Chapter 7 – Graphing Equations of Lines and Linear Models; Rates of Change
Section 2 – Rate of Change and Slope of a Line

Objectives
1. Calculate the rate of change of a quantity with respect to another quantity.
2. Explain the connection between constant rate of change and an exact linear association.
3. Use a run and the corresponding rise of a linear model to find a rate of change.
4. Describe the connection between the direction of an association and rate of change.
5. Calculate the slope of a line.
6. Explain why the slope of an increasing line is positive and the slope of a decreasing line is negative.
7. Explain why the absolute value of the slope of a line measures the steepness of the line.
8. Explain why the slope of a horizontal line is zero and the slope of a vertical line is undefined.

Vocabulary
1. rate of change
2. run/rise
3. slope
4. increasing/decreasing
5. steepness of a line
6. horizontal/vertical line

Lesson/Activity
OBJECTIVE 1 – Calculate the rate of change of a quantity with respect to another quantity.

1. If the temperature increased by 12°F over 3 hours, estimate how much the temperature increased per hour.

2. If the temperature increased from 60°F at 7 A.M. to 68°F at 11 A.M., estimate how much the temperature increased per hour.

Formula for Rate of Change

Suppose that a quantity y changes steadily from y_1 to y_2 as a quantity x changes steadily from x_1 to x_2. Then the **rate of change** of y with respect to x is the ratio of the change in y to the change in x:

$$\frac{change\ in\ y}{change\ in\ x} = \frac{y_2 - y_1}{x_2 - x_1}$$

3. The sales of cigarettes in the United States deceased approximately linearly from 400 billion cigarettes in 2003 to 280 billion cigarettes in 2013 (Source: Euromonitor International). Find the approximate rate of change of cigarettes sales.

4. A person of height 132.5 centimeters should use a ski pole of length 90 centimeters. A person of height 153.0 centimeters should use a ski pole of length 110 centimeters.
   Find the approximate rate of change of ski pole length with respect to a person's height.

OBJECTIVE 2 – Explain the connection between constant rate of change and an exact linear association.

5. Suppose that a student travels at 60 miles per hour on a road trip. Let **d** be the distance (in miles) that the student can drive in **t** hours.
   a. Identify the explanatory and response variables.
   b. Construct a table to describe the association between **t** and **d** for driving times of 0, 1, 2, 3, 4, and 5 hours.
   c. Construct a scatterplot for driving times of 0, 1, 2, 3, 4, and 5 hours.
   d. Find the slope of the linear model.
   e. Describe the four characteristics of the association. Compute and interpret **r** as part of your analysis. If there is an association, draw an appropriate model on the scatterplot.

Constant Rate of Change Implies an Exact Linear Association

If the rate of change of one variable with respect to another variable is constant, then there is an exact linear association between the variables.

6. The amounts of money a student is paid are shown in the following table for various numbers of hours worked. Let p be the student's pay (in dollars) for working t hours.

Time (hours)	Pay (dollars)
0	0
1	15
2	30
3	45
4	60
5	75

   a. Identify the explanatory and response variables.
   b. Describe the four characteristics of the association. Compute and interpret **r** as part of your analysis.
   c. Find the rate of change of pay from 2 to 3 hours.
   d. Find the rate of change of pay from 0 to 5 hours. Compare the result with the result you found in Part (c).

An Exact Linear Association Implies Constant Rate of Change

If there is an exact linear association between two variables, then the rate of change of one variable with respect to the other is constant.

OBJECTIVE 3 – Use a run and the corresponding rise of a linear model to find a rate of change.

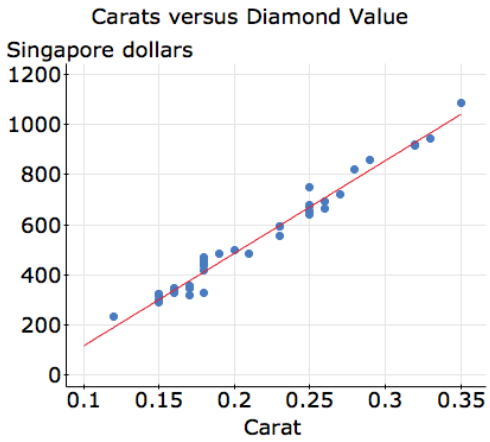
For any linear model, the **run** is the horizontal change and the **rise** is the vertical change in going from one point on the line to another point on the line.

Using Run and Rise to Find Rate of Change

Assume there is an exact linear association between an explanatory variable x and a response variable y and (x_1, y_1) and (x_2, y_2) are two distinct points of the linear model. Then the rate of change of y with respect to x is the ratio of the rise to the run in going from point (x_1, y_1) to point (x_2, y_2):

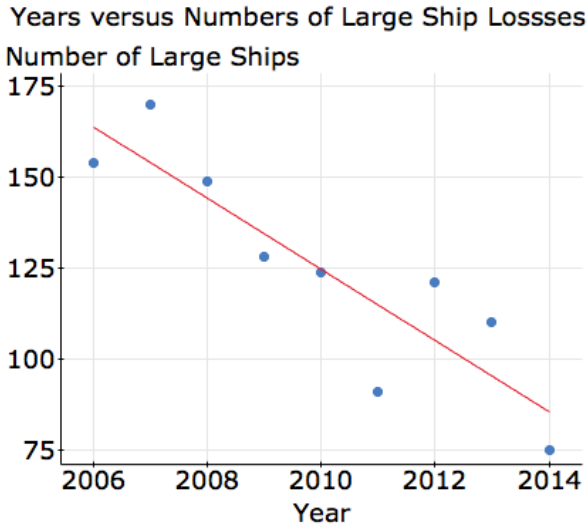
$$rate\ of\ change\ of\ y\ with\ respect\ to\ x = \frac{y_2 - y_1}{x_2 - x_1} = \frac{rise}{run}$$

7. A carat is a unit of weight for precious stones and pearls. The scatterplot and the model in the following figure describe the association between the weights (in carats) of some diamonds and their value (in Singapore dollars). Is the association positive, negative, or neither? Estimate the rate of change of value with respect to number of carats.



In Problem 7, you likely found that if one diamond weighs 1 carat more than another diamond, it will cost approximately $3720 more than the other diamond. This does **not** mean that the diamonds cost $3720 per carat.

8. The scatterplot and the model in the following figure describe the association between years and the number of large ships lost at sea due to fire, collision, storm, and machine breakdown. Is the association positive, negative, or neither? Estimate the rate of change of number of large ship losses.



OBJECTIVE 4 – Describe the connection between the direction of an association and rate of change.

Connection between the Direction of an Association and Rate of Change

- If an association between two variables is positive, then the approximate rate of change of one variable with respect to the other is positive (see Problem 7).

- If an association between two variables is negative, then the approximate rate of change of one variable with respect to the other is negative (see Problem 8).

OBJECTIVE 5 – Calculate the slope of a line.

Definition: Slope of a nonvertical line

Assume (x_1, y_1) and (x_2, y_2) are two distinct points of a nonvertical line. The **slope** of the line is the rate of change of y with respect to x. In symbols:

$$m = slope = \frac{y_2 - y_1}{x_2 - x_1} = \frac{rise}{run}$$

9. Find the slope of the line that contains the points (1, 5) and (4, 3).

It is a common error to make incorrect substitutions into the slope formula. Carefully consider why the middle and right-hand formulas are incorrect:

Correct	**Incorrect**	**Incorrect**
$m = \frac{y_2 - y_1}{x_2 - x_1}$	$m = \frac{y_2 - y_1}{x_1 - x_2}$	$m = \frac{x_2 - x_1}{y_2 - y_1}$

Find the slope of the line that contains the given points.

10. (1, 3) and (4, 9)

11. (−4, 1) and (2, −3)

12. (−5, −1) and (4, 2)

13. (−6, −3) and (−2, −5)

When working with negative coordinates, it can help to first write $\frac{(\ ) - (\ )}{(\ ) - (\ )}$ and then insert the coordinates into the appropriate parentheses.

OBJECTIVE 6 – Explain why the slope of an increasing line is positive and the slope of a decreasing line is negative.

- A line that goes upward from left to right is **increasing**. [Draw a figure.]
- A line that goes downward from left to right is **decreasing**. [Draw a figure.]

Slopes of Increasing or Decreasing Lines

An increasing line has positive slope. [Draw a figure. Also refer to Problems 10 and 12.]
A decreasing line has negative slope. [Draw a figure. Also refer to Problems 11 and 13.]

14. Find the approximate slope of the line that contains the points (−2.8, 5.9) and (−1.1, −3.7). Round the result to the second decimal place. State whether the line is increasing or decreasing.

OBJECTIVE 7 − Explain why the absolute value of the slope of a line measures the steepness of the line.

15. Compare the steepness of two lines with slopes 2 and 3.

Measuring the Steepness of a Line

The absolute value of the slope of a line measures the steepness of the line.
The steeper the line, the larger the absolute value of its slope will be.

OBJECTIVE 8 − Explain why the slope of a horizontal line is zero and the slope of a vertical line is undefined.

Draw a horizontal line and find its slope $\left(\frac{0}{run} = 0\right)$.

Draw a vertical line and explain why its slope, $\frac{run}{0}$, is undefined.

Slopes of Horizontal and Vertical Lines

A horizontal line has slope equal to zero. [Draw a figure.]
A vertical line has undefined slope. [Draw a figure.]

For Problems 16–19, find the slope of the line that contains the given points. Determine whether the line is increasing, decreasing, horizontal, or vertical.

16. (2, −3) and (2, 1)

17. (−3, −2) and (2, 8)

18. (−1, −4) and (3, −4)

19. (−5, −2) and (−1, −4)

20. Sketch an increasing line, a decreasing line, a horizontal line, and a vertical line. For each line, determine whether the slope is positive, negative, zero, or undefined.

Homework/Assessment

1, 3, 7, 9, 13, 17, 21, 23, 25, 31, 39, 43, 53, 59, 63, 65, 67, 77, 83, 97