Chapter 7 – Graphing Equations of Lines and Linear Models; Rates of Change
Section 4 – Functions

Objectives
1. Describe the meanings of relation, domain, range, and function.
2. Identify functions by using the vertical line test.
3. Describe the meaning of linear function.
4. Describe the Rule of Four for functions.
5. Use the graph of a function to find the function's domain and range.
6. Use function notation.
7. Use function notation to make predictions.
8. Determine the domain and range of a model.

Vocabulary
1. relation
2. domain/range
3. input/output
4. function
5. vertical line test
6. linear function
7. increasing/decreasing
8. function notation

Lesson/Activity
OBJECTIVE 1 − Describe the meanings of relation, domain, range, and function.
Refer to the ordered pairs (1, 4), (2, 3), (2, 5), and (3, 1) to introduce the concepts relation, domain, range, input, and output.

Definition: Relation, domain, and range
A **relation** is a set of ordered pairs.
The **domain** of a relation is the set of all values of the explanatory variable.
The **range** of the relation is the set of all values of the response variable.

- Each member of the domain is an **input**.
- Each member of the range is an **output**.

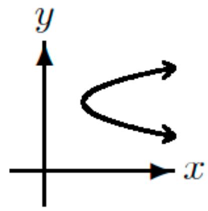
Definition: Function
A **function** is a relation in which each input leads to exactly one output.

Determine whether the given relation is a function.
1. $y = x + 5$
2. $y = 2x$
3. $y = \pm x$

4. Some ordered pairs of four relations are listed in the following table. Which of these relations could be functions? Explain.

Relation 1		Relation 2		Relation 3		Relation 4	
x	y	x	y	x	y	x	y
0	13	2	20	5	3	7	1
1	18	3	18	6	3	7	2
2	23	3	16	7	3	7	3
3	28	4	14	8	3	7	4
4	33	5	12	9	3	7	5

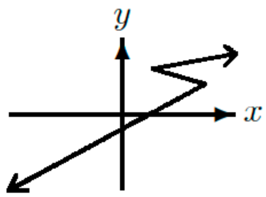
OBJECTIVE 2 – Identify functions by using the vertical line test.
Use arrows to show that there is an input that has two outputs for the relation graphed below.
Make a conclusion whether the relation is a function or not.

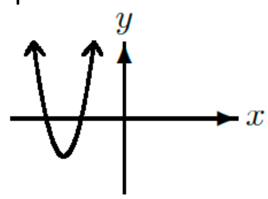


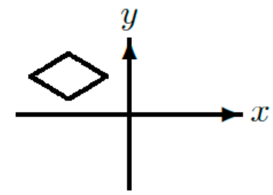
Vertical Line Test

A relation is a function if and only if each vertical line intersects the graph of the relation at no more than one point. We call this requirement the **vertical line test**.

Determine whether the following graph represents a function. Explain.







OBJECTIVE 3 – Describe the meaning of linear function.

Definition Linear Function

A **linear function** is a relation whose equation can be put into the form *y = mx + b* where *m* and *b* are constants.

The observation we have made about linear equations in two variables apply to linear functions.

1. The graph of a linear function is a nonvertical line.
2. The constant **m** is the rate of change of y with respect to x.
3. The constant **m** is the slope of the line.
4. The absolute value of the slope is a measure of the line's steepness.
5. a. If **m** > 0, The graph of the function is an increasing line.
   b. If **m** < 0, The graph of the function is a decreasing line.
   c. If **m** = 0, The graph of the function is a horizontal line.
6. The y-intercept of the line is (0, b).

Determine whether the given relation is a linear function.

5. $y = -2/5x + 3$ 6. $x = 2$ 7. $y = -4$ 8. $x = y^2$

OBJECTIVE 4 – Describe the Rule of Four for functions.

Rule of Four for Functions

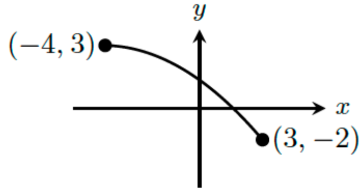
We can describe some or all the input-output pairs of a function by means of
1. an equation, 2. a graph, 3. a table, or 4. words.

9. a. Is the relation $y = 3x - 2$ a function?
   b. List some input-output pairs of $y = 3x - 2$ by using a table.
   c. Describe the input-output pairs of $y = 3x - 2$ by using a graph.
   d. Describe the input-output pairs of $y = 3x - 2$ by using words.

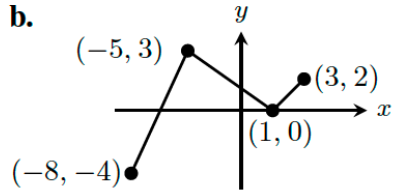
OBJECTIVE 5 – Use the graph of a function to find the function's domain and range.

10. Use the graph of the function to determine the function's domain and range.

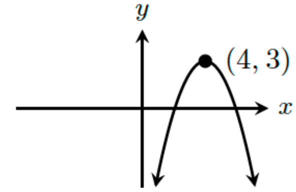
a.



b.



c.



OBJECTIVE 6 – Use function notation.

To use "*f*" as the name of the function y = 4x + 2, we use "*f*(*x*)" to represent *y*: Thus, *y* = *f*(*x*).

- We can substitute *f*(*x*) for *y* in the equation y = 4x + 2: *f*(*x*) = 4x + 2.
- To find *f*(*3*), we say we **evaluate** the function *f* at *3*.
- Warning: "*f*(*x*)" does not mean *f* times *x*.

Evaluate *f*(*x*) = 3x − 5 at the given values of *x*.

11. *f*(4)   12. *f*(0)   13. *f*(-1/3)

For *f*(*x*) = -2x + 6, *g*(*x*) = $3x^2$ − 4x, *h*(*x*) =(5x – 4)/(3x + 2), and *k*(*x*) = 5, find the following:

14. *f*(4)   15. *g*(2)   16. *h*(6)   17. *k*(3)

18. Let *f*(x) = -8.73x − 256. Find *x* when *f*(x) = 96.15. Round your result to the second decimal place.

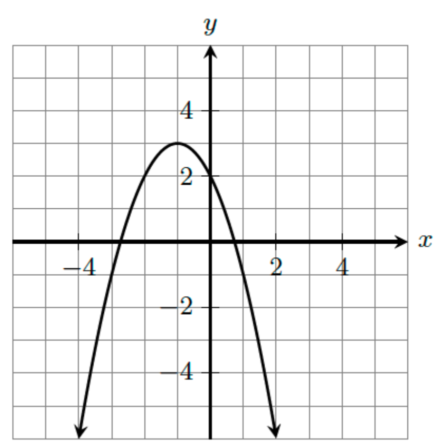
Some input-output pairs of a function *f* are shown in the following table.

x	*y*
1	4
2	3
3	2
4	3
5	4

19. Find *f*(3).

20. Find *x* when *f*(x) = 3.

21. A function *f* is graphed in the figure below.
    a. Find *f*(-1).
    b. Find x when *f*(*x*) = -1.
    c. Find the domain of *f*.
    d. Find the range of *f*.



22. Recall that we can describe some or all the input-output pairs of a function by means of an equation, a graph, a table, or words. Let *f*(x) = -4x + 7.
    a. Describe five input-output pairs of *f* by using a table.
    b. Describe the input-output pairs of *f* by using a graph.
    c. Describe the input-output pairs of *f* by using words.

OBJECTIVE 7 – Use function notation to make predictions.

Definition: Function notation

The response variable of a function *f* can be represented by the expression formed by writing the explanatory variable name within the parentheses of *f*( ):

response variable = *f*(explanatory variable)

23. Let **T** be the total fall semester cost (in dollars) of tuition and fees for part-time students who took **C** credits (units or hours) of courses at Centenary College (Source: Centenary College). The situation can be modeled by the equation T = 575C + 15.

    a. Rewrite the equation T = 575C + 15 with the function name f.

    b. Find f(6). What does it mean in this situation?

24. The percentages of American adults who smoke are shown in the following table for various years.

Year	Percent
1970	37.4
1980	33.2
1990	25.3
2000	23.1
2010	19.4
2012	18.0

Source: National Center for Health Statistics

A model is p = -0.45t + 36.83, where **p** is the percentage of Americans who smoke at **t** years since 1970.

    a. Verify that the graph of p = -0.45t + 36.83 comes close to the data points.

    b. Rewrite the equation p = -0.45t + 36.83 with the function name g.

    c. Estimate the percentage of American adults who smoked in 2005.

    d. Find the **p**-intercept of the model. What does it mean in this situation?

25. Let **p** be the percentage of gambling revenue in Nevada that is from penny slot machines at **t** years since 2008. The percentage in 2008 was 14% and increased by 2.8 percentage points per year until 2013 (Source: Nevada State Gaming Control Board).

    a. Find an equation of a model that describes this situation.

    b. Rewrite your equation using the function name f.

    c. Find *f*(5). What does it mean in this situation?

26. The number of households with cable TV subscriptions was 61.8 million in 2009 and decreased by about 1.5 million per year until 2014 (Source: IHS). Let n = g(t) be the number (in millions) of households with cable TV subscriptions at t years since 2009.

    a. Find an equation of *g*.

    b. Find *g*(4). What does it mean in this situation?

OBJECTIVE 8 − Determine the domain and range of a model.

For the **domain** and **range** of a model, we consider input-output pairs only when both the input and output make sense in the situation.

27. A store is open from 9 A.M. to 5 P.M., Mondays through Saturdays. Let I = $f(t)$ be an employee's weekly income (in dollars) from working t hours each week at $10 per hour.

    a. Find an equation of the model f.

    b. Find the domain and range of the model f.

Homework/Assessment

1, 3, 5, 13, 19, 29, 33, 39, 47, 55, 57, 61, 65, 79, 81, 85, 89, 93, 95, 101