Chapter 7 – Graphing Equations of Lines and Linear Models; Rates of Change
Section 1 – Graphing Equations of Lines and Linear Models

Objectives
1. For an equation in two variables, determine the meaning of solution, satisfy, and solution set.
2. Describe the meaning of the graph of an equation.
3. Graph equations of the form y = *m*x + *b*.
4. Describe the meaning of b in equations of the form y = *m*x + *b*.
5. Graph equations of the form y = *b* and x = *a*.
6. Describe the Rule of Four for equations.
7. Graph an equation of a linear model.

Vocabulary
1. solution/solution set
2. satisfies
3. graph
4. linear equation in two variables
5. rule of four

Lesson/Activity

OBJECTIVE 1 − For an equation in two variables, determine the meaning of solution, satisfy, and solution set.

1. Show that the equation y = x + 3 becomes a true statement when 2 is substituted for x and 5 is substituted for y.
   We say (2, 5) ***satisfies*** y = x + 3 and that (2, 5) is a ***solution*** of y = x + 3.

Definition: **Solution**, **satisfy**, and **solution set** of an equation in two variables
An ordered pair (*a*, *b*) is a **solution** of an equation in terms of *x* and *y* if the equation becomes a true statement when *a* is substituted for *x* and *b* is substituted for *y*. We say (*a*, *b*) **satisfies** the equation. The **solution set** of an equation is the set of all solutions of the equation.

2. Is (2, 7) a solution of y = 4x − 1?

3. Is (3, 5) a solution of y = 4x − 1?

OBJECTIVE 2 − Describe the meaning of the graph of an equation.

4. a. Find five solutions of y = 2x − 3 and plot them in the same coordinate system.
   b. Do the five points lie on a line? If so, sketch the line.
   c. Select another point on the line and show that the corresponding ordered pair satisfies y = 2x − 3.
   d. Select a point that doesn't lie on the line and show that the corresponding ordered pair does not satisfy y = 2x − 3.

We call the line that we sketched in Problem 4 the ***graph*** of y = 2x − 3.

Definition: Graph
The **graph** of an equation in two variables is the set of points that correspond to all solutions of the equation.

- Every point on the graph of an equation represents a solution of the equation.
- Every point not on the graph represents an ordered pair that is not a solution.

OBJECTIVE 3 − Graph equations of the form y = *m*x + *b*.

The equation y = 2x − 3 is of the form y = *m*x + *b*.

Graph of y = mx + b

The graph of an equation of the form y = *m*x + *b*, where *m* and *b* are constants, is a line.

Here are some equations whose graphs are lines:

y = 3x + 7 y = -6x + 2 y = 2x y = x − 4 y = 1

Graph the equation by hand. Also, find the y-intercept.

5. y = 3x – 5 6. y = -2x + 4 7. y = -3x 8. y = 2/3x + 1

OBJECTIVE 4 − Describe the meaning of *b* in equations of the form y = *m*x + *b*.

Substitute 0 for x into the equation y = *m*x + *b*: y = *m*(0) + *b* → y = 0 + *b* → y = *b*
The y-intercept of the graph of y = *m*x + *b* is (0, *b*).

y -intercept of the graph of y = mx + b

The graph of an equation of the form y = *m*x + *b* has y-intercept (0, *b*).

For y = 3x – 5, the y-intercept is (0, -5) and for y = -2x + 4, the y-intercept is (0, 4).
See Problems 5 and 6.

OBJECTIVE 5 − Graph equations of the form y = *b* and x = *a*.

Find five solutions of the given equation. Then graph the equation by hand.

9. x = 2 10. y = 3

Equations for Horizontal and Vertical Lines

If *a* and *b* are constants, then

- The graph of y = *b* is a horizontal line.
- The graph of x = *a* is a vertical line.

Graph the equation by hand.

11. x = 4 12. y = -2 13. x = 0 14. y = 0

Equations Whose Graphs Are Lines

If an equation can be put into the form y = *m*x + *b* or x = *a*,
where *m*, *a*, and *b* are constants, then the graph of the equation is a line.
We call such an equation a **linear equation in two variables**.

Here are some examples of linear equations in two variables:

y = -5x + 9 y = 3x – 8 y = 4 x = -2

OBJECTIVE 6 – Describe the Rule of Four for equations.

Rule of Four for Solutions of an Equation

We can describe some or all the solutions of an equation in two variables with:

1. an equation, 2. a table, 3. a graph, or 4. words.

These four ways to describe solutions are known as the **Rule of Four**.

15. a. List some solutions of $y = 4x - 3$ by using a table.
    b. Describe the solutions of $y = 4x - 3$ by using a graph.
    c. Describe the solutions of $y = 4x - 3$ by using words.
16. Consider the equation $y = x + 2$.
    a. Substitute 3 for x and show that the input $x = 3$ leads to the output $x = 5$.
    b. Graph $y = x + 2$. Then use arrows to show that the input $x = 3$ leads to the output $x = 5$.

OBJECTIVE 7 – Graph an equation of a linear model.

17. In the United Kingdom, nervous dental patients are sometimes given a sedative to calm them. To administer the sedative, the dentist performs a cannulation, which involves inserting a thin tube into a vein in a patient's hand. In an experiment involving 20 individuals, researchers tested whether the amount of pain the individuals experienced during cannulation explains the level of anxiety that the individuals predicted they would experience by undergoing cannulation again. After undergoing cannulations, the individuals rated their experiences of pain from 0 to 100 and their predicted anxieties about undergoing cannulations again from 0 to 100. The patients' ratings are shown in the following table.

Pain Rating	Anxiety Rating	Pain Rating	Anxiety Rating	Pain Rating	Anxiety Rating	Pain Rating	Anxiety Rating
37	50	23	13	29	35	69	68
63	70	38	35	34	30	14	3
8	2	60	20	41	48	12	10
10	10	38	30	75	70	49	43
70	81	42	42	8	32	48	43

Source: Pain-anxiety scatterplot (Source: Anesthetics: A Randomized, Double-Blind, Placebo-Controlled, Comparative Study of Topical Skin Analgesics and the Anxiety and Discomfort Associated with Venous Cannulation, A. F. Speirs et al.)

Let *a* be the anxiety rating a patient predicts for undergoing another cannulation after having undergone one with a pain rating of *p*.

a. Identify the explanatory and response variables.
b. Construct a scatterplot.
c. Describe and interpret the direction of the association.
d. Graph the model $a = 0.91p + 1.71$ on the scatterplot.
e. Does the model come close to the data points?
f. Use the model to predict the anxiety rating a patient would predict about undergoing another cannulation after having undergone one with a pain rating of 30.

Homework/Assessment

1, 3, 7, 9, 19, 25, 27, 35, 39, 43, 47, 49, 51, 77, 81