by which the estimate differs from the actual value.

- For an overestimate, the error is positive.

- For an underestimate, the error is negative.

5. Use the robbery model to estimate the number of bank robberies per 100 branches in 2001. Find the error in the estimate.

6. Use the robbery model to estimate the number of bank robberies per 100 branches in 2007. Find the error in the estimate.

OBJECTIVE 5 − Describe the meaning of interpolate, extrapolate, and model breakdown.

Refer to the estimates and predictions that you made in Problems 2, 3, 5, and 6 as interpolations or extrapolations.

Definition: Interpolation, extrapolation

For a situation that can be described by a model whose explanatory variable is **x**,

- We perform **interpolation** when we use a part of the model whose x-coordinates are between the x-coordinates of two data points.

- We perform **extrapolation** when we use a part of the model whose x-coordinates are not between the x-coordinates of any two data points.

7. Use the robbery model to estimate the number of bank robberies per 100 branches in 2002. Did you perform interpolation or extrapolation? Explain.

8. Use the robbery model to estimate the number of bank robberies per 100 branches in 2014. Did you perform interpolation or extrapolation? Explain.

9. Use the robbery model to estimate the number of bank robberies per 100 branches in 2030. Did you perform interpolation or extrapolation? Explain.

Definition: Model breakdown

When a model gives a prediction that does not make sense or an estimate that is not a good approximation, we say that **model breakdown** has occurred.

OBJECTIVE 6 – Find intercepts of a line.

Definition: Intercepts of a line

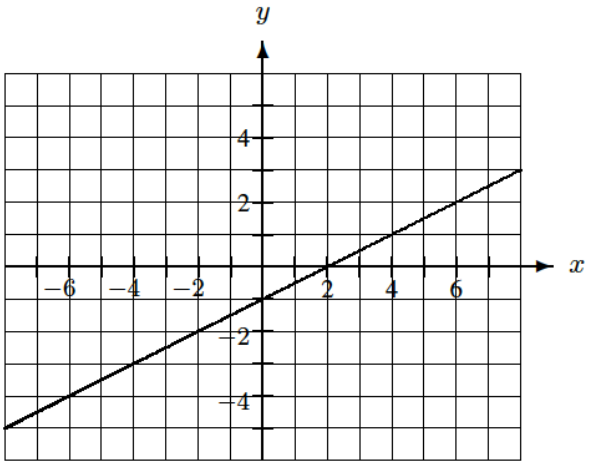
An **intercept** of a line is any point where the line and an axis (or axes) of a coordinate system intersect.

There are two types of intercepts of a line sketched on a coordinate system with an x–axis and a y–axis:

- An x–intercept of a line is a point where the line and the x–axis intersect. [Draw a figure.]

- A y–intercept of a line is a point where the line and the y–axis intersect. [Draw a figure.]

10. Refer to the following figure.



a. Find the x-intercept of the line.
b. Find the y-intercept of the line.
c. Find y when x = -6.
d. Find x when y = -3.

OBJECTIVE 7 – Find intercepts of a linear model.

11. Find the given intercept of the robbery model. What does it mean in this situation? Did you perform interpolation or extrapolation? Explain.
    a. n–intercept
    b. t–intercept

12. The percentages of cell phone users who send or receive text messages multiple times per day are shown in the following table for various age groups.

Age Group (years)	Age Used to Represent Age Group (years)	Percent
18–24	21.0	76
25–34	29.5	63
35–44	39.5	42
45–54	49.5	37
55–64	59.5	17

Source: Edison Research and Arbitron

Let **p** be the percentage of cell phone users at age **a** years who send or receive text messages multiple times per day.

a. Draw a linear model that describes the association between and **a** and **p**.
b. Predict the percentage of 25-year-old cell phone users who send or receive text messages multiple times per day. Did you perform interpolation or extrapolation? Explain.
c. Find the **p**-intercept. What does it mean in this situation? Did you perform interpolation or extrapolation? Explain.
d. Find the a-intercept. What does it mean in this situation? Did you perform interpolation or extrapolation? Explain.

OBJECTIVE 8 − Modify a model.

13. Additional research about cell phone users who send or receive text messages multiple times per day yields the data shown in the first and last rows of the following table. Use this data and the following assumptions to modify the model you found in Problem 12:

- Children 3 years old and younger do not send or receive text messages multiple times per day.
- The percentage of cell phone users who send or receive text messages multiple times per day levels off at 5% for users over 80 years in age.
- The age of the oldest cell phone users is 116 years.

Age Group (years)	Age Used to Represent Age Group (years)	Percent
12–17	14.5	75
18–24	21.0	76
25–34	29.5	63
35–44	39.5	42
45–54	49.5	37
55–64	59.5	17
over 64	70.0	7

Source: Edison Research and Arbitron

Homework/Assessment

1, 3, 5, 7, 17, 21, 25, 29, 31, 35, 37, 39, 43, 47, 51