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Recognizing symbols and notation. Fill each blank with the correct symbol.

1. subset ____________________ 2. proper subset ______________ 3. null set ______________
4. element of a set ____________ 5. not an element of a set ________ 6. strictly greater than ______
7. strictly less than ____________ 8. greater than or equal to _____ 9. less than or equal to _____

Every real number must fit into one of the three following categories: the number is positive, the number is negative, or the number is equal to zero. If $x$ is any real number, use symbols to state each of the following:

10. $x$ is positive ______________________________ 11. $x$ is negative ______________________________

Classifying numbers in the real number system. Use the roster method to list some elements of each of the following sets of numbers.

12. $N$: natural numbers ______________________________________
13. $W$: whole numbers ________________________________________
14. $Z$: integers ______________________________________________
15. $Q$: rational numbers ________________________________________
16. irrational numbers __________________________________________
17. $R$: real numbers ___________________________________________

18. Find the decimal representation of the rational number $\frac{1}{9}$ and then state whether this is a terminating decimal or a nonterminating (repeating) decimal.

$$\frac{1}{9} = \underline{} \underline{}$$

19. Find the decimal representation of the rational number $\frac{3}{8}$ and then state whether this is a terminating decimal or a nonterminating (repeating) decimal.

$$\frac{3}{8} = \underline{} \underline{}$$
Objective 1: Use Set Notation

1. Explain the difference between a subset and a proper subset.
   1. __________________________________________________________________________

2. Is the empty set a subset of every set?  
   2. _______

3. Is every set a subset of itself?  
   3. _______

4. If set $A$ is a subset of set $B$ (written $A \subseteq B$), what is the necessary condition for $A$ to be a proper subset of $B$?
   4. __________________________________________________________________________

5. Let $D = \{a, b, c\}$. List every possible subset of $D$. (Hint: there should be 8 subsets.)
   5. __________________________________________________________________________
6. Let \(X = \{-2, -1, 0, 1, 2\}\), \(Y = \{x \mid x \text{ is a natural number}\}\), \(Z = \{1, 3, 5, 7\}\), and \(W = \{-1, 0, 1\}\). Write TRUE or FALSE to each statement. See Example 3 and Example 4

(a) \(W \subset X \) ______________________________
(b) \(W \subseteq X \) ______________________________

(c) \(X \subset W \) ______________________________
(d) \(Z \subset Y \) ______________________________

(e) \(\emptyset \subset Y \) ______________________________
(f) \(\{-1, 0, 1\} \subset W \) ______________________________

(g) \(\{-1, 0, 1\} \subseteq W \) ______________________________
(h) \(X \subset Y \) ______________________________

(i) \(0 \in Y \) ______________________________
(j) \(\frac{2}{3} \notin Y \) ______________________________

(k) Every element of \(W\) is also an element of \(X\). ______________________________
(l) \(\frac{1}{2} \in Y \) ______________________________

Objective 2: Classify Numbers

7. Let \(S = \{4, -1, 1.\overline{3}, \pi, -\frac{6}{3}, 100,000, -\frac{5}{4}, \sqrt{9}, \frac{0}{2}, \sqrt{5}, 0.25, \sqrt{-16}\}\). See Example 5

(a) Rewrite \(S\), simplifying elements when possible ______________________________

List the elements of \(S\) that are:

(b) Natural numbers ______________________________
(c) Whole numbers ______________________________

(d) Integers ______________________________
(e) Rational numbers ______________________________

(f) Irrational numbers ______________________________
(g) Real numbers ______________________________
Do the Math Exercises R.2
Sets and Classification of Numbers

**In Problems 1 and 2, write each set using the roster method.**

1. \{x | x is an integer between -4 and 6\}  
2. \{x | x is a whole number less than 0\}

1. __________
2. __________

**In Problems 3 – 6, let A = \{1, 3, 5, 7, 9\}, B = \{2, 4, 6, 8\}, C = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}, and D = \{8, 6, 4, 2\}. Write TRUE or FALSE to each statement. Be sure to justify your answer.**

3. \(A \subseteq C\)  
4. \(A \subset C\)

3. __________
4. __________

5. \(B = D\)  
6. \(\emptyset \subset B\)

5. __________
6. __________

**In Problems 7 and 8, fill in the blank with the appropriate symbol, \(\in\) or \(\notin\).**

7. \(4.5 \quad \quad \{x | x is a rational number\}\)  
8. \(0 \quad \quad \{x | x is a natural number\}\)

7. __________
8. __________

**In Problems 9 and 10, list numbers in each set that are (a) Natural numbers, (b) Integers, (c) Rational numbers, (d) Irrational numbers, (e) Real numbers.**

<table>
<thead>
<tr>
<th>9. (\left{-5, \frac{4}{3}, -\frac{7}{5}, 5, \pi\right})</th>
<th>10. (\left{13, 0, -4.5656..., 2.43, \sqrt{2}, \frac{8}{7}, \sqrt{-9}\right})</th>
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<tbody>
<tr>
<td>(a)</td>
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<td>(e)</td>
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**In Problem 11, approximate the number by (a) truncating and (b) rounding to the indicated number of decimal places.**

11. \(-9.9999; 2\) decimal places

11a. _______

11b. _______

**12. On the real number line, label the points**

with coordinates: 4, 2.5, \(-\frac{5}{3}\), \(-0.5\)

11b. _______
In Problems 13 and 14, replace the question mark by <, >, or =, whichever is correct.

13. \( \frac{2}{3} \ ? \frac{2}{5} \)  
14. \( \frac{1}{5} \ ? 0.3 \)  

15. **Cisco Systems** Cisco systems stock lost $0.37 in a recent trading day. Express this loss as a rational number. *(SOURCE: Yahoo! Finance)*

16. Use your calculator to express \( \frac{8}{7} \) rounded to three decimal places.

17. Use your calculator to express \( \frac{8}{7} \) truncated to three decimal places.

18. Express \( \frac{2}{3} \) as a decimal. Is this decimal terminating or repeating?

19. Express \( -\frac{4}{25} \) as a decimal. Is this decimal terminating or repeating?

20. *(a)* Are there any real numbers that are both rational and irrational? *(b)* Are there any real numbers that are neither? *(c)* Explain your reasoning.

20a. _______  
20b. _______  
20c. ___________________________________________
Definitions and vocabulary. In your own words, write a brief definition for each of the following.

1. absolute value _________________________________________________________________________

2. product _______________________________________________________________________________

3. quotient ______________________________________________________________________________

4. sum __________________________________________________________________________________

5. difference _____________________________________________________________________________

Division Properties. For any nonzero real number $a$,

6. $\frac{0}{a} =$ _____________________

7. $\frac{a}{a} =$ _____________________

8. $\frac{0}{a} =$ _____________________

In Problems 9 and 10, identify the (a) divisor; (b) dividend; and (c) quotient.

9. $\frac{41}{2}$  (a) _____; (b) _____; (c) _____

10. $\frac{5}{20}$  (a) _____; (b) _____; (c) _____

In Problems 11 – 14, perform the indicated operation. Express all rational numbers in lowest terms.

11. $\frac{3}{4} + \frac{1}{2}$ 11. __________

12. $\frac{9}{5} - \frac{4}{5}$ 12. __________

13. $\frac{5}{8} \times \frac{12}{25}$ 13. __________

14. $\frac{25}{12} \div \frac{30}{27}$ 14. __________
Guided Practice R.3
Operations on Signed Numbers; Properties of Real Numbers

**Objective 2: Add and Subtract Signed Numbers**

1. To add two real numbers that are the same sign, add their absolute values.
   
   (a) If both numbers are positive, the sign of the sum is __________.  
      1a. __________
   
   (b) If both numbers are negative, the sign of the sum is __________.  
      1b. __________

2. To add two real numbers that are different signs, subtract the absolute value of the smaller number from the absolute value of the larger number. The sign of the sum is __________.  
   2. __________

**Objective 3: Multiply and Divide Signed Numbers**

3. To multiply (or divide) two real numbers, we multiply (or divide) the numbers and determine the sign of the product (or quotient) according to the following rule. Complete the chart, where + indicates that the sign of the number is positive and − indicates the sign of the number is negative.

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**Objective 4: Perform Operations on Fractions** (See textbook Example 10)

4. Find the least common denominator of the rational numbers \( \frac{7}{12} \) and \( \frac{5}{18} \). Then rewrite each rational number with the least common denominator.

**Step 1:** Factor each denominator.

- List the prime factors of 12: (a) __________
- List the prime factors of 18: (b) __________

**Step 2:** Write down the common factor(s) between each denominator. Then copy the remaining factors.

- List the common factors: (c) __________
- List all remaining factors: (d) __________

**Step 3:** Multiply the factors listed in Step 2. The product is the least common denominator.

- Multiply the factors listed in (c) and (d): (e) __________

**Step 4:** Rewrite \( \frac{7}{12} \) and \( \frac{5}{18} \) with a denominator of 36.

- To rewrite \( \frac{7}{12} \) with a denominator of 36, we multiply the fraction by \( \frac{3}{3} \) because \( a \cdot 1 = a \).
Multiply to find a fraction equivalent to \( \frac{5}{18} \) whose denominator is 36.

\[ \frac{5}{18} \cdot \frac{\_}{\_} = \frac{\_}{36} \]

**Objective 5: Know the Associative and Distributive Properties**

5. Write an example for each of the properties of the real number system.

(a) Identity Property of Addition

(b) Identity Property of Multiplication

(c) Inverse Property of Addition

(d) Inverse Property of Multiplication

(e) Double Negative Property

(f) Commutative Property of Addition

(g) Commutative Property of Multiplication

(h) Multiplication Property of Zero

(i) Associative Property of Addition

(j) Associative Property of Multiplication

(k) Distributive Property

6. Use the Distributive Property to remove the parentheses. *See textbook Example 13*

(a) \( 3(2x - 5) \)  
(b) \( -12(3y - 1) \)  
(c) \( (5n + 2) \cdot 4 \)  
(d) \( \frac{2}{3}(6x - 24) \)
## Do the Math Exercises R.3

**Operations on Signed Numbers; Properties of Real Numbers**

1. In Problems 1 and 2, determine (a) the additive inverse and (b) the multiplicative inverse of the given number.

   1. \(-\frac{1}{5}\)
      
      1a. ________  
      1b. ________

   2. 10
      
      2a. ________  
      2b. ________

2. Use the Distributive Property to remove the parentheses: \(-\frac{2}{3}(3x + 15)\)
   
   3. _______

3. Use the Reduction Property to simplify: \(\frac{40}{16}\)
   
   4. _______

4. In Problems 5 – 16, perform the indicated operation. Express all rational numbers in lowest terms.

   5. \(|-4| + 12\)
      
      5. _______

   6. \(7 \cdot (-15)\)
      
      6. _______

   7. \(-\frac{7}{3}\left(-\frac{12}{35}\right)\)
      
      7. _______

   8. \(-\frac{10}{3} \div \frac{15}{21}\)
      
      8. _______

   9. \(\frac{18}{7}\)
      
      9. _______

   10. \(\frac{8 - 18}{5}\)
       
       10. _______

   11. \(-\frac{17}{45} - \frac{23}{24}\)
       
       11. _______

   12. \(\frac{2}{3} \div 8\)
       
       12. _______
Do the Math Exercises R.3

13. \[ |−5.4 + 10.5| \]  
14. \[ \frac{20}{5} \] \quad \frac{4}{4}  

15. \[ −\frac{5}{2} - 4 \]  
16. \[ −\frac{5}{6} - \frac{1}{4} + \frac{5}{24} \]  

In Problems 17 – 20, state the property that is being illustrated.

17. \[ 3 + (4 + 5) = (3 + 4) + 5 \]  
18. \[ 5 \cdot \frac{1}{5} = 1 \]  

19. \[ 5 \cdot \frac{4}{4} = 5 \]  
20. \[ (x + 2) \cdot 3 = 3 \cdot (x + 2) \]  

21. Peaks and Valleys In Louisiana, the highest elevation is Driskill Mountain (535 feet above sea level); the lowest elevation is New Orleans (8 feet below sea level). What is the difference between the highest and lowest elevation?

22. Why Is the Product of Two Negatives Positive? In this problem we use the Distributive Property to illustrate why the product of two negative real numbers is positive.

(a) Express the product of any real number \( a \) and 0.  

22a. \[ \]  

(b) Use the Additive Inverse Property to write 0 from part (a) as \( b + (−b) \).  

22b. \[ \]  

(c) Use the Distributive Property to distribute the \( a \) into the expression in part (b).  

22c. \[ \]  

(d) Suppose that \( a < 0 \) and \( b > 0 \). What can be said about the product \( ab \)? Now, what must be true regarding the product \( a(−b) \) in order for the sum to be zero?

22d. \[ \]
Definitions and vocabulary. Fill in the blank for each of the following.

1. Integer exponents provide a shorthand device for representing repeated multiplications of a real number. In the expression $4^3$,

   (a) $4$ is called the _________

   (b) $3$ is called the _________

   (c) To evaluate the expression, multiply _________ = _____.

2. Consider the expression $3x$.

   (a) Another word for raising the expression $3x$ to the second power is to say $3x$ _________.

   (b) This is written _________.

3. Consider the expression $y + 1$.

   (a) Another word for raising the expression $y + 1$ to the third power is to say $y + 1$ _________.

   (b) This is written _________.

4. Identify the base for each expression.

   (a) $-5^2$ _________  (b) $(−2x)^4$ _________

5. In Problems 5 – 6, perform the indicated operation. Express all rational numbers in lowest terms.

   5. $\left( -\frac{3}{2} \right) \left( -\frac{3}{2} \right) \left( -\frac{3}{2} \right)$

6. $-\frac{5}{6} + \left( -\frac{4}{9} \right)$
Objective 1: Evaluate Real Numbers with Exponents

1. Evaluate each expression. (See textbook Examples 1 and 2)

   (a) \(6^2\)  
   (b) \((-6)^2\)  
   (c) \(-6^2\)

   1a. __________  
   1b. __________  
   1c. __________

   (d) \(-(-6)^2\)  
   (e) \(-\left(\frac{4}{5}\right)^3\)  
   (f) \(-\left(\frac{4}{5}\right)^4\)

   1d. __________  
   1e. __________  
   1f. __________

2. Raising a negative number to an even exponent results in a __________ number. 2. __________

3. Raising a negative number to an odd exponent results in a __________ number. 3. __________

Objective 2: Use the Order of Operations to Evaluate Expressions

4. Complete the table:

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<tr>
<th>Order of Operations</th>
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<tbody>
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<td>(a) (P)</td>
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<tr>
<td>(b) (E)</td>
</tr>
<tr>
<td>(c) (M/D)</td>
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<tr>
<td>(d) (A/S)</td>
</tr>
</tbody>
</table>
5. Evaluate: \( \frac{3 \cdot (-2)^2}{-3^2 - 6 \cdot 3} \)  
*(See textbook Example 7)*

**Step 1:** Evaluate exponents.  
(a) \( \frac{3 \cdot (-2)^2}{-3^2 - 6 \cdot 3} = \) 
_______________________________

**Step 2:** Multiply.  
(b) ____________________________________________________________________

**Step 3:** Subtract.  
(c) ____________________________________________________________________

**Step 4:** Divide out the common factor.  
(d) ____________________________________________________________________
Do the Math Exercises R.4
Order of Operations

In Problems 1 – 18, evaluate each expression.

1. \((-3)^2 + (-2)^2\) 
2. \(-5 + 3 \cdot 12\)

3. \(-4^2 - (-4)^2\) 
4. \(3[15 - (7 - 3)]\)

5. \(2 + 5(8 - 5)\) 
6. \(\frac{5 - (-7)}{4}\)

7. \([6 \cdot 2 - 5 \cdot 3]\) 
8. \(\frac{12 - 4}{-2}\)

9. \(\frac{2 \cdot 4 - 5}{4^2 + (-2)^2} + \frac{3^2}{2^3}\) 
10. \(3[2 + 3(1 + 5)]\)

11. \(\frac{4}{4^2 - 1} - \frac{3}{5(7 - 5)}\) 
12. \((3^2 - 3)(3 - (-3)^3)\)
13. \(-2(5 - 2) - (-5)^3\)  
14. \(-2(4 + |2 \cdot 3 - 5^2|)\)  

15. \(|6(3 \cdot 2 - 10)|\)  
16. \(|4(2 \cdot 5 + (-3)4)|\)  

17. \(\frac{3(5 + 2^2)}{2 \cdot 3^3}\)  
18. \(\left(\frac{2}{3}\right)^2 \cdot \left(\frac{1+2^3}{2^3 - 2}\right)\)  

19. Geometry The surface area of a right circular cylinder whose radius is 6 inches and height is 10 inches is given approximately by \(2 \cdot \pi \cdot 6^2 + 2 \cdot \pi \cdot 6 \cdot 10\) in\(^2\) where \(\pi \approx 3.1416\). Evaluate this expression rounded to the nearest hundredth.

20. Insert parentheses in order to make the statement true:
   (a) \(3 + 5 \cdot 6 - 3 = 18\)  
   (b) \(3 + 5 \cdot 6 - 3 = 24\)  

21. Explain why \(\frac{2 + 7}{2 + 9} \neq \frac{7}{9}\).  


Five-Minute Warm-Up R.5
Algebraic Expressions

1. Use the Distributive Property to remove the parentheses.

(a) \(-5(2x - 3)\)  
1a. ________

(b) \((4a + 3) \cdot 9\)  
1b. ________

(c) \(\frac{2}{3}(6n - 15)\)  
1c. ________

(d) \(-\frac{5}{4}(\frac{8}{5}z - \frac{24}{25})\)  
1d. ________

2. Evaluate each expression.

(a) \(12 - 4 \cdot 5\)  
2a. ________

(b) \(\frac{2 - 6}{4 + 2(-5)}\)  
2b. ________

(c) \(\frac{2}{3} \cdot 6^2 - 4 + 2^3\)  
2c. ________

(d) \(\frac{3 \cdot (-2) + 5}{6} - \frac{(-2)^2 + 8}{9}\)  
2d. ________
Objective 1: Translate English Expressions into Mathematical Language

1. Express each English phrase using an algebraic expression. (See textbook Example 1)

   (a) The product of \(-2\) and some number \(x\) \hspace{1cm} (b) 6 less than half of a number \(y\)

   1a. __________ \hspace{1cm} 1b. __________

   (c) 8 divided into a number \(m\) \hspace{1cm} (d) A number \(x\) subtracted from 25

   1c. __________ \hspace{1cm} 1d. __________

Objective 2: Evaluate Algebraic Expressions

2. Evaluate the expression \(-x^2 - 4x + 5\) for \(x = -3\). (See textbook Example 2)

   2. __________

Objective 3: Simplify Algebraic Expressions by Combining Like Terms

3. Simplify each expression by combining like terms. (See textbook Examples 4 and 5)

   (a) \(5v + v - 6v\) \hspace{1cm} (b) \(4x^2 + 2x^2\)

   3a. __________ \hspace{1cm} 3b. __________

4. Simplify each algebraic expression. (See textbook Example 6 and 7)

   (a) \(3(x - 2) + 5\) \hspace{1cm} (b) \(\frac{5}{3}(6x + 9) - 4x - \frac{1}{2}(8x - 2)\)

   4a. __________ \hspace{1cm} 4b. __________
Objective 4: Determine the Domain of a Variable

5. The set of values that a variable may assume is called the __________ of the variable.  5. __________

6. In the expression $\frac{1}{x}$, what value(s) for $x$ must be excluded from the domain?  6. __________

7. In general, we exclude any values of the variable which causes _________________________________

8. Determine which of the numbers are in the domain of the variable. (See textbook Example 8)

8a. __________  

(a) $\frac{12}{2x - 4}$  

$x = 0; \ x = -2; \ x = 2; \ x = 4$

8b. __________  

(b) $\frac{x - 3}{x + 5}$  

$x = 3; \ x = 5; \ x = -5; \ x = 0$
Do the Math Exercises R.5
Algebraic Expressions

In Problems 1 – 4, evaluate each expression for the given value of the variable.

1. \( y^2 - 4y + 5; \ y = 3 \)

2. \(-2z^2 + z + 3; \ z = -4 \)

3. \(|4z - 1|; \ z = -\frac{5}{2} \)

4. \(\frac{3 - 5z}{z - 4}; \ z = 4 \)

In Problems 5 – 14, simplify each expression by combining like terms.

5. \( 5y + 2y \)

6. \( 8x - 9x + 1 \)

7. \( \frac{3}{10}y + \frac{4}{15}y \)

8. \(-x - 3x^2 + 4x - x^2 \)

9. \( 10y + 3 - 2y + 6 + y \)

10. \( 3(2y + 5) - 6(y + 2) \)

11. \( \frac{1}{2}(20x - 14) + \frac{1}{3}(6x + 9) \)

12. \(-4(w - 3) - (2w + 1) \)

13. \( \frac{4}{3}(5y + 1) - \frac{2}{5}(3y - 4) \)

14. \( 0.4(2.9x - 1.6) - 2.7(0.3x + 6.2) \)
In Problems 15 – 18, express each English phrase using an algebraic expression.

15. The difference of 10 and a number \( y \)

16. The ratio of a number \( x \) and 5

17. A number \( z \) increased by 30

18. Twice some number \( x \) decreased by the ratio of a number \( y \) and 3

19. Which of the following are in the domain of the variable? 
\[
\frac{x + 1}{x^2 + 5x + 4}
\]

(a) \( x = 5 \) 
(b) \( x = -1 \) 
(c) \( x = -4 \) 
(d) \( x = 0 \)

20. **Area of a Triangle** The algebraic expression \( \frac{1}{2}(h+2)h \) represents the area of a triangle whose base is two centimeters longer than its height \( h \). See the figure. Evaluate the algebraic expression for \( h = 10 \) centimeters.

21. Write an English phrase that would translate into the given mathematical expression.

(a) \( 2x - 3 \)

(b) \( 2(x - 3) \)
Five-Minute Warm-Up 1.1
Linear Equations in One Variable

1. Determine the additive inverse of $-\frac{1}{2}$.  
   1. __________

2. Determine the multiplicative inverse of 6.  
   2. __________

3. Use the Reduction Property to simplify the expression: $\frac{2}{3} \cdot \frac{3}{2} x$  
   3. __________

4. Find the Least Common Denominator of $\frac{5}{12}$ and $\frac{7}{15}$.  
   4. __________

5. Use the Distributive Property to remove the parenthesis: $-3(2x - 5)$  
   5. __________

6. What is the coefficient of $-x$?  
   6. __________

7. Simplify by combining like terms: $-2(3x - 1) + 5 + 7x$  
   7. __________

8. Simplify: $6\left(\frac{x}{3} + \frac{5x}{6}\right)$  
   8. __________

9. Evaluate the expression $\frac{2}{5}(5x - 10) + 3x$ when $x = -3$.  
   9. __________

10. Is $x = -1$ in the domain of $\frac{6}{1 - x}$?  
    10. __________
Guided Practice 1.1
Linear Equations in One Variable

Objective 1: Determine Whether a Number Is a Solution to an Equation

1. How do you determine if a value for a variable is a solution to an equation?

Objective 2: Solve Linear Equations (See textbook Example 5)

2. The goal in solving any linear equation is to get the variable by itself with a coefficient of 1, that is, to isolate the variable. List 5 steps you might use when solving a linear equation in one variable.

   a. ____________________________________________________________________________

   b. ____________________________________________________________________________

   c. ____________________________________________________________________________

   d. ____________________________________________________________________________

   e. ____________________________________________________________________________

3. Solve the linear equation \( \frac{2x + 3}{2} - \frac{x - 1}{4} = \frac{x + 5}{12} \).

   **Step 1:** Remove all parentheses using the Distributive Property.

   Determine the LCD: \( (a) \)

   Use the Multiplication Property of Equality: \( (b) \)

   Use the Distributive Property: \( (c) \)

   Divide out the common factors: \( (d) \)

   (e) Use ____________________________ \( 12x + 18 - 3x + 3 = x + 5 \)

   **Step 2:** Combine like terms on each side of the equation.

   Combine like terms: \( (f) \)
**Step 3:** Use the Addition Property of Equality to get all variables on one side of the equation and all constants on the other side.

Subtract \(x\) from both sides: \(g\) \(9x + 21 \underline{\_{\phantom{2}}} = x + 5 _\underline{\phantom{2}}\)

Simplify: \(h\) 

Subtract 21 from both sides: \(i\) \(8x + 21 \underline{\_{\phantom{2}}} = 5 _\underline{\phantom{2}}\)

Simplify: \(j\) 

**Step 4:** Use the Multiplication Property of Equality to get the coefficient on the variable to equal 1.

Divide both sides by 8: \(k\) 

**Step 5:** Check Verify the solution.

Substitute your value for \(x\) into the original equation:

\[
\frac{2x + 3}{2} - \frac{x - 1}{4} = \frac{x + 5}{12}
\]

Simplify:

State the solution set: \(l\) 

**Objective 3:** Determine Whether an Equation Is a Conditional Equation, an Identity, or a Contradiction

4. Solve the linear equation \(-2(3x - 1) = 4 - 6(x + 3)\). State whether the equation is an identity, contraction or conditional equation. (See textbook Example 7)

(a) Is the last statement of the solution true or false? ______________________________

(b) Therefore the solution set is __________________________________________

(c) Is this equation an identity, contradiction or a conditional equation? __________________

5. Solve the linear equation \(3 - 5(2x + 1) - 9 = -4x + 5 - 2(3x + 8)\). State whether the equation is an identity, contraction or conditional equation. (See textbook Example 8)

(a) Is the last statement of the solution true or false? ______________________________

(b) Therefore the solution set is __________________________________________

(c) Is this equation an identity, contradiction or a conditional equation? __________________
Do the Math Exercises 1.1
Linear Equations in One Variable

In Problems 1 and 2, determine if the stated value for the variable is a solution to the equation.

1. \(-4x - 3 = -15; x = 3\)
2. \(3(t + 1) - t = 4t + 9; t = -3\)

1. ___________
2. ___________

In Problems 3 – 14, solve each linear equation. Be sure to verify your solution.

3. \(-6x - 5 = 13\)
4. \(-5z + 3 = -3z + 1\)

3. ___________
4. ___________

5. \(4(z - 2) = 12\)
6. \(\frac{3x}{2} + \frac{x}{6} = -\frac{5}{3}\)

5. ___________
6. ___________

7. \(\frac{r}{2} + 2(r - 1) = \frac{5r}{2} + 4\)
8. \(\frac{2x + 1}{3} - \frac{6x - 1}{4} = -\frac{5}{12}\)

7. ___________
8. ___________

9. \(\frac{3x + 1}{4} - \frac{7x - 4}{2} = \frac{26}{3}\)
10. \(0.3z + 0.8 = -0.1\)

9. ___________
10. ___________
Do the Math Exercises 1.1

11. \( \frac{4}{5}(y - 4) + 3 = \frac{2}{3}(y + 1) + \frac{4}{15} \)

12. \( 7(x + 2) = 5(x - 2) + 2(x + 12) \)

13. \( \frac{z - 4}{6} - \frac{2z + 1}{9} = \frac{1}{3} \)

14. \( 3b - 3(b + 1) = 5(b - 1) - 5b \)

15. Find \( a \) such that the solution set of \( ax + 6 = 20 \) is \( \{7\} \).

16. Find \( a \) such that the solution set of \( ax + 3 = 2x + 3(x + 1) \) is the set of all real numbers.

In Problems 17 and 18, determine which values of the variable must be excluded from the domain.

17. \( \frac{-3}{5x + 8} \)

18. \( \frac{-2x + 7}{4(x - 3) + 2} \)

19. Explain the difference between the directions “solve” and “simplify.”
Five-Minute Warm-Up 1.2
An Introduction to Problem Solving

Use your experience or refer to Section R.5 to make a list of words that correspond to each operation.

1. Add (+) 2. Subtract (−) 3. Multiply (∙) 4. Divide (/)


In Problems 5 – 12, express each English phrase as an algebraic expression.

5. The sum of 3 and 2x 6. 5 times the sum of y and 1

7. The sum of 5 times a number z and 20 8. The ratio of twice a number n and 3

9. 12 less than half of a number x 10. The difference of x and 15

11. Write 5% as a decimal. 12. Write 0.075 as a percent.
Guided Practice 1.2
An Introduction to Problem Solving

Objective 1: Translate English Sentences into Mathematical Statements

1. List the five categories of problems we will be modeling in this course. Give a brief description of each.

_______________________________________________________________________________________
_______________________________________________________________________________________
_______________________________________________________________________________________
_______________________________________________________________________________________
_______________________________________________________________________________________

2. List six steps for solving problems with mathematical models.

_______________________________________________________________________________________
_______________________________________________________________________________________
_______________________________________________________________________________________
_______________________________________________________________________________________
_______________________________________________________________________________________
_______________________________________________________________________________________

Objective 2: Model and Solve Direct Translation Problems

3. If \( n \) represents the first of three unknown consecutive integers, how would you represent the next two integers? (a) _______________ (b) _______________

4. If \( x \) represents the first of four unknown consecutive even integers, how would you represent the next three even integers? (a) _______________ (b) _______________ (c) _______________

5. If \( p \) represents the first of four unknown consecutive odd integers, how would you represent the next three odd integers? (a) _______________ (b) _______________ (c) _______________

6. What is the formula for simple interest? Identify what each of the variables represent.

_______________________________________________________________________________________
**Objective 3: Model and Solve Mixture Problems** *(See textbook Example 8)*

7. **Candy Store** Valentine’s Day is coming up so Andy decided to buy chocolates for his co-workers at $4.50 per pound and truffles for his girlfriend at $7.50 per pound. If he purchased a total of 11 pounds of candy and spent $58.50, how many pounds of each type did he buy?

(a) Eleven pounds of candy were purchased. If \( p \) represents the number of pounds of chocolates purchased and the rest of the candies were truffles, write an expression that represents the pounds of truffles. 

(b) Complete the following table.

<table>
<thead>
<tr>
<th></th>
<th>Price $/Pound</th>
<th>Number of Pounds</th>
<th>Revenue</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chocolates</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Truffles</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Blend</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(c) Write and solve an equation that models Andy’s purchase.

(d) Answer the question.

**Objective 4: Model and Solve Uniform Motion Problems** *(See textbook Example 9)*

8. **Boats** Two boats are traveling toward the same port from opposite directions. They are 63 miles apart and one boat is traveling 6 mph faster than the other. If the boats both reach the port in 4 hours and 30 minutes, find the speed for each boat.

(a) What do you know about the distances travelled by the two boats?

(b) Complete the following table.

<table>
<thead>
<tr>
<th></th>
<th>Rate, mph</th>
<th>Time, hours</th>
<th>Distance, miles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slower boat</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Faster boat</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(c) Write and solve an equation that models the distance travelled by the boats.

(d) Answer the question.
Do the Math Exercises 1.2
An Introduction to Problem Solving

1. 50 is 90% of what?  
2. 90 is what percent of 120?

In Problems 3 – 6, translate each of the following English statements into a mathematical statement. Then solve the equation.

3. The difference between 10 and a number \( z \) is 6.  
4. The sum of two times \( y \) and 3 is 16.

5. Five times a number \( x \) is equivalent to the difference of three times \( x \) and 10.  
6. 40% of a number equals the difference between the number and 10.

7. Number Sense Pattie is thinking of two numbers. She says that one of the numbers is 8 more than the other number and the sum of the numbers is 56. What are the numbers?

8. Consecutive Integers The sum of four consecutive odd integers is 104. Find the integers.
9. **Investments**  Johnny is a shrewd 8-year old. For Christmas, his grandparents gave him $10,000. Johnny decides to invest some of the money in a savings account that pays 2% per annum and the rest in a stock fund paying 10% per annum. Johnny wants his investments to yield 7% per annum. How much should Johnny put into each account?

10. **Coins**  Diana has been saving nickels and dimes. She opened up her piggy bank and determined that it contained 48 coins worth $4.50. Determine how many nickels and dimes were in the piggy bank.

11. **Antifreeze**  The cooling system of a car has a capacity of 15 liters. If the system is currently filled with a mixture that is 40% antifreeze, how much of this mixture should be drained and replaced with pure antifreeze so that the system is filled with a solution that is 50% antifreeze?

12. **Candy**  “Sweet Tooth!” candy store sells chocolate-covered almonds for $6.50 per pound and chocolate-covered peanuts for $4.00 per pound. The manager decides to make a bridge mix that combines the almonds with the peanuts. She wants the bridge mix to sell for $5.00 per pound, and there should be no loss in revenue from selling the bridge mix rather than the almonds and peanuts alone. How many pounds of chocolate-covered almonds and chocolate-covered peanuts are required to create 50 pounds of bridge mix?

13. **Cars**  Two cars leave from the same location and travel in opposite directions along a straight road. One car travels 30 miles per hour while the other travels at 45 miles per hour. How long will it take the two cars to be 255 miles apart?
Five-Minute Warm-Up 1.3
Using Formulas to Solve Problems

1. **List for the formulas for each of the following geometric figures.**

<table>
<thead>
<tr>
<th>(a) Square</th>
<th>(b) Rectangle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area:</td>
<td>Area:</td>
</tr>
<tr>
<td>Perimeter:</td>
<td>Perimeter:</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(c) Triangle</th>
<th>(d) Trapezoid</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area:</td>
<td>Area:</td>
</tr>
<tr>
<td>Perimeter:</td>
<td>Perimeter:</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(e) Parallelogram</th>
<th>(f) Circle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area:</td>
<td>Area:</td>
</tr>
<tr>
<td>Perimeter:</td>
<td>Circumference:</td>
</tr>
</tbody>
</table>

2. In Problems 2 – 6, (a) round and then (b) truncate each decimal to the indicated number of places.

   2. 15.96145; 3 decimal places
   
      2a. __________
      2b. __________

   3. −0.098; 2 decimal places
   
      3a. __________
      3b. __________

   4. 9.55; nearest whole number
   
      4a. __________
      4b. __________

   5. 100.73; nearest tenth
   
      5a. __________
      5b. __________

   6. $5.76715; nearest cent
   
      6a. __________
      6b. __________
Guided Practice 1.3
Using Formulas to Solve Problems

**Objective 1: Solve for a Variable in a Formula**

1. Explain the meaning of the expression “solve for the variable”.

2. Solve the formula for the indicated variable.  
   *(See textbook Examples 2 and 3)*
   
   (a) \( V = \frac{1}{3}Bh \) for \( h \)  
   (b) \( A = P + Prt \) for \( P \)

   2a. 
   2b. 

**Objective 2: Use Formulas to Solve Problems** *(See textbook Examples 4 and 5)*

3. The sum of the measures of the interior angles of a triangle is 180°. The measure of the first angle is 15° less than the second. The measure of the third angle is 45° more than half of the second. Find the measure of each interior angle of the triangle.

(a) **Step 1**: Identify  What formula is needed to solve this problem? 

(b) **Step 2**: Name  
   If \( x \) represents the value of the second angle, what expression represents the value of the first angle? 

   What expression represents the value of the third angle? 

(c) **Step 3**: Translate  Substitute the values from Step 2 into the formula from Step 1.

   \[ x^\circ + y^\circ + z^\circ = 180^\circ \]
(d) Step 4: Solve

(e) Step 5: Check Substitute the value you found for \( x \) into your equation. Does the sum of the angles equal 180°?

(f) Step 6: Answer the Question \( m\angle x = \) _________; \( m\angle y = \) _________; \( m\angle z = \) _________

Note: \( m\angle x \) means the measure of angle \( x \).

4. The perimeter \( P \) of a rectangle is given by the formula \( P = 2l + 2w \) where \( l \) is the length and \( w \) is the width. Solve the equation for \( w \) and then use this equation to find the width of a rectangle whose length is 13.5 cm and whose perimeter is 40 cm.

(a) Solve the formula for \( w \).

(b) Use part (a) to find the width, \( w \), if the length of the rectangle is 13.5 cm and the perimeter of the rectangle is 40 cm. 4a. __________ 4b. __________

(c) Notice that when solving an applied problem for an unknown in which some of the values of the variables are given, such as in Problem 4, it is possible to solve the equation for the unknown, \( w \). Using the new equation, found in 4(a), substitute the given values to answer the question.

It is also possible to substitute the known values into the original equation, \( P = 2l + 2w \), and then solve for \( w \). Now solve Problem 4 using this approach. Which method do you prefer? ______________________________
In Problems 1 – 6, solve the formula for the indicated variable.

1. **Direct Variation:** \( y = kx \) for \( k \)  

2. **Algebra:** \( y = mx + b \) for \( m \)  

3. **Conversion:** \( F = \frac{9}{5} C + 32 \) for \( C \)  

4. **Geometry:** \( A = \frac{1}{2} bh \) for \( h \)  

5. **Sequences:** \( S - rS = a - ar^a \) for \( S \)  

6. **Trapezoid:** \( A = \frac{1}{2} (B + b) \) for \( b \)  

In Problems 7 – 10, solve for \( y \).

7. \( 4x + 2y = 20 \)  

8. \( \frac{2}{3} x - \frac{5}{2} y = 5 \)  

9. \( 4.8x - 1.2y = 6 \)  

10. \( \frac{2}{5} x + \frac{1}{3} y = 8 \)  

**11. Maximum Heart Rate**  
The model \( M = -0.85A + 217 \) was developed by Miller to determine the maximum heart rate \( M \) of an individual who is age \( A \).  

(a) Solve the model for \( A \).  

(b) What is the age of an individual whose maximum heart rate is 160?
12. Supplementary Angles  Two angles are supplementary if the sum of the measures of the angles is 180°. If one angle is twice the measure of its supplement, find the measures of the two angles.

13. Complementary Angles  Two angles are complementary if the sum of the measures of the angles is 90°. If one angle is 30° less than twice its complement, find the measures of the two angles.

14. Angles in a Triangle  The sum of the measures of the interior angles in a triangle is 180°. The measure of the second angle is 3 times the measure of the first angle. The measure of the third angle is 20° more than the measure of the first angle. Find the measure of each of the interior angles.

15. Critical Thinking  Suppose that you wish to install a window with the dimensions given in the figure.

![Window diagram]

(a) What is the area of the opening of the window?

(b) What is the perimeter of the window?

(c) If glass costs $8.25 per square foot, what is the cost of the glass for the window?
Five-Minute Warm-Up 1.4
Linear Inequalities in One Variable

In Problems 1 – 6, replace the question mark by <, >, or = to make the statement true.

1. \(-0.5 \ ? \ 0\)  
2. \(-6.5 \ ? \ -7\)  
3. \(\frac{11}{12} \ ? \ \frac{13}{16}\)  
4. \(2.625 \ ? \ \frac{5}{8}\)  
5. \(-1\frac{3}{8} \ ? \ -2\frac{1}{4}\)  
6. \(0.0033 \ ? \ 0.0034\)

7. One inequality symbol that is not used in this section is \(\neq\) as the symbols used in this section represent strict and nonstrict inequalities.

(a) List the two symbols which represent strict inequalities.  

(b) List the two symbols that represent nonstrict (or weak) inequalities.  

(c) Use inequality notation to express a variable, \(x\), is between \(\frac{1}{2}\) and \(\frac{3}{8}\).
Guided Practice 1.4
Linear Inequalities in One Variable

Objective 1: Represent Inequalities Using the Real Number Line and Interval Notation

1. Which of the following, if any, is appropriate use of inequality notation?
   (a) \( 5 < x < -1 \)  
   (b) \( -1 \leq x \leq 1 \)
   1. 

Objective 2: Understand the Properties of Inequalities

2. True or False The Addition Property of Inequality states that the direction of the inequality does not change regardless of the quantity that is added to each side of the inequality.
   2. 

3. True or False When multiplying both sides of an inequality by a negative number, we reverse the direction of the inequality symbol.
   3. 

4. True or False When multiplying both sides of an inequality by a positive number, we reverse the direction of the inequality symbol.
   4. 

Objective 3: Solve Linear Inequalities (See textbook Examples 5 and 6)

5. Solve the inequality \( 5x + 2 > -13 \). Graph the solution set.

   Step 1: We isolate the term containing the variable.
   \[ 5x + 2 > -13 \]
   Subtract 2 from both sides:
   \[ (a) \quad 5x + 2 > -13 \]
   Simplify:
   \[ (b) \quad \text{________} \]

   Step 2: Now we want to get a coefficient of 1 on the variable.
   Divide both sides by 5:
   \[ (c) \quad \text{________} \]
   Simplify:
   \[ (d) \quad \text{________} \]
   Write the answer in set-builder notation:
   \[ (e) \quad \text{________} \]
   Write the answer using interval notation:
   \[ (f) \quad \text{________} \]
   Graph the solution set:
   \[ (g) \quad \text{________} \]
Guided Practice 1.4

6. Solve the inequality \(3x + 7 \geq 7x - 1\). Graph the solution set.

\[
\begin{align*}
3x + 7 & \geq 7x - 1 \\
\text{Subtract 7 from both sides:} & \quad (a) \ 3x + 7 \quad \quad \geq \quad 7x - 1 \\
\text{Simplify:} & \quad (b) \\
\text{Subtract } 7x \text{ from both sides:} & \quad (c) \ 3x \quad \quad \geq \quad 7x - 8 \\
\text{Simplify:} & \quad (d) \\
\text{Divide both sides by } -4 \text{. Don’t forget to change the direction of the inequality symbol. Simplify:} & \quad (e) \\
\text{Write the solution using set-builder notation:} & \quad (f) \\
\text{Write the solution using interval notation:} & \quad (g) \\
\text{Graph the solution set:} & \quad (h) 
\end{align*}
\]

7. Write the appropriate inequality symbol for each phrase.

<table>
<thead>
<tr>
<th>Phrase</th>
<th>Inequality Symbol</th>
<th>Phrase</th>
<th>Inequality Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) At least</td>
<td>(a) []</td>
<td>(b) No less than</td>
<td>(b) []</td>
</tr>
<tr>
<td>(c) More than</td>
<td>(c) []</td>
<td>(d) Greater than</td>
<td>(d) []</td>
</tr>
<tr>
<td>(e) No more than</td>
<td>(e) []</td>
<td>(f) At most</td>
<td>(f) []</td>
</tr>
<tr>
<td>(g) Fewer than</td>
<td>(g) []</td>
<td>(h) Less than</td>
<td>(h) []</td>
</tr>
</tbody>
</table>

8. **Commission** Nghiep sells digital cameras. His annual salary is $25,000 plus a commission of 20% on all of his sales. What is the value of the digital cameras Nghiep needs to sell so that his annual salary will be at least $36,000?

(a) **Step 1: Identify** What key word(s) indicates that this requires an inequality? ________________
(b) **Step 2: Name** Let \(v\) represent the value of the digital cameras sold.
(c) **Step 3: Translate** Write an inequality that represents the minimum annual salary. _________________
(d) **Step 4: Solve** _________________
(e) **Step 5: Check** When you substitute your values into the inequality, is the statement true? ___________
(f) **Step 6: Answer the Question** ________________
In Problems 1 – 4, solve each linear inequality. Express your answer in set-builder notation. Graph the solution set.

1. \( x + 6 < 9 \)
2. \( 5x - 4 \leq 16 \)

3. \( -7x < 21 \)
4. \( 8x + 3 \geq 5x - 9 \)

In Problems 5 – 12, solve each linear inequality. Express your answer in interval notation.

5. \( 3x + 4 \geq 5x - 8 \)
6. \( 3(x - 2) + 5 > 4(x + 1) + x \)

7. \( 2.3x - 1.2 > 1.8x + 0.4 \)
8. \( \frac{2x - 3}{3} > \frac{4}{3} \)

9. \( \frac{2}{3} - \frac{5}{6}x > 2 \)
10. \( \frac{5}{6}(3x - 2) - \frac{2}{3}(4x - 1) < -\frac{2}{9}(2x + 5) \)
11. \[ \frac{2}{5} x + \frac{3}{10} \leq \frac{1}{2} \]

12. \[ \frac{x}{12} \geq \frac{x}{2} - \frac{2x + 1}{4} \]

13. Find the set of all \( x \) such that the difference between 3 times \( x \) and 2 is less than 7.

14. Find the set of all \( y \) such that the sum of twice \( y \) and 3 is greater than 13.

15. **Computing Grades** In order to earn an \( A \) in Mrs. Padilla’s Intermediate Algebra course, Mark must obtain an average score of at least 90. On his first four exams Mark scored 94, 83, 88, and 92. The final exam counts as two test scores. What score does Mark need on the final to earn an \( A \) in Mrs. Padilla’s class?

16. **Moving Trucks** A 15 foot moving truck from Budget costs $39.95 per day plus $0.65 per mile. If your budget only allows for you to spend $125.75, what is the maximum number of miles you can drive? (SOURCE: Budget)

17. Explain why we never mix inequalities as in \( 4 < x > 7 \).
1. Plot the following points on the real number line: \( \frac{3}{2}, -2, 0, 3 \).

2. Determine which of the following are solutions to the equation \( 4x - 5(x + 1) = 6 \).

   (a) \( x = -1 \)  
   (b) \( x = -11 \)  
   (c) \( x = 1 \)  
   (d) \( x = 11 \)

3. Evaluate the expression \( -x^2 - 3x + 4 \) for the given values of the variable.

   (a) \( x = 3 \)  
   (b) \( x = -2 \)  
   (c) \( x = 0 \)

4. Solve the equation \( 4x - 3y = -12 \) for \( y \).

5. Evaluate each of the following absolute values.

   (a) \( |-12| \)  
   (b) \( |0| \)  
   (c) \( |125| \)
Guided Practice 1.5
Rectangular Coordinates and Graphs of Equations

Objective 1: Plot Points in the Rectangular Coordinate System

1. Label each quadrant and axis in the rectangular or Cartesian coordinate system.

2. What name do we give the ordered pair \((0, 0)\)? ___

In Problems 3 and 4, circle one answer for each underlined choice.

3. To plot the ordered pair \((-3, 2)\), you would move ___ units up, down, left, or right from the origin?

4. To plot the ordered pair \((-6, -2)\), you would move ___ units down from the origin.

Objective 3: Graph an Equation Using the Point-Plotting Method (See textbook Example 3)

5. Graph the equation \(y = 3x - 1\) by plotting points.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y = 3x - 1)</th>
<th>((x, y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) -2</td>
<td>(3(__) - 1 = __))</td>
<td>((-2, __))</td>
</tr>
<tr>
<td>(b) -1</td>
<td>(3(__) - 1 = __))</td>
<td>((-1, __))</td>
</tr>
<tr>
<td>(c) 0</td>
<td>(3(__) - 1 = __))</td>
<td>((0, __))</td>
</tr>
<tr>
<td>(d) 1</td>
<td>(3(__) - 1 = __))</td>
<td>((1, __))</td>
</tr>
<tr>
<td>(e) 2</td>
<td>(3(__) - 1 = __))</td>
<td>((2, __))</td>
</tr>
</tbody>
</table>

Step 1: We want to find all the points \((x, y)\) that satisfy the equation. To determine these points we choose values of \(x\) and use the equation to determine the corresponding values of \(y\).

Step 2: Draw the axes in the Cartesian plane and plot the points listed in the third column. Now connect the points to obtain the graph of the equation (a line).
**Objective 4: Identify the Intercepts from the Graph of an Equation** *(See textbook Example 6)*

6. The point(s), if any, where a graph crosses or touches the x-axis is called the x-intercept. Suppose a graph crosses the x-axis at a point \( a \) and touches the x-axis at a point \( b \). Write the x-intercept(s) as an ordered pair. 

7. The point(s), if any, where a graph crosses or touches the y-axis is called the y-intercept. Suppose a graph crosses the y-axis at a point \( c \). Write the y-intercept(s) as an ordered pair.

---

**Objective 5: Interpret Graphs**

8. \((1, 5)\) is a point on the graph of \( 3x - y = -2 \). If the x-axis represents the number of picnic tables manufactured and sold and the y-axis represents the profit (in tens of dollars) from the sale of those tables, describe the meaning of the ordered pair \((1, 5)\)

9. \((2, 8)\) is a point on the graph of \( y = -x^2 + 3x + 6 \). If the x-axis represents the time (in seconds) after a ball leaves the hand of a thrower and the y-axis represents the height (in feet) above the ground, describe the meaning of the ordered pair \((2, 8)\).
1. Determine the coordinates of each of the points plotted. Tell in which quadrant or on what coordinate axis each point lies.

![Graph with points A, B, and C labeled]

1. A __________  
   B __________  
   C __________

2. Use the graph above to plot each of the following points. Tell in which quadrant or on which coordinate axis each point lies.

   - D(0, 5)  
   - E(2, 3)  
   - F(1, 0)

   2. D __________  
      E __________  
      F __________

3. Determine which of the following are points on the graph of the equation \(-4x + 3y = 18\).

   - (a) (1, 7)  
   - (b) (0, 6)  
   - (c) (-3, 10)  
   - (d) \(\left(\frac{3}{2}, 4\right)\)

   3. __________

4. Determine which of the following are points on the graph of the equation \(x^2 + y^2 = 1\).

   - (a) (0, 1)  
   - (b) (1, 1)  
   - (c) \(\left(\frac{1}{2}, \frac{1}{2}\right)\)  
   - (d) \(\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)\)

   4. __________
5. The graph of the equation is given. List the intercepts of the graph.

5. \( x \)-intercept: __________________

\( y \)-intercept: __________________

6. \( y = x - 2 \)  

7. \( y = x^2 - 2 \)  

8. \( y = |x| + 1 \)  

9. \( 3x + y = -2 \)  

10. If \((a, -2)\) is a point on the graph of \( y - 3x = 5 \), what is \( a \)?  

10. __________

11. If \((-2, b)\) is a point on the graph of \( y = -2x^2 + 3x + 1 \), what is \( b \)?  

11. __________
In Problems 1 and 2, solve each linear equation.

1. $-6x + 36 = 0$

2. $5y + 2 = 0$

In Problems 3 and 4, solve each equation for $y$.

3. $5x - 2y = -20$

4. $\frac{2}{3}x - \frac{1}{2}y = 1$

In Problem 5 and 6, evaluate each expression.

5. $\frac{-3 - (-7)}{6 - 8}$

6. $\frac{-1 - 7}{-6 - (-2)}$

In Problems 7 and 8, use the Distributive Property to simplify each expression.

7. $-3(x - 5)$

8. $\frac{5}{4}(x - 8)$
Guided Practice 1.6
Linear Equations in Two Variables

Objective 1: Graph Linear Equations Using Point Plotting

1. When a linear equation in two variables is written in the form \( Ax + By = C \) where \( A, B, \) and \( C \) are real numbers and both \( A \) and \( B \) cannot be 0, we say the linear equation is written in \( \underline{________________________} \) form.

Objective 2: Graph Linear Equations Using Intercepts

2. To find the \( x \)-intercept(s), if any, of the graph of an equation, let \( _______ \) in the equation and solve for \( x \).

3. To find the \( y \)-intercept(s), if any, of the graph of an equation, let \( _______ \) in the equation and solve for \( y \).

Objective 4: Find the Slope of a Line Given Two Points

4. The ratio of the rise (vertical change) to the run (horizontal change) is called the \( \underline{________________________} \) of the line.

5. The slope of a line which passes through the ordered pairs \( (x_1, y_1) \) and \( (x_2, y_2) \) is given by the formula:

\[
m = \underline{________________________}
\]

6. Properties of Slope  Describe the graph of the line with the following slope:

(a) Positive \( \underline{________________________} \)  \( \text{ (b) Negative } \underline{________________________} \)

(c) Zero \( \underline{________________________} \)  \( \text{ (d) Undefined } \underline{________________________} \)

Objective 7: Use the Point-Slope Form of a Line  (See textbook Example 10)

7. What equation do we use to write the point-slope form of a line with slope \( m \) and containing \( (x_1, y_1) \)?

\[ \underline{________________________} \]

8. Find the equation of a line whose slope is \( -2 \) and that contains the point \( (4, -1) \).

(a) Identify values:

\[ m = \underline{________________________}; \ x_1 = \underline{________________________}; \ y_1 = \underline{________________________} \]

(b) Point-slope form of a line:

\[ \underline{________________________} \]

(c) Substitute the values into (b):

\[ \underline{________________________} \]

(d) Simplify:

\[ \underline{________________________} \]
Objective 9: Find the Equation of a Line Given Two Points *(See textbook Example 12)*

9. Find the equation of the line through the points \((2, -2)\) and \((-2, 6)\). If possible, write the equation in slope-intercept form. Graph the line.

**Step 1:** Find the slope of the line containing the points. 

**Step 2:** Use the point-slope form of a line to find the equation.

**Step 3:** Solve the equation for \(y\).

---

**Step 1:** Find the slope of the line containing the points. 

**Step 2:** Use the point-slope form of a line to find the equation.

**Step 3:** Solve the equation for \(y\).
In Problems 1 – 4, graph each linear equation in two variables.

1. \( x + y = -3 \)

2. \( \frac{1}{4}x + \frac{1}{5}y = 1 \)

3. \( x = 2 \)

4. \( y = -4 \)

In Problems 5 and 6, determine the slope of the line containing the two points.

5. \((3, -1)\) and \((-2, 11)\)

6. \((4, 1)\) and \((4, -3)\)

7. Graph the line containing the point \(P(-1, 4)\) and having slope \(m = 2\).

In Problems 8 – 10, find an equation of the line with the given slope and containing the given point. Express your answer in slope-intercept form.

8. \(m = -1; (0, 0)\)

9. \(m = 4; (2, -1)\)
Do the Math Exercises 1.6

10. \(m = -\frac{4}{3}, (1, -3)\)

11. Find the equation of the line containing \((-5, 1)\) and \((1, -1)\). Express your answer in slope-intercept form.

In Problems 12 – 15, find the slope and \(y\)-intercept of each line.

12. \(y = 3x + 2\)

13. \(2x + y = 3\)

14. \(x = 3\)

15. \(y = -4\)

16. Find a linear function \(g\) such that \(g(1) = 5\) and \(g(5) = 17\). What is \(g(-3)\)?

17. Find a linear function \(F\) such that \(F(2) = 5\) and \(F(-3) = 9\). What is \(F\left(-\frac{3}{2}\right)\)?

18. What type of line has one \(x\)-intercept but no \(y\)-intercept?

19. What type of line has one \(y\)-intercept but no \(x\)-intercept?
Five-Minute Warm-Up 1.7
Parallel and Perpendicular Lines

1. Determine the reciprocal of 1. 2. Determine the reciprocal of $-5$.

3. Determine the reciprocal of $-\frac{3}{2}$. 4. Determine the reciprocal of $\frac{1}{3}$.

5. Solve for $y$: $-7x + 3y = 9$ 6. Find the slope of the line: $4x + y = 3$

7. Find the slope of the line: $x = -7$ 8. Find the slope of the line: $y = 3$
Guided Practice 1.7
Parallel and Perpendicular Lines

Objective 1: Define Parallel Lines

1. In your own words, write a definition of parallel lines.

2. Two lines are parallel if and only if they have the same ______________ and different ______________.

3. If \( L_1 : y = m_1x + b_1 \) and \( L_2 : y = m_2x + b_2 \) and \( L_1 \) is parallel to \( L_2 \), then \( m_1 _____ m_2 \) and \( b_1 _____ b_2 \).

Objective 2: Find Equations of Parallel Lines  (See textbook Example 2)

4. Find an equation for the line that is parallel to \( 3x - 4y = 2 \) and contains the point \((4, -1)\). Graph the lines in the Cartesian plane.

Step 1: Find the slope of the given line by putting the equation in slope-intercept form.

\( \text{(a)} \) ______________________

Step 2: Use the point-slope form of a line with the given point and the slope found in Step 1 to find the equation of the parallel line.

Slope of the line found in Step 1: \( \text{(b)} \) ______________________

Slope of the parallel line: \( \text{(c)} \) ______________________

Identify values: \( \text{(d)} m = ____ ; x_1 = ____ ; y_1 = ____ \)

Point-slope form of a line: \( \text{(e)} \) ______________________

Substitute values into (e): \( \text{(f)} \) ______________________

Step 3: Put the equation in slope-intercept form by solving for \( y \).

Solve for \( y \): \( \text{(g)} \) ______________________

Graph both lines in the Cartesian plane: \( \text{(h)} \)
Objective 3: Define Perpendicular Lines

5. In your own words, write a definition of perpendicular lines.

6. Two lines are perpendicular if and only if the product of their slopes is ______.

7. If \( L_1 : y = m_1x + b_1 \) and \( L_2 : y = m_2x + b_2 \) and \( L_1 \) is perpendicular to \( L_2 \), then \( m_1 \cdot m_2 = \) ____ or \( m_1 = \)____.

Objective 4: Find Equations of Perpendicular Lines (See textbook Example 5)

8. Find an equation of the line that is perpendicular to the line \( 3x - y = 6 \) and contains the point \((-9, 1)\).
Write the equation in slope-intercept form.

**Step 1:** Find the slope of the given line by putting the equation in slope-intercept form.

**Step 2:** Find the slope of the perpendicular line.

Slope of the line found in Step 1: \( m = \)____
Slope of the perpendicular line: \( m = \)____
Identify values: \( m = \)____; \( x_1 = \)____; \( y_1 = \)____

**Step 3:** Use the point-slope form of a line with the given point and the slope found in Step 2 to find the equation of the perpendicular line.

Point-slope form of a line: \( y - y_1 = m(x - x_1) \)
Substitute values into (e):

**Step 4:** Put the equation in slope-intercept form by solving for \( y \).

Solve for \( y \): \( y = \)____
Graph both lines in the Cartesian plane:
Do the Math Exercises 1.7
Parallel and Perpendicular Lines

1. Complete the following chart:

<table>
<thead>
<tr>
<th>Slope of the given line</th>
<th>Slope of a line parallel to the given line</th>
<th>Slope of a line perpendicular to the given line</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. 10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b. 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c. 2/3</td>
<td></td>
<td>-2/3</td>
</tr>
<tr>
<td>d. 0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>e. 0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In Problems 2 and 3, without graphing determine whether the given linear equations are parallel, perpendicular, or neither.

2. \( L_1 : -3x - y = 3 \)
   \( L_2 : x - 3y = 12 \)
   2.

3. \( L_1 : 10x - 3y = 5 \)
   \( L_2 : 5x + 6y = 3 \)
   3.

4. Determine whether the lines containing the following pairs of points are parallel, perpendicular, or neither.
   \( L_1 : (1, -3); (5, -4) \) and \( L_2 : (-1, -3); (-2, -7) \)
   4.

In Problems 5 and 6, find an equation of the line with the given properties. Express your answer in slope-intercept form.

5. Parallel to \( y = -3x + 1 \) through the point \((2, 5)\)
   5.

6. Perpendicular to \( y = 4x + 3 \) through the point \((4, 1)\)
   6.
Do the Math Exercises 1.7

In Problems 7 – 12, find an equation of the line with the given properties. Express your answer in slope-intercept form.

7. Parallel to \( x = -2 \) through the point \( (2, 5) \)  
8. Perpendicular to \(-2x + 5y - 3 = 0\) through the point \( (2, -3) \)

7. ___________  
8. ___________

9. Parallel to \( 2x + y = 5 \) through the point \((-4, 3)\)  
10. Perpendicular to \( y = 8 \) through the point \((2, -4)\)

9. ___________  
10. ___________

11. Perpendicular to \( 3x + y = 1 \) through the point \((3, -1)\)  
12. Parallel to the line \( x + 4y = 2 \) and through the point \((-7, 2)\)

11. ___________  
12. ___________

13. Find \( B \) so that \(-6x + By = 3\) is perpendicular to \( 2x - 3y = 8 \).

13. ___________

14. Geometry In geometry, we learn that a parallelogram is a quadrilateral in which both pairs of opposite sides are parallel. Determine whether the points \( A = (-2, -1), \ B = (4, 1), \ C = (5, 5), \) and \( D = (-1, 3) \) are the vertices of a parallelogram. What is the slope of each of the sides of the quadrilateral?

14. slope of:

\[ \frac{\overline{AB}}{} \]  
\[ \frac{\overline{BC}}{} \]  
\[ \frac{\overline{CD}}{} \]  
\[ \frac{\overline{AD}}{} \]  

Is ABCD a parallelogram? ___________
Five-Minute Warm-Up 1.8
Linear Inequalities in Two Variables

In Problems 1 and 2, determine whether the given value satisfies the inequality. (Yes or No)

1. \(-4x + 1 \geq -3; \quad x = 1\)

2. \(-y - 15 < 3(2y - 1); \quad y = -2\)

In Problems 3 – 6, solve the inequality in one variable.

3. \(2x - 2 \geq 3 + x\)

4. \(8 - 4(2 - x) \leq -2x\)

5. \(n + 3 > \frac{5}{2}(n - 6)\)

6. \(\frac{a}{6} < 2 + \frac{a}{3}\)
Guided Practice 1.8
Linear Inequalities in Two Variables

**Objective 1: Determine Whether an Ordered Pair Is a Solution to a Linear Inequality**

1. **True or False** To determine if an ordered pair is a solution to the linear inequality, substitute the values for the variables into the inequality. If a true statement results, then the ordered pair is a solution to the inequality.  

   1. _________

**Objective 2: Graph Linear Inequalities** *(See textbook Example 2)*

2. To graph any linear inequality in two variables, you must first graph the corresponding equation.

   (a) If the inequality is strict (< or >), you should use a _______________ line.

   (b) If the inequality is nonstrict (≤ or ≥), you should use a _______________ line.

3. The graph of a line separates the xy-plane into two ___________________________________.

4. If a test point satisfies the inequality, then every ordered pair that lies in that half plane also satisfies the inequality. To represent this solution set, we ________________ the half plane containing the test point.

5. Graph the linear inequality 3x − 4y > 12.

   **Step 1:** We replace the inequality symbol with an equal sign and graph the corresponding line.

   Write the equation:  
   (a) _________

   Identify the x-intercept:  
   (b) _________

   Identify the y-intercept:  
   (c) _________

   Graph the line. Is the line connecting these points solid or dashed?  
   (d) _________

   **Step 2:** We select any test point that is not on the line and determine whether the test point satisfies the inequality. When the line does not contain the origin, it is usually easiest to choose the origin, (0, 0), as the test point.

   Select a test point:  
   (e) _______  
   3x − 4y > 12

   Substitute the values for the variables into the inequality. Is the statement true or false?  
   (f) _________

   Which half plane should be shaded; the half plane containing the test point or the opposite half plane?  
   (g) _________

   **Exercise 5 continued…**
Graph the solution set by plotting the intercepts, graphing the line, and shading the appropriate half plane.

3x – 4y > 12

**Objective 3: Solve Problems Involving Linear Inequalities** *(See textbook Example 4)*

6. **Salesperson** Juanita sells two different computer models. For each Model A computer sold she makes $45 and for each Model B computer sold she makes $65. Juanita set a monthly goal of earning at least $4000.

(a) Write a linear inequality that describes Juanita’s options for making her sales goal.

**Step 1: Identify** We want to determine how many of each model should be sold so that she will earn at least $4000. This requires an inequality in two variables.

**Step 2: Name the Unknowns** Let $x$ represent the number of Model A sold and let $y$ represent the number of Model B sold.

**Step 3: Translate** If Model A sells for $45 each and Model B sells for $65 each, write an inequality the represents her total income greater than or equal to $4000.

(a) ___________________

(b) Will Juanita makes her sales goal if she sells 50 Model A and 28 Model B computers? ____________

(c) Will Juanita makes her sales goal if she sells 41 Model A and 33 Model B computers? ____________
Do the Math Exercises 1.8
Linear Inequalities in Two Variables

In Problems 1 and 2, determine which, if any, of the following points are solutions to the linear inequality.

1. \[ 2x + y > -3 \]
   (a) (2, −1)  (b) (1, −3)  (c) (−5, 4)

2. \[ 2x - 5y \leq 2 \]
   (a) (1, 2)  (b) (3, 0)  (c) (−3, −2)

In Problems 3 – 8, graph each inequality.

3. \[ y < -2 \]

4. \[ x \geq -3 \]

5. \[ y \geq -\frac{4}{3}x + 5 \]

6. \[ -4x + y \geq -5 \]

7. \[ \frac{x}{3} - \frac{y}{4} > 1 \]

8. \[ -4x + 6y > 24 \]
In Problems 9 – 13, translate each statement into a linear inequality.

9. One number, \(x\), is at most 12 more than a second number, \(y\).

10. The sum of two numbers, \(x\) and \(y\), is at least \(-3\).

11. Budget Constraints  Johnny can spend no more than $3.00 that he got from his grandparents. He goes to the candy store and wants to buy gummy bears that cost $0.10 each and suckers that cost $0.25 each.

   (a) Write a linear inequality that describes Johnny’s options for buying candy. Let \(g\) represent the number of gummy bears and \(s\) represent the number of suckers.

   (b) Can Johnny buy 18 gummy bears and 5 suckers?

   (c) Can Johnny buy 19 gummy bears and 4 suckers?

12. Fund Raising  For a fund-raiser, the Math club agrees to sell candy bars and candles. The club’s profit will be 50¢ for each candy bar it sells and $2.00 for each candle it sells. The club needs to earn at least $1000 in order to pay for an upcoming field trip.

   (a) Write a linear inequality that describes the combination of candy bars and candles that must be sold. Let \(x\) represent the number of candy bars sold and \(y\) represent the number of candles sold.

   (b) Will selling 500 candy bars and 350 candles earn enough for the trip?

   (c) Will selling 600 candy bars and 400 candles earn enough for the trip?

13. Determine the linear inequality whose graph is:
1. Write each inequality using interval notation.

   (a) \(-1 < x \leq 0\)       (b) \(4 \leq x < 7\)

   1a. __________

   1b. __________

2. Write each inequality using interval notation.

   (a) \(x \leq -4\)       (b) \(x > -2\)

   2a. __________

   2b. __________

3. Plot the ordered pairs in the rectangular coordinate system.

   \(A(0, -3)\); \(B(-2, -4)\); \(C(1, -1)\); \(D(2, 0)\); \(E(-4, 3)\)

4. Graph the equation \(5x + 3y = -15\).

5. Graph the equation \(y = -x^2 + 2\) by plotting points.
Guided Practice 2.1
Relations

Objective 1: Understand Relations

1. In your own words, write a definition for a relation.

Objective 2: Find the Domain and the Range of a Relation (See textbook Example 4)

2. The domain is set of all ______________ and is set of _____ -coordinates for the relation which is defined by the set of ordered pairs \((x, y)\).

3. The range is set of all ______________ and is set of _____ -coordinates for the relation which is defined by the set of ordered pairs \((x, y)\).

4. Name four different ways to represent a relation.

________________     __________________     __________________     __________________

In Problems 5 and 6, the figure shows the graph of a relation. Determine (a) the domain and (b) the range of the relation.

5. __________________ 6. __________________

5a. __________ 5b. __________ 6a. __________ 6b. __________
Objective 3: Graph a Relation Defined by an Equation (See textbook Example 5)

7. Graph the relation \( x = y^2 - 3 \). Use the graph to determine (a) the domain and (b) the range of the relation.

<table>
<thead>
<tr>
<th>( x = y^2 - 3 )</th>
<th>( y )</th>
<th>( (x, y) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( -2 )</td>
<td>( y )</td>
<td>( (x, y) )</td>
</tr>
<tr>
<td>( -1 )</td>
<td>( y )</td>
<td>( (x, y) )</td>
</tr>
<tr>
<td>( 0 )</td>
<td>( y )</td>
<td>( (x, y) )</td>
</tr>
<tr>
<td>( 1 )</td>
<td>( y )</td>
<td>( (x, y) )</td>
</tr>
<tr>
<td>( 2 )</td>
<td>( y )</td>
<td>( (x, y) )</td>
</tr>
</tbody>
</table>

7a. __________

7b. __________
Write the relation as a set of ordered pairs. Then identify the domain and range.

1. _______________________________
   domain ___________________________
   range ____________________________

In Problems 2 – 4, identify the domain and the range of each relation.

2. \( \{(−2, 6), (−1, 3), (0, 0), (1, −3), (2, 6)\} \)
   domain ___________________________
   range ____________________________

3. \( \{(−2, −8), (−1, −1), (0, 0), (1, 1), (2, 8)\} \)
   domain ___________________________
   range ____________________________

4. \( \{(−3, 0), (0, 3), (3, 0), (0, −3)\} \)
   domain ___________________________
   range ____________________________

In Problems 5 – 8, identify the domain and range of the relation from the graph.

5. domain ___________________________
   range ____________________________

6. domain ___________________________
   range ____________________________
In Problems 9 – 14, identify the domain and range of each relation. You may want to draw a graph by point-plotting or by using your graphing calculator.

9. \( y = x - 2 \)
   - Domain: ___________________________
   - Range: ___________________________

10. \( y = x^2 - 2 \)
    - Domain: ___________________________
    - Range: ___________________________

11. \( y = -2x^2 + 8 \)
    - Domain: ___________________________
    - Range: ___________________________

12. \( y = |x| - 2 \)
    - Domain: ___________________________
    - Range: ___________________________

13. \( x = y^2 + 2 \)
    - Domain: ___________________________
    - Range: ___________________________

14. \( y = x^3 - 4 \)
    - Domain: ___________________________
    - Range: ___________________________

15. Bob Villa wishes to put a new window in his home. He wants the perimeter of the window to be 100 feet. The graph shows the relation between the width, \( x \), of the opening and the area of the opening.

   (a) Determine the domain and the range of the relation. 

   (b) Explain why the domain obtained in part (a) is reasonable.
In Problems 1 and 2, evaluate the expression for the given value of the variable.

1. \(-\frac{4}{5}x - 3\) for \(x = -25\)  
2. \(-2x^2 + 3x - 1\) for \(x = -1\)

3. Express the inequality \(x > -16\) using interval notation.

4. Express the interval \((-\infty, 100]\) using set-builder notation.

5. For the following set of ordered pairs, list (a) the domain and (b) the range.
   
   \(\{(−2, 1), (−4, 3), (−6, 5), (−8, 7)\}\)
   
   5a. domain _______________________
   5b. range ________________________

In Problems 6 – 8, evaluate each expression.

6. \(\frac{2^2 - 5}{-5}\)  
7. \(-4(-1)^2 + 6(-1) + 5\)  
8. \(\frac{4 - (-20)}{-2^3}\)

9. The volume of a right circular cylinder is given by the formula \(V = \pi r^2 (r + 3)\) where \(r\) is the radius of the cylinder and whose height is 3 inches more than the radius. Find the volume of a cylinder whose radius is 2.5 inches. Round your answer to the nearest tenth of a cubic inch.
**Objective 1: Determine Whether a Relation Expressed as a Map or Ordered Pairs Represents a Function**

1. Explain what a *function* is. Be sure to include the terms *domain* and *range* in your explanation.

2. Determine whether the relation represents a function. If the relation is a function, then state its domain and range. *(See textbook Example 2)*

   (a) \(\{(3, -6), (4, -1), (5, -6)\}\)

   (b) \(\{(4, 5), (3, -3), (4, -1)\}\)

**Objective 2: Determine Whether a Relation Expressed as an Equation Represents a Function**

3. The symbol \(\pm\) is a shorthand device and is read “plus or minus.” Write the two equations that are represented by \(y = \pm 2x\) and then determine whether the equation \(y = \pm 2x\) is a function.

   (a) \(y = \pm 2x\) means \(y = _____\) and also \(y = _____\). (b) Is \(y\) a function of \(x\)?

4. Determine whether each equation shows \(y\) as a function of \(x\). *(See textbook Examples 3 and 4)*

   (a) \(y = 9\)

   (b) \(x + y^2 = 2\)

   4a. ________

   4b. ________

**Objective 3: Determine Whether a Relation Expressed as a Graph Represents a Function**

5. We use the Vertical Line Test to determine whether the graph of a relation is a function. In your own words, state the *Vertical Line Test (VLT)*.

6. Which of the following are graphs of functions? *(See textbook Example 5)*

   (a) ________

   (b) ________

   (c) ________

   (d) ________
Objective 4: Find the Value of a Function

7. For the function \( y = f(x) \), the variable \( x \) is called the ______________ variable, because it can be assigned any of the values in the ______________.

8. For the function \( y = f(x) \), the variable \( y \) is called the ______________ variable, because it can be assigned any of the values in the ______________.

In Problems 9 – 12, find the value of each function. (See textbook Example 6 and Example 7)

9. \( f(x) = 2x^2 - 5x; \quad f(-4) \)  
10. \( g(x) = -3x + 2; \quad g(2a) \)

11. \( h(t) = 4; \quad h(5) \)  
12. \( F(z) = -3z + 4; \quad F(k + 1) \)

Objective 5: Find the Domain of a Function

13. The domain of a function is the set of all inputs for which the function gives an output that is a real number or makes sense. When identifying the domain of a function we exclude values of the variable that causes division by __________.

14. Find the domain of the function: \( f(x) = \frac{2x}{2x + 1} \) (See textbook Example 8)

Objective 6: Work with Applications of Functions

15. The number \( N \) of trucks produced at a certain factory in 1 day after \( t \) hours of operation is given by \( N(t) = 80t - 4t^2 \), where \( 0 \leq t \leq 8 \). (See textbook Example 10)

(a) Identify the dependent variable.  
15a. __________

(b) Identify the independent variable.  
15b. __________

(c) Evaluate \( N(5) \) and explain the meaning of \( N(5) \).  
15c. __________
Do the Math Exercises 2.2
An Introduction to Functions

In Problems 1 – 3, determine whether the relation represents a function (Yes or No).

1. Grade on exam

<table>
<thead>
<tr>
<th>Grade</th>
<th>Study time (hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>5</td>
</tr>
<tr>
<td>B</td>
<td>6</td>
</tr>
<tr>
<td>C</td>
<td>4</td>
</tr>
<tr>
<td>D</td>
<td>3.5</td>
</tr>
</tbody>
</table>

1. ___________

2. \{(-2, 3), (-2, 1), (-2, -3), (-2, 9)\}

2. ___________

3. \{(-5, 3), (-2, 1), (5, 1), (7, -3)\}

3. ___________

In Problems 4 – 7, determine whether each equation shows y as a function of x. (Yes or No)

4. \(y = -6x + 3\)

4. ___________

5. \(y = \pm 4x\)

5. ___________

6. \(y = x^2 + 2\)

6. ___________

7. \(y^2 = x\)

7. ___________

In Problems 8 – 11, determine whether the graph is that of a function (Yes or No).

8. ___________

9. ___________

10. ___________

11. ___________
Do the Math Exercises 2.2

In Problems 12 and 13, find the following values for each function.

(a) \( f(3) \)      (b) \( f(-2) \)      (c) \( f(2x) \)      (d) \( f(x+2) \)

12. \( f(x) = 3x + 1 \)  13. \( f(x) = -2x - 3 \)

12a. 
12b. 
12c. 
12d. 
13a. 
13b. 
13c. 
13d. 

In Problems 14 and 15, find the domain of each function.

14. \( f(x) = -2x^2 + x + 1 \)  15. \( h(q) = \frac{3q^2}{q + 2} \)

14. 
15. 

16. If \( f(x) = -2x^2 + 5x + C \) and \( f(-2) = -15 \), what is the value of \( C \)?

16. 

17. If \( f(x) = \frac{-x + B}{x - 5} \) and \( f(3) = -1 \), what is the value of \( B \)?

17. 

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In Problems 1 and 2, solve each equation.

1. \(-4x + 2 = 0\)  
2. \(-18 + 3y = 0\)  

1. ________  
2. ________  

3. Graph the equation \(y = -\frac{4}{3}x + 2\).  
4. Graph the equation \(y = x^2 - 4\) by plotting points.

In Problems 5 and 6, determine the (a) domain and (b) the range from the graph.

5.  
6.  

5a. ________  
5b. ________  
6a. ________  
6b. ________
Guided Practice 2.3
Functions and Their Graphs

**Objective 1: Graph a Function**

1. Complete the table and graph the function \( f(x) = |2x - 4| \). *(See textbook Example 1)*

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
<th>((x, f(x)))</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Objective 2: Obtain Information from the Graph of a Function**

2. Use the graph of the function to answer parts (a) – (c). *(See textbook Example 2)*

(a) Determine the domain of the function. 2a. __________

(b) Determine the range of the function. 2b. __________

(c) Identify the x-intercept(s). 2c. __________

(d) Identify the y-intercept(s). 2d. __________
3. Consider the function \( f(x) = -\frac{5}{2}x + 8 \). (See textbook Example 4)
   (a) Is the point \((4, -2)\) on the graph of the function? 3a. 
   (b) If \( x = 6 \), what is \( f(x) \)? What point is on the graph of the function? 3b. 
   (c) If \( f(x) = 3 \), what is \( x \)? What point is on the graph of \( f \)? 3c. 

4. The zeros of a function are also the ___-intercepts of the graph of the function. To find the zeros of a function we set the function equal to _________ and solve for \( x \).

5. Find the zeros of the function \( f \) whose graph is shown below. (See textbook Example 5) 5. 

![Graph of a function with points (-2, 0), (2, 0), and (0, -3).]

Objective 3: Know Properties and Graphs of Basic Functions

6. List the 6 basic functions described in Table 3 and briefly describe each graph.
   (a) ____________________________
   (b) ____________________________
   (c) ____________________________
   (d) ____________________________
   (e) ____________________________
   (f) ____________________________
Do the Math Exercises 2.3
Functions and Their Graphs

In Problems 1 – 5, find the domain of each function.

1. \( G(x) = -8x + 3 \) 
2. \( H(x) = \frac{x + 5}{2x + 1} \)

3. \( s(t) = 2t^2 - 5t + 1 \) 
4. \( H(q) = \frac{1}{6q + 5} \)

5. \( f(x) = \frac{4x - 9}{7} \)

In Problems 6 and 7, graph each function.

6. \( F(x) = x^2 + 1 \) 
7. \( H(x) = |x + 1| \)

In Problems 8 and 9, find (a) the domain, (b) the range, (c) the intercepts, if any, and (d) the zeros, if any.

8. 

9. 

8a. _________
8b. _________
8c. _________
8d. _________
9a. _________
9b. _________
9c. _________
9d. _________
10. Use the table of values for the function \( G \) to answer the following:

<table>
<thead>
<tr>
<th>( x )</th>
<th>( G(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5</td>
<td>-3</td>
</tr>
<tr>
<td>-4</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>7</td>
<td>5</td>
</tr>
</tbody>
</table>

(a) What is \( G(3) \)?

(b) For what number(s) is \( G(x) = 5 \)?

(c) What is the \( x \)-intercept of the graph of \( G \)?

(d) What is the \( y \)-intercept of the graph of \( G \)?

(e) Are there any zeros of the function \( G \)? If so, name the zero(s).

11. Use the function \( f(x) = 3x + 5 \) to answer the following:

(a) Is the point \((-2, 1)\) on the graph of the function?

(b) If \( x = 4 \), what is \( f(x) \)? What point is on the graph of the function?

(c) If \( f(x) = -4 \), what is \( x \)? What point is on the graph of the function?

(d) Is \( x = 0 \) a zero of the function?

12. Geometry

The volume \( V \) of a sphere as a function of its radius \( r \) is given by

\[
V(r) = \frac{4}{3} \pi r^3
\]

(a) What is the domain of this function?

(b) Find the volume of the sphere whose radius is \( 4\frac{1}{2} \) cm. Round your answer to one decimal place.

13. A piecewise-defined function is a function defined by more than one equation. For example, the absolute value function \( f(x) = |x| \) is actually defined by two equations:

\[ f(x) = x \text{ if } x \geq 0 \text{ and } f(x) = -x \text{ if } x < 0. \]

We can combine these equations into one piecewise-defined function written as

\[
f(x) = \begin{cases} 
  x & \text{for } x \geq 0 \\
  -x & \text{for } x < 0 
\end{cases}
\]

To evaluate \( f(3) \), we recognize that \( 3 \geq 0 \), so we use the rule \( f(x) = x \) and obtain \( f(3) = 3 \). To evaluate \( f(-4) \), we recognize that \( -4 < 0 \), so we use the rule \( f(x) = -x \) and obtain \( f(-4) = -(-4) = 4 \).

Given \( f(x) = \begin{cases} 
  x + 3 & \text{for } x < 0 \\
  -2x + 1 & \text{for } x \geq 0 
\end{cases} \), find each of the following:

(a) \( f(3) \)

(b) \( f(-2) \)

(c) \( f(0) \)
Five-Minute Warm-Up 2.4
Linear Functions and Models

In Problems 1 – 2, graph each linear equation.

1. \( \frac{2}{3}x - \frac{1}{2}y = -2 \)

2. \( x = -1 \)

3. Find the equation of the line through \((-1, 2)\) and \((2, -7)\).

4. \( 6.5 - 1.5(x - 4) + 4x = 10.25 \)

5. \( -\frac{7}{8}(2y + 3) \leq \frac{5}{4}(y - 2) \)
Guided Practice 2.4
Linear Functions and Models

Objective 1: Graph Linear Functions

1. A ______ function is a function of the form \( f(x) = mx + b \) where \( m \) and \( b \) are real numbers. The graph of a linear function is called a ________.

2. To graph a linear function, we use the same technique used to graph a linear equation written in slope-intercept form, \( y = mx + b \), where \( m \) is the ________ and \((0, b)\) is the ________.

3. Graph the linear function \( f(x) = \frac{3}{2}x - 2 \). (See textbook Example 1)
   
   (a) Identify the \( y \)-intercept. 3a. ______
   
   (b) Identify the slope. 3b. ______
   
   (c) Graph the function.

Objective 2: Find the Zero of a Linear Function

4. To find the zero of a linear function \( f(x) = mx + b \), we solve the equation ____________________.

5. Perimeter of a Rectangle  In a given rectangle, the length is 3 ft less than twice the width. If \( x \) represents the width of the rectangle, the perimeter can be calculated by the function \( P(x) = 2x + 2(2x - 3) \). (See textbook Example 3)
   
   (a) What is the implied domain of the function? 5a. ______
   
   (b) What is the perimeter of a rectangle whose width is 12 ft? 5b. ______
   
   (c) What is the width of a rectangle whose perimeter is 84 ft? 5c. ______
   
   (d) For what width of the rectangle will the perimeter exceed 12 feet? 5d. ______
**Objective 3: Build Linear Models from Verbal Descriptions**

6. The linear cost function is \( C(x) = ax + b \), where \( b \) represents the \__________\ costs of operating a

   business and \____\ represents the costs associated with manufacturing one additional item.  \( \text{(See textbook Example 4)} \)

7. Some companies use *straight-line depreciation* to depreciate their assets so that the value of the asset

   declines by a constant amount each year. To calculate the amount the asset depreciates each year, divide the

   total cost by the number of years of useful life.

A cab company bought a new car for $22,500 and plans to drive it until there is no scrap value. The life of a

   car in the cab fleet is 5 years.  \( \text{(See textbook Example 5)} \)

   (a) By how much does the car depreciate each year?  \( 7a. \) \__________

   (b) This rate can be expressed as the slope of a linear function. Is this slope positive

       or negative?  \( 7b. \) \__________

   (c) The *book value* is the value of the asset at a particular time. To find the book value,

       we take the original value and deduct the amount of depreciation after a given time.

       Write a linear function that expresses the book value \( V \) of the car as a function of its

       age, \( x \).  \( 7c. \) \__________

   (d) What is the implied domain of the linear function?  \( 7d. \) \__________

   (e) What is the book value after 4 years?  \( 7e. \) \__________

   (f) When will the book value of the car be $10,125?  \( 7f. \) \__________

   (g) What is the independent variable?  \( 7g. \) \__________

   (h) What is the dependent variable?  \( 7h. \) \__________

**Objective 4: Build Linear Models from Data**

8. How many points are needed to determine the equation of a line?  \( 8. \) \__________

9. If the data appears to be linearly related, we select two points on the line of best fit. The linear model

   can be determined by finding the equation of the line through the two points as described in Section 1.6,

   Example 12.

   (a) To find the equation of the line, you must first determine the \__________.

   (b) Second, use the point-slope form of a line, \__________\, to find the equation.
Do the Math Exercises 2.4
Linear Functions and Models

In Problems 1 – 4, graph each linear function.

1. \( f(x) = 2x - 4 \)
2. \( g(x) = -\frac{3}{2}x + 1 \)
3. \( h(x) = \frac{1}{4}x + 2 \)
4. \( F(x) = 3 \)

In Problems 5 and 6, find the zeros of the linear function.

5. \( f(x) = 3x - 24 \) \hspace{1cm} 6. \( g(x) = -\frac{3}{2}x + 6 \)

7. Suppose that \( f(x) = \frac{4}{3}x + 5 \) and \( g(x) = \frac{1}{3}x + 1 \).
   (a) Solve \( f(x) = g(x) \).
   \hspace{1cm} 7a. \hspace{0.5cm} \\
   (b) What is the value of \( f \) at the solution?
   \hspace{1cm} 7b. \hspace{0.5cm} \\
   (c) What is the value of \( g \) at the solution?
   \hspace{1cm} 7c. \hspace{0.5cm} \\
   (d) Solve \( f(x) > g(x) \).
   \hspace{1cm} 7d. \hspace{0.5cm} 
8. Graph \( f(x) = \frac{4}{3}x + 5 \) and \( g(x) = \frac{1}{3}x + 1 \) on the same Cartesian plane. Label the intersection point.

9. (a) Find a linear function \( g \) such that \( g(1) = 5 \) and \( g(5) = 17 \). (b) What is \( g(-3) \)?

9a. 
9b. 

10. Birth Rate A multiple birth is any birth with 2 or more children born. The birth rate is the number of births per 1,000 women. The birth rate \( B \) of multiple births as a function of age \( a \) is given by the function \( B(a) = 1.73a - 14.56 \) for \( 15 < a < 44 \). [Source: Centers for Disease Control]

(a) What is the independent variable?  
10a. 

(b) What is the dependent variable?  
10b. 

(c) What is the domain of this linear function?  
10c. 

(d) What is the multiple birth rate of women who are 22 years of age according to the model?  
10d. 

(e) What is the age of women whose multiple birth rate is 49.45?  
10e. 

11. A strain of \( E. coli \) Beu 397-reca441 is placed into a Petri dish at 30° Celsius and allowed to grow. The population is estimated by means of an optical device in which the amount of light that passes through the Petri dish is measured. The data below was collected. Do you think that a linear function could be used to describe the relation between the two variables? Why or Why not?

<table>
<thead>
<tr>
<th>Time, ( x )</th>
<th>Population, ( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.09</td>
</tr>
<tr>
<td>2.5</td>
<td>0.18</td>
</tr>
<tr>
<td>3.5</td>
<td>0.26</td>
</tr>
<tr>
<td>4.5</td>
<td>0.35</td>
</tr>
<tr>
<td>6</td>
<td>0.50</td>
</tr>
</tbody>
</table>
Five-Minute Warm-Up 2.5
Compound Inequalities

1. Use (a) set-builder notation and (b) interval notation to list the set of all real numbers \( x \) such that \(-4 \leq x < -1\). (c) Graph the inequality.

   1a. __________  
   1b. __________

2. Graph the inequality \( x < -3 \).

3. Use interval notation to express the inequality shown in the graph.

   3. __________

4. Solve \( 2(x + 4) = x - (3x + 4) \).

   4. __________

5. Solve \( \frac{3}{8}x + 1 \leq -\frac{5}{4} \).

   5. __________

6. Solve \(-2(1 - x) + 2x < 7x + 4 \).

   6. __________
Guided Practice 2.5
Compound Inequalities

Objective 1: Determine the Intersection or Union of Two Sets
1. The intersection of two sets $A$ and $B$, denoted __________, is the set of all elements that belong to both set $A$ and set $B$. The word __________ implies intersection.

2. The union of two sets $A$ and $B$, denoted __________, is the set of all elements that are in set $A$ or in set $B$ or in both $A$ and $B$. The word __________ implies union.

3. Suppose $A = \{x|x > -2\}$, $B = \{x|x \leq 1\}$, and $C = \{x|x \geq 3\}$. (See textbook Example 2)
   (a) Determine $A \cap B$. Graph the solution set on a real number line.

   **Step 1:** Graph $A = \{x|x > -2\}$
   
   **Step 2:** Graph $B = \{x|x \leq 1\}$

   **Step 3:** Identify where the graph of the inequalities overlap.

   (b) Determine $B \cup C$. Graph the solution set on a real number line.

   **Step 1:** Graph $B = \{x|x \leq 1\}$

   **Step 2:** Graph $C = \{x|x \geq 3\}$

   **Step 3:** Identify the solution as the set that is in either $B$ or $C$.

Objective 2: Solve Compound Inequalities Involving “and” (See textbook Example 3)

4. Solve $5x - 1 < 9$ and $5x \geq -20$. Graph the solution set.

   **Step 1:** Solve each inequality separately.
   
   Add 1 to both sides:
   $5x - 1 < 9$
   (a) _________________
   (b) _________________

   Divide both sides by 5:
   $5x \geq -20$
   (c) _________________

   **Step 2:** Find the intersection of the solution sets, which will represent the solution set to the compound inequality.

   Graph (b):
   
   (d) _________________

   Graph (c):
   
   (e) _________________

   Graph the intersection (overlap):
   
   (f) _________________
5. If \(a < b\), then we can write \(a < x\) and \(x < b\) more compactly as \_______________.

6. Solve \(-8 \leq 5x - 3 \leq 7\) and graph the solution set. (*See textbook Example 6*)

Our goal is to get the variable by itself in the “middle” with a coefficient of 1. Remember that the inequality is always written in order, the smaller number on left and the larger number on the right side of the inequality.

\(-8 \leq 5x - 3 \leq 7\)

(a) Add 3 to all three parts:

\(_8^{\phantom{1}} \leq 5x \leq 7^{\phantom{1}}\)

(b) Simplify:

\______________

(c) Divide all three parts by 5 and simplify:

\______________

(d) Graph the solution set:

\[-4 -2 0 2 4\]

**Objective 3: Solve Compound Inequalities Involving “or” (See textbook Example 7)**

7. Solve \(-\frac{3}{2}x + 6 > 9\) or \(7x - 10 > 4\). Write the solution set in interval notation.

**Step 1:** Solve each inequality separately.

(a) \(-\frac{3}{2}x + 6 > 9\)

Subtract 6 from both sides:

\______________

(b) \(7x - 10 > 4\)

Multiply both sides by \(-\frac{2}{3}\):

Don’t forget to reverse the direction of the inequality.

\______________

Add 10 to both sides:

\______________

(c) \(7x - 10 > 4\)

Divide both sides by 7:

\______________

**Step 2:** Find the union of the solution sets, which will represent the solution set to the compound inequality.

Graph (b):

\[-4 -2 0 2 4\]

Graph (d):

\[-4 -2 0 2 4\]

Graph the union:

\[-4 -2 0 2 4\]

Write the solution in interval notation:

\______________
Do the Math Exercises 2.5
Compound Inequalities

In Problems 1 – 3, use \( A = \{4, 5, 6, 7, 8, 9\} \), \( B = \{1, 5, 7, 9\} \), and \( C = \{2, 3, 4, 6\} \) to find each set.

1. \( A \cup C \)  
   2. \( A \cap C \)  
   3. \( B \cap C \)

In Problems 4 and 5, use \( E = \{x \mid x \leq 2\} \) and \( F = \{x \mid x \geq -2\} \) to find each of the following.

4. \( E \cap F \)  
5. \( E \cup F \)

In Problems 6 – 17, solve each compound inequality. Express your answer in interval notation.

6. \( x \leq 5 \) and \( x > 0 \)  
7. \( x < 0 \) or \( x \geq 6 \)

8. \( 7x + 2 \geq 9 \) and \( 4x + 3 \leq 7 \)  
9. \( x + 3 \leq 5 \) or \( x - 2 \geq 3 \)

10. \( -12 < 7x + 2 \leq 6 \)  
11. \( 3x \geq 7x + 8 \) or \( x < 4x - 9 \)

12. \( -\frac{4}{5}x - 5 > 3 \) or \( 7x - 3 > 4 \)  
13. \( 0 < \frac{3}{2}x - 3 \leq 3 \)
Do the Math Exercises 2.5

14. \( x - \frac{3}{2} \leq \frac{5}{4} \) and \( -\frac{2}{3}x - \frac{2}{9} < \frac{8}{9} \)  

15. \(-3 < -4x + 1 < 17\)

16. \(-4 \leq \frac{4x - 3}{3} < 3\)

17. \(-15 < -3(x + 2) \leq 1\)

In Problem 18, use the Addition Property and/or Multiplication Properties to find a and b.

18. If \(-4 < x < 3\), then \(a < 2x - 7 < b\).

19. Diastolic Blood Pressure  Blood pressure is measured using two numbers. One of the numbers measures diastolic blood pressure. The diastolic blood pressure represents the pressure while the heart is resting between beats. In a healthy person, the diastolic blood pressure should be greater than 60 and less than 90. If we let the variable \(x\) represent a person’s diastolic blood pressure, express the diastolic blood pressure of a healthy person using compound inequality.

20. Heating Bills  For usage above 300 kilowatt-hours, the non-space heat winter energy charge for Illinois Power residential service was $23.12 plus $0.05947 per kilowatt-hour over 300. During one winter, a customer’s charge ranged from a low of $50.28 to a high of $121.43. Over what range of values did electric usage vary (in kilowatt-hours)?
Five-Minute Warm-Up 2.6
Absolute Value Equations and Inequalities

1. Evaluate each expression.
   (a) $|−12|$  
   (b) $|0|$  
   (c) $\frac{3}{4}$  
   (d) $|−5.2|$  
   1a. __________  
   1b. __________  
   1c. __________  
   1d. __________

2. Express the distance between the origin, 0, and 45 as an absolute value.
   2. ___________

3. Express the distance between the origin, 0, and $−12$ as an absolute value.
   3. __________

4. Solve each equation.
   (a) $−3x + 7 = −5$  
   (b) $4(x + 1) = x + 5x − 10$  
   4a. __________  
   4b. __________

5. Solve each inequality.
   (a) $6x − 10 < 8x + 2$  
   (b) $\frac{1}{2}(3x − 1) ≤ \frac{2}{3}(x + 3)$  
   5a. __________  
   5b. __________
Guided Practice 2.6
Absolute Value Equations and Inequalities

Objective 1: Solve Absolute Value Equations

1. If $a$ is a positive real number and if $u$ is any algebraic expression, then $|u| = a$ is equivalent to __________________ or __________________.

2. When solving absolute value equations the first step is to ________________________________.

3. Solve the equation $|3x - 1| - 5 = -3$. (See textbook Example 2)

   **Step 1:** Isolate the expression containing the absolute value.
   Add 5 to both sides:

   $|3x - 1| - 5 = -3$
   (a) ______________________________

   **Step 2:** Rewrite the absolute value equation as two equations: $u = a$ and $u = -a$, where $u$ is the algebraic expression in the absolute value symbol. Here $u = 3x - 1$ and $a = 2$.

   (b) __________________ or __________________

   **Step 3:** Solve each equation.

   (c) $x = _______ \text{ or } x = _______$

   **Step 4:** Check. Verify each solution.
   Substitute your values for $x$ into the original equation. If the statement is true, then the value is a solution of the absolute value equation. If the statement is false, delete the value from the solution set.

   (d) solution set: __________________

4. If $u$ and $v$ are any algebraic expressions, then $|u| = |v|$ is equivalent to __________________ or __________________.

Objective 2: Solve Absolute Value Inequalities Involving $<$ or $\leq$

5. If $a$ is a positive real number and if $u$ is any algebraic expression, then

   $|u| < a$ is equivalent to __________________________.

   $|u| \leq a$ is equivalent to __________________________.
6. Solve the inequality $|4x - 3| \leq 9$. Write the solution set in interval notation. (See textbook Example 6)

**Step 1:** The inequality is in the form $|u| \leq a$ where $u = 4x - 3$ and $a = 9$. We rewrite the inequality as a compound inequality that does not involve absolute value.

$|4x - 3| \leq 9$  

Use the fact that $|u| \leq a$ means $-a \leq u \leq a$:

(a) ______________________

**Step 2:** Solve the resulting compound inequality.

Add 3 to all three parts:

(b) ______________________

Divide all three parts of the inequality by 4:

(c) ______________________

Graph the solution:

(d) ______________________

Write the solution in interval notation:

(e) ______________________

**Objective 3: Solve Absolute Value Inequalities Involving > or ≥**

7. If $a$ is a positive real number and if $u$ is any algebraic expression, then

$|u| > a$ is equivalent to __________ or __________.

$|u| \geq a$ is equivalent to __________ or __________.

8. Solve the inequality $3|8x + 3| > 9$. Write the solution set in interval notation. (See textbook Example 9)

**Step 1:** The inequality is in the form $|u| > a$ where $u = 8x + 3$ and $a = 3$. We rewrite the inequality as a compound inequality that does not involve absolute value.

$3|8x + 3| > 9$  

Isolate the absolute value:

(a) ______________________

Rewrite the inequality:

(b) ______________________

**Step 2:** Solve each inequality separately.

(c) __________ or __________

**Step 3:** Find the union of the solution sets of each inequality.

Graph the solution:

(d) ______________________

Write the solution in interval notation:

(e) ______________________
Do the Math Exercises 2.6
Absolute Value Equations and Inequalities

In Problems 1 and 2, solve each absolute value equation.

1. \( \left| \frac{2x - 3}{5} \right| = 2 \)
2. \( 3|y - 4| + 4 = 16 \)

In Problems 3 – 6, solve each absolute value inequality. Express your answer in set-builder notation.

3. \( |y + 4| < 6 \)
4. \( |-3x + 2| - 7 \leq -2 \)

5. \( |x + 4| \geq 7 \)
6. \( 3|x| + 8 > 2 \)

In Problems 7 – 14, solve each absolute value equation or inequality.

7. \( |2x + 1| = x - 3 \)
8. \( |3 - 5x| < | -7 | \)

9. \( |3x - 4| = -9 \)
10. \( |-9x + 2| \geq -1 \)
11. \[ |4x + 3| = 1 \]  
12. \[ |4x - 3| > 1 \]

13. \[ |7x + 5| + 4 < 3 \]  
14. \[ |4y + 3| \geq -3 \]

In Problems 15 and 16, write each statement as an absolute value inequality.

15. \( x \) differs from \(-4\) by less than 2  
16. twice \( x \) differs from 7 by more than 3

17. **Gestation Period**  
The length of human pregnancy is about 266 days. It can be shown that a mother whose gestation period \( x \) satisfies the inequality \[ \left| \frac{x - 266}{16} \right| > 1.96 \] has an unusual length of pregnancy. Determine the length of pregnancy that would be considered unusual.

18. Explain why the solution set of \[ |5x - 3| > -5 \] is the set of all real numbers.
Five-Minute Warm-Up 3.1
Systems of Linear Equations in Two Variables

1. Evaluate $5x - 2y$ for $x = 3, \ y = -1$.

2. Determine whether the point $\left(8, -\frac{4}{3}\right)$ is on the graph of the equation $x - 3y = 12$.

3. Graph the linear equation $y = -\frac{2}{3}x + 4$.

4. Find the equation of the line parallel to $x - y = 2$ containing the point $(-3, -2)$.

5. Determine the slope and $y$-intercept of $x - 3y = -9$.

6. What is the additive inverse of 15?

7. Solve: $4x - 2(5x - 1) = -4$
Guided Practice 3.1
Systems of Linear Equations in Two Variables

Objective 1: Determine Whether an Ordered Pair Is a Solution of a System of Linear Equations

1. Complete the following chart which describes the solutions to a system of linear equations in two variables.

<table>
<thead>
<tr>
<th>Number of Solutions</th>
<th>Classification</th>
<th>Graph of the Two Lines</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) no solution</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(b) infinitely many solutions</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(c) exactly one solution</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Objective 3: Solve a System of Two Linear Equations by Substitution (See textbook Example 4)

2. Solve the following system by substitution:

\[
\begin{align*}
5x + 2y &= -5 \quad (1) \\
3x - y &= -14 \quad (2)
\end{align*}
\]

Step 1: Solve one of the equations for one of the unknowns. It is easiest to solve equation (2) for \( y \) since the coefficient of \( y \) is -1.

Equation (2):
Subtract 3\( x \) from both sides:
Multiply both sides by -1:

\[
\begin{align*}
3x - y &= -14 \\
(a) & \quad \underline{\phantom{0}} \\
(b) & \quad \underline{\phantom{0}}
\end{align*}
\]

Step 2: Substitute your expression for \( y \) in equation (1).

Equation (1):
Substitute the expression from equation (2) into equation (1):

\[
\begin{align*}
3x - y &= -14 \\
(c) & \quad \underline{\phantom{0}} \\
(d) & \quad \underline{\phantom{0}}
\end{align*}
\]

Step 3: Solve the equation for \( x \).

Distribute the 2:
Combine like terms:
Subtract 28 from both sides:
Divide both sides by 11:

\[
\begin{align*}
3x - y &= -14 \\
(e) & \quad \underline{\phantom{0}} \\
(f) & \quad \underline{\phantom{0}} \\
(g) & \quad \underline{\phantom{0}} \\
(h) & \quad \underline{\phantom{0}}
\end{align*}
\]

Step 4: Substitute the value for \( x \) into the equation from Step 1(b) and then solve for \( y \).

Step 5: Check your answer in both of the original equations. If both equations yield a true statement, you have the correct answer.

Write the ordered pair that is the solution to the system.

\[
\begin{align*}
(i) y &= \underline{\phantom{0}} \\
(j) & \quad \underline{\phantom{0}}
\end{align*}
\]
Objective 4: Solve a System of Two Linear Equations by Elimination (See textbook Example 6)

3. Solve the following system by elimination:
\[
\begin{align*}
2x + y &= -4 \\
3x + 5y &= 29
\end{align*}
\]

**Step 1:** Our first goal is to get the coefficients on one of the variables to be additive inverses. In looking at this system, we can make the coefficients of \(y\) be additive inverses by multiplying equation (1) by \(-5\).

Multiply both sides of (1) by \(-5\), use the Distributive Property, and then write the equivalent system of equations.

\[
\begin{align*}
2x + y &= -4 \\
3x + 5y &= 29
\end{align*}
\]

\[
\begin{align*}
(1) & \\
(2)
\end{align*}
\]

**Step 2:** We now add equations (1) and (2) to eliminate the variable \(y\) and then solve for \(x\).

Add (1) and (2):

\[
(1) \quad (2)
\]

Divide both sides by \(-7\):

\[
(1) \quad (2)
\]

**Step 3:** Substitute your value for \(x\) into either equation (1) or equation (2). We will use equation (1) as it looks like less work.

Equation (1):

\[
2x + y = -4
\]

Substitute your value for \(x\).

\[
(1)
\]

Solve for \(y\).

\[
(1)
\]

**Step 4:** Check your answer in both of the original equations. If both equations yield a true statement, you have the correct answer.

Write the ordered pair that is the solution to the system.

\[
(1) \quad (2)
\]

Objective 5: Identify Inconsistent Systems

4. Algebraically, what occurs when you solve an inconsistent system of equations? ________________

_______________________________________________________________________________________

Objective 6: Write the Solution of a System with Dependent Equations

5. Algebraically, what occurs when you solve a dependent system of equations? ________________

_______________________________________________________________________________________

6. The following system is consistent and dependent.
\[
\begin{align*}
-x + 3y &= 1 \\
2x - 6y &= -2
\end{align*}
\]

Express the solution using set-builder notation. 6. ___________
Do the Math Exercises 3.1

Systems of Linear Equations in Two Variables

In Problems 1 and 2, determine whether the given ordered pairs are solutions of the system of linear equations.

1. \[
\begin{align*}
  x - 2y &= -11 \\
  3x + 2y &= -1
\end{align*}
\]
   \(a\) \((-5, 3)\) \(b\) \((-3, 4)\)

2. \[
\begin{align*}
  -3x + y &= 5 \\
  6x - 2y &= 6
\end{align*}
\]
   \(a\) \((-2, -1)\) \(b\) \((2, 0)\)

In Problems 3 and 4, solve the system of equations by graphing.

3. \[
\begin{align*}
  y &= -2x + 4 \\
  y &= 2x - 4
\end{align*}
\]

4. \[
\begin{align*}
  -x + 2y &= -9 \\
  2x + y &= -2
\end{align*}
\]

In Problems 5 and 6, solve the system of equations using substitution.

5. \[
\begin{align*}
  y &= \frac{1}{2}x \\
  x - 4y &= -4
\end{align*}
\]

6. \[
\begin{align*}
  3x + 2y &= 0 \\
  6x + 2y &= 5
\end{align*}
\]

In Problems 7 and 8, solve the system of equations using elimination.

7. \[
\begin{align*}
  6x - 4y &= 6 \\
  -3x + 2y &= 3
\end{align*}
\]

8. \[
\begin{align*}
  x + 2y &= -\frac{8}{3} \\
  3x - 3y &= 5
\end{align*}
\]
Do the Math Exercises 3.1

In Problems 9 – 11, solve the system of equations by any method.

9. \[
\begin{align*}
2x + y &= -1 \\
-3x - 2y &= 7
\end{align*}
\]

10. \[
\begin{align*}
y &= \frac{1}{2}x + 2 \\
x - 2y &= -4
\end{align*}
\]

11. \[
\begin{align*}
\frac{1}{3}x - \frac{1}{2}y &= -5 \\
\frac{4}{5}x + \frac{6}{5}y &= 1
\end{align*}
\]

In Problems 12 and 13, use slope-intercept form to determine the number of solutions the system has.

12. \[
\begin{align*}
4x - 2y &= 8 \\
-10x + 5y &= 5
\end{align*}
\]

13. \[
\begin{align*}
2x - y &= -5 \\
-4x + 3y &= 9
\end{align*}
\]

14. Rhombus  A rhombus is a parallelogram whose adjacent sides are congruent. Consider the rhombus with vertices \((-1, 3), (3, 6), (3, 1),\) and \((-1, -2)\) to find the following.

(a) Find the equation of the line for the diagonal through the points \((-1, 3)\) and \((3, 1)\).  14a. _________

(b) Find the equation of the line for the diagonal through the points \((-1, -2)\) and \((3, 6)\). 14b. _________

(c) Find the point of intersection of the diagonals.  14c. _________
Five-Minute Warm-Up 3.2
Problem Solving: Systems of Two Linear Equations Containing Two Unknowns

In Problems 1 – 4, translate each sentence into a mathematical statement.

1. Twelve more than a number $x$ is 5.  
   1. __________

2. Six less than a number $y$ is two times $y$.  
   2. __________

3. Twice the difference of the length, $l$, and 6 is equivalent to the width, $w$.  
   3. __________

4. The difference of twice the height, $h$, and 10 is the same as the quotient of 2 and $w$.  
   4. __________

5. If a total of $12,000 is to be invested in stocks and bonds and $s$ represents the amount in stocks, write an algebraic expression for the amount invested in bonds.  
   5. __________

6. Suppose that you have a credit card balance of $2,500 and the credit card company charges 18% annual interest on outstanding balances. How much interest will you pay after 1 month?  
   6. __________

7. Write a linear cost function if the fixed costs are $4000 and the cost of production is $42 per unit. Let $x$ represent the number of units produced.  
   7. __________
Guided Practice 3.2
Problem Solving: Systems of Two Linear Equations Containing Two Unknowns

Objective 1: Model and Solve Direct Translation Problems

1. An adult ticket to the amusement park costs $26, and a child’s ticket to the amusement park costs $18.50. A group of 13 friends purchased adult and child’s tickets and paid $278. How many tickets for children were purchased? (See textbook Example 2)

(a) Sept 1: Identify What do we want to find in the problem? _______________________________________

(b) Step 2: Name Let a represent the number of adult tickets purchased and c represent the number of child’s tickets purchased.

Step 3: Translate Write a system of equations that can be used to solve this problem. Let equation (1) represent the number of tickets purchased and equation (2) represent the total cost of the tickets.

(c) Step 4: Solve

(d) Step 5: Check When you substitute the values you found in Step 4 into your equations from Step 3, does each equation yield a true statement? Is your answer reasonable?

(e) Step 6: Answer ______________________________________________________________________

Objective 2: Model and Solve Geometry Problems

2. A rectangular parking lot has a perimeter of 125 feet. The length of the parking lot is 10 feet more than the width. What are the dimensions of the parking lot? (See textbook Example 3)

(a) Write the formula from Geometry needed to solve this problem. ________________

(b) Write an equation in two variables that relates the length to the width of the parking lot. ________________

(c) Use the equations from (a) and (b), using l for the length and w for the width of the parking lot, to write a system of two linear equations containing two unknowns that will solve this problem. Be sure your equations are in standard form.

(d) Solve the system. ________________

(e) Check your result and then answer the question. ______________________________________________________________________
Objective 3: Model and Solve Mixture Problems

3. A backpacker wishes to mix peanuts worth $2 per pound and trail mix worth $5 per pound to make 10 pounds of a cheaper trail mix to take on the family backpacking trip to the Sierra Mountains. If the backpacker could find the blend in the grocery store, it would sell for $3.20 per pound. How many pounds of each should be in the mixture if it is worth $3.20 per pound? (See textbook Example 6)

(a) Complete the table from the information given, using \( p \) for the number of pounds of peanuts and \( t \) for the number of pounds of trail mix.

<table>
<thead>
<tr>
<th>Number of Pounds</th>
<th>Price per Pound</th>
<th>Total Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peanuts</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Trail Mix</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Blend</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Write a system of equations that models this problem.

\[
\begin{align*}
12p + 5t & = 60 \\
2p + 3t & = 32
\end{align*}
\]

(c) Solve the system and answer the question.

Objective 4: Model and Solve Uniform Motion Problems

4. Some uniform motion problems involve currents such as wind, water currents, or even standing on a moving walkway in an airport. If \( r \) represents the traveling speed without any current and \( c \) represents the rate of the current, write an expression for each of the following rates.

(a) speed when traveling against the current

(b) speed when traveling with the current

5. A cyclist can go 36 miles with the wind blowing at her back in 3 hours. On the return trip, after 4 hours, the cyclist still has 4 miles remaining to return to the starting point. Find the speed of the cyclist and the speed of the wind. (See textbook Example 7)

(a) Complete the table from the information given, using \( c \) for speed of the cyclist and \( w \) for speed of the wind.

<table>
<thead>
<tr>
<th>Distance</th>
<th>Rate</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>With the wind</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Against the wind</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Write a system of equations that models this problem.

\[
\begin{align*}
3c + w & = 36 \\
4c + w & = 40
\end{align*}
\]

(c) Solve the system and answer the question.

Objective 5: Find the Intersection of Two Linear Functions

6. Geometrically, when you set two functions equal to each other and solve, what are you finding?

7. In business, what is a break-even point?
Problem Solving: Systems of Two Linear Equations Containing Two Unknowns

1. The sum of two numbers is 25. The difference of two numbers is 3. Find the numbers.

2. The sum of four times a first number and a second number is 68. If the first number is decreased by twice the second number the result is −1. Find the numbers.

3. **Making Change** Johnny has $6.75 in dimes and quarters. He has 8 more dimes than quarters. How many dimes does Johnny have?

4. **Perimeter** The perimeter of a rectangle is 260 centimeters. If the width of the rectangle is 15 centimeters less than the length, what are the dimensions of the rectangle?

5. **Investments** Marge and Homer have $80,000 to invest. Their financial advisor has recommended that they diversify by placing some of the money in stocks and some in bonds. Based upon current market conditions, he has recommended that three times the amount in bonds should equal two times the amount invested in stocks. How much should be invested in bonds?
6. **Candy** A candy store sells chocolate-covered almonds for $6.50 per pound and chocolate-covered peanuts for $4.00 per pound. The manager decides to make a bridge mix that combines the almonds with the peanuts. She wants the bridge mix to sell for $6.00 per pound, and there should be no loss in revenue from selling the bridge mix versus the almonds and peanuts alone. How many pounds of chocolate-covered almonds and chocolate-covered peanuts are required to create 50 pounds of bridge mix?

7. **Pharmacy** A doctor’s prescription calls for the creation of pills that contain 10 units of vitamin B₁₂ and 13 units of vitamin E. Your pharmacy stocks two powders that can be used to make these pills: Powder A contains 20% vitamin B₁₂ and 40% vitamin E; Powder B contains 50% vitamin B₁₂ and 30% vitamin E. How many units of each powder should be mixed in each pill?

8. **Against the Wind** A Piper Arrow can fly 510 miles in 3 hours with a tail wind. Against this same wind, the plane can fly 390 miles in 3 hours. Find the airspeed of the plane. What is the impact of the wind on the plane?

9. **Runners** Enrique leaves his house and starts to run at an average speed of 6 miles per hour. Half an hour later, Enrique’s younger (and faster) brother leaves the house to catch up to Enrique running at an average speed of 8 miles per hour. How long will it take for Enrique’s brother to run half the distance that Enrique has run?
1. Evaluate $2x - 5y - 8z$ for $x = -7$, $y = 4$, $z = -3$.

2. \[
\begin{align*}
    x - y &= 10 \\
    x + y &= -20
\end{align*}
\]

3. \[
\begin{align*}
    5x - y &= 3 \\
    -10x + 2y &= 2
\end{align*}
\]

4. \[
\begin{align*}
    4x + y &= 3 \\
    8x + 2y &= 6
\end{align*}
\]

In Problems 2 – 4, solve the system of equations using elimination.
Objective 1: Solve Systems of Three Linear Equations

1. Geometrically, what does an equation in three variables represent? ________________________________

2. A system of three linear equations containing three variables has one of the following possible solutions:
   (a) Exactly one solution is a ___________________ system with ___________________ equations.
   (b) No solution is an ___________________ system.
   (c) Infinitely many solutions is a ___________________ system with ___________________ equations.

3. Use the method of elimination to solve the system:
   \[
   \begin{align*}
   x + 3y + 3z &= 9 \quad (1) \\
   3x + 5y + 4z &= 8 \quad (2) \\
   5x + 3y + 7z &= 9 \quad (3)
   \end{align*}
   \]
   (See textbook Example 2)

Step 1: Our goal is to eliminate the same variable from two of the equations. In looking at the system, we notice that we can use equation (1) to eliminate the variable \(x\) from equations (2) and (3). We can do this by multiplying equation (1) by -3 and adding the result to equation (2). The equation that results becomes equation (4). Why do we do this? Because the coefficients on \(x\) will be additive inverses and adding the equations eliminates the variable \(x\). We also multiply equation (1) by -5 and add the result to equation (3). The equation that results becomes equation (5).

<table>
<thead>
<tr>
<th>Step 1</th>
<th>Multiply equation (1) by -3:</th>
<th>(a) __________________________</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Equation (2):</td>
<td>(b) __________________________</td>
</tr>
<tr>
<td></td>
<td>Add (a) and (b):</td>
<td>(c) __________________________ (4)</td>
</tr>
<tr>
<td></td>
<td>Multiply equation (1) by -5:</td>
<td>(d) __________________________</td>
</tr>
<tr>
<td></td>
<td>Equation (3):</td>
<td>(e) __________________________</td>
</tr>
<tr>
<td></td>
<td>Add (d) and (c):</td>
<td>(f) __________________________ (5)</td>
</tr>
</tbody>
</table>

Step 2: We now concentrate on equations (4) and (5), treating them as a system of two equations containing two variables. It is easiest to eliminate the variable \(y\) by multiplying equation (4) by -3 and adding the result to equation (5). This results in an equation in one variable, equation (6).

<table>
<thead>
<tr>
<th>Step 2</th>
<th>Multiply equation (4) by -3:</th>
<th>(g) __________________________</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Equation (5):</td>
<td>(h) __________________________</td>
</tr>
<tr>
<td></td>
<td>Add (g) and (h):</td>
<td>(i) __________________________ (6)</td>
</tr>
</tbody>
</table>

Step 3: We solve equation (6) for \(z\). On line (i), divide both sides by 7: (j) __________________________

Step 4: Back-substitute your value for \(z\) into either equation (4) or equation (5) to solve for \(y\). (k) \(y = __________________________

Step 5: Back-substitute your values for \(y\) and \(z\) into one of the equations (1), (2), or (3). Solve for \(x\). (l) \(x = __________________________

continued next page
**Step 6: Check** your answer in all three of the original equations. If all equations yield a true statement, you have the correct answer.

Write the ordered triple that is the solution to the system.

(m) ___________________

**Objective 2: Identify Inconsistent Systems**

4. Whenever you solve a system of equations and end up with a false statement such as $0 = -7$, you have an ______________ system. We say that the solution to the system is ______________.

**Objective 3: Write the Solution with a System of Dependent Equations**

5. Whenever you solve a system of equations and end up with a true statement such as $3 = 3$ or $0 = 0$, you have a __________________________ system.

6. Typically, when writing the solution set, we express the values of $x$ and $y$ in terms of $z$, although this is not required. We know that

$$\begin{align*}
2x - y &= 2 \\
-x + 5z &= 3 \\
-y + 10z &= 8
\end{align*}$$

is a dependent system. *(See textbook Example 5)*

(a) Solve equation (2) for $x$: ______________________

(b) Solve equation (3) for $y$: ______________________

(c) Express the solution to the system: $\{(x, y, z) | x = ________, y = ________, z \text{ is any real number}\}$

**Objective 4: Model and Solve Problems Involving Three Linear Equations**

7. **Theater Revenues** A theater has 600 seats, divided into orchestra, main floor, and balcony seating. Orchestra seats sell for $80, main floor seats for $60, and balcony seats for $25. If all the seats are sold, the total revenue to the theater is $33,500. One evening, all of the orchestra seats were sold, $\frac{3}{5}$ of the main seats were sold and $\frac{4}{5}$ of the balcony seats were sold. The total revenue collected was $24,640. How many are there of each kind of seat? *(See textbook Example 6)*

(a) Write an equation that expresses the total number of seats in the theater if $a$ represents the number of orchestra, $b$ represents the number of main floor, and $c$ represents the number of balcony seats:

(b) Write an equation that calculates the total revenue from all seats:

(c) Write an equation that calculates the revenue when a portion of the seats are sold:
Do the Math Exercises 3.3
Systems of Linear Equations in Three Variables

Determine whether the given ordered triples are solutions of the system of linear equations.

1. \[
\begin{align*}
2x + y - 2z &= 6 \\
-2x + y + 5z &= 1 \\
2x + 3y + z &= 13
\end{align*}
\]

(a) (3, 2, 1) \hspace{1cm} (b) (10, −4, 5)

\hspace{1cm} 1a. \hspace{0.5cm} \hspace{1cm} 1b. \hspace{1cm} \hspace{1cm}

In Problems 2 – 6, solve each system of three linear equations containing three unknowns.

2. \[
\begin{align*}
x + 2y - z &= 4 \\
2x - y + 3z &= 8 \\
-2x + 3y - 2z &= 10
\end{align*}
\]

2. \hspace{1cm}

3. \[
\begin{align*}
x - y + 3z &= 2 \\
-2x + 3y - 8z &= -1 \\
2x - 2y + 4z &= 7
\end{align*}
\]

3. \hspace{1cm}

4. \[
\begin{align*}
x &= -3z = -3 \\
3y + 4z &= -5 \\
3x - 2y &= 6
\end{align*}
\]

4. \hspace{1cm}
Do the Math Exercises 3.3

5. \[
\begin{align*}
  x - y + 2z &= 3 \\
  2x + y - 2z &= 1 \\
  4x - y + 2z &= 0
\end{align*}
\]

5. __________

6. \[
\begin{align*}
  x + y + z &= 4 \\
  2x + 3y - z &= 8 \\
  x + y - z &= 3
\end{align*}
\]

6. __________

7. Curve Fitting The function \( f(x) = ax^2 + bx + c \) is a quadratic function where \( a, b, \) and \( c \) are constants.

(a) If \( f(-1) = 6 \), then \( 6 = a(-1)^2 + b(-1) + c \) or \( a - b + c = 6 \). Find two additional linear equations if \( f(1) = 2 \), and \( f(2) = 9 \). 7a. __________

(b) Use the three linear equations found in part (a) to determine \( a, b, \) and \( c \). 7b. __________

(c) What is the quadratic function that contains the points \((-1, 6), (1, 2), \) and \((2, 9)\)? 7c. __________

8. Nutrition Antonio is on a special diet that requires he consume 1325 calories, 172 grams of carbohydrates, and 63 grams of protein for lunch. He wishes to have a Broccoli and Cheese Baked Potato, Chicken BLT Salad, and a medium Coke. Each Broccoli and Cheese Baked Potato has 480 calories, 80 g of carbohydrates, and 9 g of protein. Each Chicken BLT Salad has 310 calories, 10 g for carbohydrates, and 33 g for protein. Each Coke has 140 calories, 37 g of carbohydrates, and 0 g of protein. How many servings of each does Antonio need? 8. __________
Five-Minute Warm-Up 3.4
Using Matrices to Solve Systems

1. Determine the coefficients of \(-2x + y - 3z\).
   1. ___________

2. Evaluate \(-x - 5y + 11z\) for \(x = 2, y = -5, z = -1\).
   2. ___________

In Problems 3 and 4, solve for the indicated variable.

3. \(7x - 5y = 10\) for \(y\)
   3. ___________

4. \(\frac{3}{2}x + \frac{2}{3}y = -2\) for \(x\)
   4. ___________

In Problems 5 and 6, use the Distributive Property to remove the parentheses.

5. \(-3(2x - 9y + z)\)
   5. ___________

6. \(-\frac{5}{4}(8x - 4y + 12z)\)
   6. ___________

7. If \(f(x) = -x^2 - 5x + 7\), find the value of each function.
   (a) \(f(4)\)
   (b) \(f(-3)\)
   7a. ___________
   7b. ___________
**Guided Practice 3.4**

**Using Matrices to Solve Systems**

**Objective 1: Write the Augmented Matrix of a System**

1. A **matrix** is a rectangular array of numbers, meaning that the order of the numbers in the matrix is relevant. The size of the matrix, called the **dimension**, is denoted as the number of rows by the number of columns. If a matrix has 3 rows and 4 columns, we say the dimension of the matrix is $3 \times 4$.

   Find the dimension: $\begin{bmatrix} -1 & 8 & 3 & -7 \\ -5 & 0 & 1 & 2 \end{bmatrix}$  
   1. ___________

2. An **augmented matrix** can be used to represent a system of linear equations. Each row is created from one of the equations in the system and each column represents the coefficients of one of the variables. The vertical bar is the equal sign and the last column represents the constants. Be sure each equation is written in ______________ form, filling in the coefficient of any missing variables with ____.

3. Write the system of equations as an augmented matrix. *(See textbook Example 1)*

   \[
   \begin{align*}
   2x + y + z &= 3 \\
   4y - 7z &= -1 \\
   x + 3y &= 0
   \end{align*}
   \]
   3. ________________

**Objective 2: Write the System from the Augmented Matrix**

4. Write the system of linear equations corresponding to the augmented matrix. *(See textbook Example 2)*

   \[
   \begin{bmatrix} 1 & 1 & 2 \\ -3 & 1 & 10 \end{bmatrix}
   \]
   4. ________________

**Objective 3: Perform Row Operations on a Matrix**

5. There are three basic row operations. These are similar to the types of operations that we used to solve systems of equations earlier in this chapter. List the row operations for matrices.

   (a) ____________________________________________

   (b) ____________________________________________

   (c) ____________________________________________

6. The notation $-3r_1$ means multiply row 1 by $-3$. The notation $R_2 = r_1 + r_2$ means replace row 2 with the sum of row 1 plus row 2. Perform the following row operations and write the new augmented matrix. *(See textbook Example 3)*

   (a) \[
   \begin{bmatrix} 2 & -1 & 5 \\ 1 & -7 & 4 \end{bmatrix} R_2 = -2r_2 \rightarrow \]

   (b) \[
   \begin{bmatrix} 2 & -1 & 5 \\ 1 & -7 & 4 \end{bmatrix} R_1 = r_2 + r_1 \rightarrow \]

**Objective 4: Solve Systems Using Matrices**

7. When is a matrix in **row echelon form**?
8. Solve the following system using matrices:
\[
\begin{align*}
x + y + z &= 1 \\
2x + 2y &= 6 \\
3x + 4y - z &= 13
\end{align*}
\] (See textbook Example 6)

**Step 1:** Write the augmented matrix of the system. We will use the row operations from Objective 3 to solve the system.

**Step 2:** We want the entry in row 1, column 1 to be 1. This is already done.

**Step 3:** We want the entry in row 2, column 1 to be zero. We also want the entry in row 3, column 1 to be a zero. We use row operation #3 to accomplish this. The entries in row 1 remain unchanged.

\[R_2 = -2R_1 + r_2 \rightarrow R_3 = -3R_1 + r_3 \rightarrow\]

(e) _______________________________

**Step 4:** We want the entry in row 2, column 2 to be a 1. This is accomplished by interchanging rows 2 and 3, row operation #1.

(c) _______________________________

**Step 5:** We want the entry in row 3, column 2 to be zero. This is already accomplished.

Step 6: We want the entry in row 3, column 3 to be a 1. We use row operation #2 to accomplish this.

\[R_3 = -\frac{1}{2} r_3 \rightarrow\]

(d) _______________________________

**Step 7:** The augmented matrix is in row echelon form. Write the system of equations corresponding to the augmented matrix and solve. We know the value of \(z\). Substitute into equation (2) and solve for \(y\). Then use these values and substitute into equation (1) to solve for \(x\).

State the solution as an ordered triple.

(e) _______________________________

**Step 8:** Check

We leave it to you to verify the solution.

---

**Objective 5: Solve Consistent Systems with Dependent Equations and Solve Inconsistent Systems**

9. State the solution to the system represented by the augmented matrix:
\[
\begin{bmatrix}
1 & 4 & 2 \\
0 & 0 & 0 \\
\end{bmatrix}
\] (See textbook Example 7)

10. State the solution to system represented by the augmented matrix:
\[
\begin{bmatrix}
1 & 0 & -1 & -2 \\
0 & 1 & 5 & -9 \\
0 & 0 & 0 & -3 \\
\end{bmatrix}
\] (See textbook Example 8)
Write the augmented matrix of the given system of equations.

1. \[
\begin{align*}
6x + 4y + 2 &= 0 \\
-x - y + 1 &= 0
\end{align*}
\]

Perform each row operation on the given augmented matrix.

2. \[
\begin{bmatrix}
1 & -1 & 1 & 6 \\
-2 & 1 & -3 & 3 \\
3 & 2 & -2 & -5
\end{bmatrix}
\]

(a) \(R_2 = 2r_1 + r_2\) followed by

(b) \(R_3 = -3r_1 + r_3\)

In Problems 3 – 6, solve each system of equations using matrices. If a system has no solution, say that it is inconsistent.

3. \[
\begin{align*}
5x - 2y &= 3 \\
-15x + 6y &= -9
\end{align*}
\]

4. \[
\begin{align*}
3x + 3y &= -1 \\
2x + y &= 1
\end{align*}
\]
5. \[
\begin{align*}
-x + 2y + z &= 1 \\
2x - y + 3z &= -3 \\
x + 5y + 6z &= 2
\end{align*}
\]

6. \[
\begin{align*}
2x - y + 2z &= 13 \\
x + 2y - z &= -14 \\
3x + y - 2z &= -13
\end{align*}
\]

7. **Finance** Marlon has $12,000 to invest. He decides to place some of the money into a savings account paying 2\% annual interest, some in Treasury bonds paying 4\% annual interest and some in a mutual fund paying 9\% annual interest. Marlon would like to earn $440 per year in income. In addition, Marlon wants his investment in the savings account to be $4,000 more than the amount in Treasury bonds. How much should Marlon invest in each investment category?
5. Five-Minute Warm-Up 3.5
   Determinants and Cramer's Rule

1. Evaluate: $-3 \cdot 5 - 2 \cdot (-7)$
   1. __________

2. Simplify each expression.
   (a) $\frac{13}{0}$
   (b) $\frac{0}{45}$
   2a. __________
   2b. __________

3. Simplify: $\frac{-12}{-10}$
   3. __________

4. Evaluate $x - 2y + z$ for $x = -\frac{3}{2}$, $y = \frac{2}{5}$, $z = \frac{3}{4}$.
   4. __________

In Problems 5 and 6, solve each equation.

5. $-8 - 3x = 1$
   5. __________

6. $\frac{9}{7}z - 4 = 32$
   6. __________

7. Solve:
   \[
   \begin{align*}
   -6 - 2(3x - 6y) &= 0 \\
   6 - 12(x - 2y) &= 0 
   \end{align*}
   \] 7. __________
**Guided Practice 3.5**
Determinants and Cramer’s Rule

**Objective 1: Evaluate the Determinant of a $2 \times 2$ Matrix**

1. The notation $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ denotes the determinant for the matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$. We use the definition $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$ to evaluate the determinant and find a single number representing the array.

Evaluate each determinant. *(See textbook Example 1)*

(a) $\begin{vmatrix} 3 & 8 \\ -1 & 2 \end{vmatrix}$

(b) $\begin{vmatrix} -7 & 2 \\ 9 & 1 \end{vmatrix}$

1a. __________  
1b. __________

**Objective 2: Use Cramer’s Rule to Solve a System of Two Equations**

2. If the number of unknowns is the same as the number of equations in a system, we can use Cramer’s Rule to solve the system. We first set up several different determinants.

   (a) $D$ is a determinant in which each entry is a _______________ of the variables in the system.

   (b) $D_x$ is a determinant in which the first column (coefficients of $x$) is replaced by ________________.

   (c) $D_y$ is a determinant in which the _________________ column (coefficients of __) is replaced by the constants on the right side of the equal sign.

3. Given the linear equations $-5y + 3x = 9$ and $2 - x - 2y = 0$, write the system with each equation expressed in standard form.

   3. ___________

4. Using your system above, determine each of the following. *(See textbook Example 2)*

   (a) $D =$

   (b) $D_x =$

   (c) $D_y =$

5. According to Cramer’s Rule, the solution to the system of equations can be found evaluating the determinants $D$, $D_x$, and $D_y$ and simplifying the following ratios.

   (a) $x = \frac{?}{?}$

   (b) $y = \frac{?}{?}$

**Objective 3: Evaluate the Determinant of a $3 \times 3$ Matrix**

6. We cannot use the process from Objective 1 to find the value of a $3 \times 3$ determinant. Do you see why? Instead we use a process called expansion by minors. Although it is possible to expand by any row or column, typically we use the first row entries to be the coefficients of the corresponding minors. Be careful with the operations between the terms as they alternate in the expansion and can change when expanding by a different row or column. *(See textbook Example 3)*
Guided Practice 3.5

Set up, but do not evaluate, the expansion by row 1:

\[
\begin{vmatrix}
1 & -1 & 2 \\
5 & -3 & 3 \\
1 & 4 & -2
\end{vmatrix}
\]

Objective 4: Use Cramer’s Rule to Solve a System of Three Equations (See textbook Example 5)

7. Solve the following system using Cramer’s Rule:

\[
\begin{align*}
x + 2y - z &= -3 \\
2x - 4y + z &= -7 \\
-2x + 2y - 3z &= 4
\end{align*}
\]

Step 1: Find the determinant of the coefficients of the variables, \(D\).

Write the determinant, \(D\):

(a) __________________________

Evaluate \(D\):

(b) __________________________

Step 2: Because \(D \neq 0\), we continue by writing and evaluating each of the determinants \(D_x, D_y,\) and \(D_z\).

Write and evaluate \(D_x\):

(c) __________________________

Write and evaluate \(D_y\):

(d) __________________________

Write and evaluate \(D_z\):

(e) __________________________

Step 3: Solve for each of the variables.

(f) \[ x = \frac{D_x}{D} = \frac{?}{?} = \frac{?}{?} = \frac{?}{?} \]

(g) \[ y = \frac{D_y}{D} = \frac{?}{?} = \frac{?}{?} = \frac{?}{?} \]

(h) \[ z = \frac{D_z}{D} = \frac{?}{?} = \frac{?}{?} = \frac{?}{?} \]

Step 4: Check your answer.

We leave it to you to verify the solution.

State your solution as an ordered triple:

(i) __________________________

8. If \(D = 0\) Cramer’s Rules does not apply. If at least one of the determinants \(D_x, D_y,\) or \(D_z\) is different from 0, then the system is ___________________ and the solution set is ___________________.

9. If \(D = 0\) Cramer’s Rule does not apply. If all of the determinants \(D_x, D_y,\) and \(D_z\) equal 0, then the system is ___________________ and ___________________ and there are ___________________ solutions.
Do the Math Exercises 3.5
Determinants and Cramer's Rule

In Problems 1 – 3, find the value of each determinant.

1. \[
\begin{vmatrix}
5 & 3 \\
2 & 4
\end{vmatrix}
\]

2. \[
\begin{vmatrix}
-8 & 5 \\
-4 & 3
\end{vmatrix}
\]

3. \[
\begin{vmatrix}
-2 & 1 & 6 \\
-3 & 2 & 5 \\
1 & 0 & -2
\end{vmatrix}
\]

In Problems 4 – 7, solve each system of equations using Cramer’s Rule, if possible.

4. \[
\begin{cases}
2x + 4y = -6 \\
3x + 2y = 7
\end{cases}
\]

5. \[
\begin{cases}
3x - 6y = -2 \\
x + 2y = 4
\end{cases}
\]

6. \[
\begin{cases}
x + y - z = 6 \\
x + 2y + z = 6 \\
-x - y + 2z = -7
\end{cases}
\]
Do the Math Exercises 3.5

7. \[
\begin{align*}
    x - 2y - z &= 1 \\
    2x + 2y + z &= 3 \\
    6x + 6y + 3z &= 6
\end{align*}
\]

8. Solve for \( x \): \[
\begin{vmatrix}
    -2 & x \\
    3 & 4
\end{vmatrix} = 1
\]

9. **Geometry: Area of a Triangle** Given the points \( A = (-1, -1) \), \( B = (3, 2) \), and \( C = (0, 6) \), find the area of the triangle \( ABC \).
Five-Minute Warm-Up 3.6
Systems of Linear Inequalities

1. Determine whether \( x = -5 \) satisfies the inequality \(-3x - 10 \geq 5\).

2. Determine whether the ordered pair \((4, -2)\) satisfies the linear inequality \(2x - 5y < 10\).

3. Solve the inequality: \(-\frac{1}{2}(3x + 1) > \frac{2}{3}(3 - x)\)

4. Graph the linear inequality \(3x - 4y > 0\).
Guided Practice 3.6
Systems of Linear Inequalities

Objective 2: Graph a System of Linear Inequalities

1. The only way to show the solution of a system of linear inequalities is by _________________.

2. To graph the inequality $x - 8y > 4$, we use a ________________ line as the boundary.
   (Refer to Section 1.8 in textbook)

3. To graph the inequality $3x + 2y \geq -6$, we use a ________________ line as the boundary.

4. To graph a linear inequality, we first graph the equation to determine the boundary line. This line divides the plane into two half-planes. To decide which half-plane to shade, we use a test point such as $(0, 0)$, provided that this point does not lie on the line. If the test point satisfies the inequality, we shade the half-plane that contains the point. If the test point does not satisfy the inequality, we shade _________________.

5. When graphing a system of linear inequalities, we are looking for the ordered pairs that satisfy both inequalities simultaneously. Therefore, the solution of the system is the ________________ of the graphs of the linear inequalities.

6. Graph the system: $\begin{cases} 2x + 3y > -3 \\ -3x + y \leq 2 \end{cases}$ (See textbook Example 2)
   
   **Step 1:** Graph the inequality $2x + 3y > -3$ on the graph at the right.

   **Step 2:** Graph the inequality $-3x + y \leq 2$ on the graph at the right.

   **Step 3:** Looking at what you graphed in Steps 1 and 2, determine where the shaded regions overlap.
Objective 3: Solve Problems Involving Systems of Linear Inequalities

7. Party Planning  Alexis and Sarah are planning a barbeque for their friends. They plan to serve grilled fish and carne asada and want to spend at most $40 on the meat. The fish sells for $8 per pound and the carne asada is $5 per pound. Since most of their friends do not eat red meat, they plan to buy at least twice as much fish as carne asada. Let $x$ represent the amount of carne asada purchased and $y$ represent the amount of fish purchased. (See textbook Example 6)

(a) Write an inequality that describes how much Alexis and Sarah will spend on meat. 7a. 

(b) Write an inequality that describes the amount of carne asada that will be purchased relative to the amount of fish that will be purchased. 7b. 

(c) Since Alexis and Sarah will purchase a positive quantity of meat, write the two inequalities that describe this constraint. 7c. 

(d) Graph the system of inequalities.

(e) Identify the vertices of the polygon. 7e. 

Do the Math Exercises 3.6
Systems of Linear Inequalities

In Problems 1 and 2, determine which of the points, if any, satisfies the system.

1. \[ \begin{align*}
    x + y & \geq 2 \\
    -3x + y & \leq 10 \\
\end{align*} \]
   (a) \((-3, 6)\)  (b) \((4, 1)\)

2. \[ \begin{align*}
    5x + 2y & < 10 \\
    4x - 3y & < 24 \\
\end{align*} \]
   (a) \((1, 3)\)  (b) \((1, 1)\)

In Problems 3 – 6, graph each system of inequalities.

3. \[ \begin{align*}
    x + y & \geq 2 \\
    -3x + y & \leq 10 \\
\end{align*} \]

4. \[ \begin{align*}
    -x + \frac{1}{3}y & < 3 \\
    \frac{4}{3}x + y & \geq 4 \\
\end{align*} \]

5. \[ \begin{align*}
    4x + 3y & > -9 \\
    -8x - 6y & > 12 \\
\end{align*} \]

6. \[ \begin{align*}
    y & \leq 4 \\
    x & \geq -1 \\
\end{align*} \]
Graph the system of linear inequalities. Tell whether the graph is bounded or unbounded and label the corner points.

7. \[
\begin{align*}
x + y &\geq 8 \\
x + 3y &\geq 12 \\
x &\geq 0 \\
y &\geq 0 \\
\end{align*}
\]

8. **Mixing Nuts** You’ve Got to Be Nuts is a store that specializes in selling nuts. The owner finds that she has excess inventory of 100 pounds (1600 ounces) of cashews and 120 pounds (1920 ounces) of peanuts. She decides to make two types of 1 pound nut mixes from the excess inventory. A premium mix will contain 12 ounces of cashews and 4 ounces of peanuts while the standard mix will contain 6 ounces of cashews and 6 ounces of peanuts.

(a) Use \(x\) to denote the number of premium mixes and \(y\) denote the number of standard mixes. Write a system of linear inequalities that describe the possible number of each kind of mix.

(b) Find the corner points of the graph.
Five-Minute Warm-Up 4.1
Adding and Subtracting Polynomials

In Problems 1 and 2, identify the coefficient.
1. \(-x^2\) \[1. \text{_______}
2. \(5y^4\) \[2. \text{_______}

3. Combine like terms: \(3x^2 + 8x - 6x + x^2\) \[3. \text{_______}

4. Use the Distributive Property to remove the parentheses: \(-8(6x + 2)\) \[4. \text{_______}

5. Simplify: \(8p - 3(p - 4)\) \[5. \text{_______}

6. Combine like terms: \(x^2y - 2xy^2 + 3xy^2 - x^2y\) \[6. \text{_______}

In Problems 7 and 8, find each function value.
7. \(f(x) = -11x + 5; f(-2)\) \[7. \text{_______}
8. \(g(x) = -x^2 + 3x - 1; g(-4)\) \[8. \text{_______}
Guided Practice 4.1
Adding and Subtracting Polynomials

**Objective 1: Define Monomial and Determine the Coefficient and Degree of a Monomial**

1. For our study of polynomials, we begin with some definitions regarding monomials. (See textbook Example 1)

   (a) In your own words, what is a monomial? __________________________________________________________________________

   (b) What is the coefficient of a monomial? __________________________________________________________________________

   (c) How do you determine the degree of a monomial in one variable? __________________________________________________________________________

   (d) How do you determine the degree of a monomial in more than one variable? (See textbook Example 3) __________________________________________________________________________

**Objective 2: Define Polynomial and Determine the Degree of a Polynomial**

2. Next, we move on to some definitions used with polynomials. (See textbook Example 4)

   (a) In your own words, define polynomial. __________________________________________________________________________

   (b) When is a polynomial in standard form? __________________________________________________________________________

   (c) How do you determine the degree of a polynomial? __________________________________________________________________________

3. Some polynomials have special names. Always simplify the polynomial first, if possible, before determining if the polynomial has one of the following specific names:

   (a) a polynomial with exactly one term is a ____________________________________________________

   (b) a polynomial with exactly two terms is a ____________________________________________________

   (c) a polynomial with exactly three terms is a ____________________________________________________

   (d) a polynomial with more than three terms is simply called a _______________________________________

**Objective 3: Simplify Polynomials by Combining Like Terms**

4. In your own words, define like terms. __________________________________________________________________________
5. To add two polynomials, we need to combine the like terms of the polynomials. The parentheses are included to indicate the first polynomial added to a second polynomial.

Simplify using horizontal addition: \((9x^2 - x + 5) + (3x^2 + 1)\) (See textbook Example 6)

**Step 1:** Remove parenthesis. 
(a) ______________________

**Step 2:** Rearrange terms. 
(b) ______________________

**Step 3:** Use the distributive property in reverse to group the coefficients of the like terms. 
(c) ______________________

**Step 4:** Simplify. 
(d) ______________________

6. Simplify by subtracting the polynomials: \((6z^3 + 2z^2 - 5) - (-3z^3 + 9z^2 - z + 1)\) (See textbook Example 8)

**Step 1:** Distribute the \(-1\). 
(a) ______________________

**Step 2:** Rearrange terms. 
(b) ______________________

**Step 3:** Combine like terms. 
(c) ______________________

**Objective 5: Add and Subtract Polynomial Functions**

7. The *sum* of two functions \(f\) and \(g\) is defined by \((f + g)(x) = f(x) + g(x)\). This is read “the function \(f\) plus \(g\) of \(x\) is equal to \(f\) of \(x\) plus \(g\) of \(x\”). (See textbook Example 11)

Let \(f\) and \(g\) be two polynomial functions defined as \(f(x) = 6x^2 - 5x + 1\) and \(g(x) = -4x^2 + 5x - 7\), find

(a) \(f(-4)\)  
(b) \(g(-4)\)  
(c) \(f(-4) + g(-4)\)  
(d) \((f + g)(x)\)  
(e) \((f + g)(-4)\)

7a. _____________
7b. _____________
7c. _____________
7d. _____________
7e. _____________
Do the Math Exercises 4.1
Adding and Subtracting Polynomials

In Problems 1 – 3, determine the coefficient and degree of each monomial.
1. $5x^4$  
2. $-12xy$  
3. $-7$

In Problems 4 – 7, determine whether the algebraic expression is a polynomial (Yes or No). If it is a polynomial, determine the degree and state if it is a monomial, binomial or trinomial. If it is a polynomial with more than 3 terms, identify the expression as polynomial.
4. $\frac{1}{x}$
5. $-12$
6. $4y^2 + 6y - 1$
7. $4mn^3 - 2m^2n^3 + mn^8$

In Problems 8 – 12, simplify each polynomial by adding or subtracting, as indicated. Express your answer as a single polynomial in standard form.
8. $10y^4 - 6y^4$
9. $(8 - t^3) - (1 + 3t + 3t^2 + t^3)$
10. $(-5xy^2 + 3xy - 9y^2) - (5xy^2 + 7xy - 8y^2)$
11. $\left(\frac{3}{4}y^3 - \frac{1}{8}y + \frac{2}{3}\right) + \left(\frac{1}{2}y^3 + \frac{5}{12}y - \frac{5}{6}\right)$
12. $\left(2w^3 - w^2 + 6w - 5\right) + \left(-3w^3 + 5w^2 + 9\right)$
In Problems 13 – 15, given $f(x) = -2x^3 + 3x - 1$, find each the following values.

13. $f(0)$  
14. $f(2)$  
15. $f(-3)$

In Problems 16 and 17, for the given functions $f$ and $g$, find the following functions.

16. $f(x) = 4x + 3$; $g(x) = 2x - 3$; find $(f + g)(x)$.  
17. $f(x) = 3x^2 + x + 2$; $g(x) = x^2 - 3x - 1$; find $(f - g)(1)$.

In Problems 18 and 19, perform the indicated operations.

18. Add $2x^3 - 3x^2 - 5x + 7$ to $x^3 + 3x^2 - 6x - 4$.  
19. Subtract $2q^3 - 3q^2 + 7q - 2$ from $-5q^3 + q^2 + 2q - 1$.

20. The polynomial function $I(a) = -54.42a^2 + 4853.38a - 46,106.66$ can be used to approximate the average per-capita income, $I$, of a U.S. resident in 2009, where $a$ is the age of the individual.

(a) Use the function to estimate the average income of a 20 year old in 2009.  
(b) Use the function to estimate the average income of a 55 year old in 2009.
In Problems 1 – 6, simplify each expression.

1. \( x^8 \cdot x \)

2. \( 7y^3 \cdot 3y^4 \)

3. \( (4z)^3 \)

4. \( (-2a^2)^4 \)

5. \( \left( \frac{5}{2}x \right)^2 \)

6. \( \frac{4xy}{3xy} \cdot \frac{x^3y^4}{2xy^3} \)

7. Use the Distributive Property to remove the parentheses: \( 7(3a - 2b) \)

8. Simplify: \( \frac{2}{3} \left( \frac{6x}{5} + \frac{9}{8} \right) \)
Guided Practice 4.2
Multiplying Polynomials

**Objective 1: Multiply a Monomial and a Polynomial**

1. To multiply a monomial and a polynomial we use __________________________.

2. It is important that you are familiar with the Laws of Exponents. If you need more practice multiplying monomials or using the Product Rule for Exponents, review Getting Ready for Chapter 4: Laws of Exponents and Scientific Notation.

   Multiply and simplify:  \(2ab^2 \left(-a^2 - 7ab + 3b^2\right)\)

   (See textbook Example 2)

**Objective 2: Multiply Two Binomials**

3. When multiplying two binomials, you have two options. These two methods are:

   (a) ____________________________________________ (b) ____________________________________________

Practice both methods and then select the method that you prefer for multiplying the binomials in the exercise set and the remainder of the course.

4. Use the distributive property to find the product:  \((9x - 2)(3x + 5)\)

   (See textbook Example 3)

5. Use FOIL to find the product:  \((4a - 7b)(3a + b)\)

   (See textbook Example 4)

**Objective 3: Multiply Two Polynomials**

6. Find the product:  \((3x - 1)(2x^2 + x - 5)\)  (See textbook Example 5)

   \((3x - 1)(2x^2 + x - 5)\)

   Step 1: Distribute the binomial 3\(x - 1\) to each term in the trinomial.

   (a) ____________________________________________

   Step 2: Distribute.

   (b) ____________________________________________

   Step 3: Combine like terms.

   (c) ____________________________________________
Objective 4: Multiply Special Products

7. Some products occur frequently and are given special names. One special product is called the difference of squares. It states that \((A - B)(A + B) = \) _________________.

(See textbook Example 6)

8. Another special product is called the square of binomials. It states that (a) \((A + B)^2 = \) _______________ and (b) \((A - B)^2 = \) _______________. (See textbook Example 8)

Objective 5: Multiply Polynomial Functions

9. The product of two functions \(f\) and \(g\) is defined by \((f \cdot g)(x) = f(x) \cdot g(x)\). This is read “the function \(f\) times \(g\) of \(x\) is equal to \(f\) of \(x\) times \(g\) of \(x\)”. (See textbook Example 10)

Let \(f\) and \(g\) be two polynomial functions defined as 
\(f(x) = 7x - 2\) and \(g(x) = x^2 - 5\), find

(a) \(f(-1)\) \hspace{1cm} (b) \(g(-1)\) \hspace{1cm} (c) \(f(-1) \cdot g(-1)\) \hspace{1cm} 9a. ___________

9b. ___________

9c. ___________

(d) \((f \cdot g)(x)\) \hspace{1cm} (e) \((f \cdot g)(-1)\) \hspace{1cm} 9d. ___________

9e. ___________
In Problems 1 – 3, find the product.

1. $(9a^3b^2)(-3a^2b^3)$

2. $\left( \frac{12}{5}x^2y \right) \left( \frac{15}{4}x^4y^3 \right)$

3. $-3mn^3(4m^2 - mn + 5n^2)$

In Problems 4 – 12, find the product of the polynomials.

4. $(z - 8)(z + 3)$

5. $(2 - 7y)(5 + 2y)$

6. $\left( \frac{3}{2}y + 4 \right) \left( \frac{4}{3}y - 1 \right)$

7. $(3m - 5n)(m + 2n)$

8. $(3p^2 - 5p + 3)(7p - 2)$

9. $(xy - 2)(x^2 + 2xy + 4y^2)$

10. $(7p - 3q)^2$

11. $(m^2 - 2n^3)(m^2 + 2n^3)$

12. $[(m + 4) - n]^2$
Do the Math Exercises 4.2

In Problems 13 and 14, given the functions \( f(x) = (x + 5) \) and \( g(x) = x^2 - 2x + 3 \), find

13. \((f \cdot g)(x)\)

14. \((f \cdot g)(3)\)

In Problems 15 and 16, given the function \( f(x) = -2x^2 + x - 5 \), find

15. \(f(x + 2)\)

16. \(f(x + h) - f(x)\)

In Problems 17 – 23, simplify the expression.

17. \(-3x(x - 3)^2\)

18. \((4x + 3)(3x - 7)\)

19. \((2y + 3)(4y^2 - 6y + 9)\)

20. \(\left(3x + \frac{1}{3}\right)^2\)

21. \((z - 3)^3\)

22. \((2a + b - 5)(4a - 2b + 1)\)

23. \((a + 2)(a - 2)(a^2 - 4) - (a + 3)(a^2 - 3)\)
Five-Minute Warm-Up 4.3
Dividing Polynomials; Synthetic Division

In Problems 1 – 6, simplify each expression.

1. \( \frac{r^3}{r^2} \)  
2. \( \frac{63x^8}{27x^4} \)

3. \( 25z^0 \)  
4. \( \frac{8a^4b}{16ab^3} \)

5. \( \left( \frac{9s}{2t^2} \right)^2 \)  
6. \( \left( \frac{xy^2z}{2xz^2} \right)^3 \left( \frac{4xyz}{y^3} \right)^2 \)

7. Add: \( \frac{4}{9} + \frac{8}{9} \)

1. \( \text{__________} \)  
2. \( \text{__________} \)
3. \( \text{__________} \)  
4. \( \text{__________} \)
5. \( \text{__________} \)  
6. \( \text{__________} \)
7. \( \text{__________} \)
Objective 2: Divide Polynomials Using Long Division

1. Find the quotient and remainder when \((x^3 - 2x^2 + x + 6)\) is divided by \((x + 1)\).  

(See textbook Example 3)

Step 1: Divide the leading term of the dividend, \(x^3\), by the leading term of the divisor, \(x\). Enter the result over the term \(x^3\).

(a) \(\frac{x^3}{x} = \)________

Step 2: Multiply \(\underline{\phantom{0}}\) by \(x + 1\). Be sure to vertically align like terms.

(b) \(x + 1) \frac{x^3 - 2x^2 + x + 6}{\phantom{0}}\)________

Step 3: Subtract your product from the dividend.

(c) \(x + 1) \frac{x^3 - 2x^2 + x + 6}{x^2}\)________

Subtract and continue with Steps 4 and 5:

Step 4: Repeat Steps 1 – 3 treating \(-3x^2 + x + 6\) as the dividend and dividing \(x\) into \(-3x^2\) to obtain the next term in the quotient.

Step 5: Repeat Steps 1 – 3 treating \(4x + 6\) as the dividend and dividing \(x\) into \(4x\) to obtain the next term in the quotient.

When the degree of the remainder is less than the degree of the divisor, you are finished dividing.

Express the answer as the Quotient + \(\frac{\text{Remainder}}{\text{Divisor}}\).

(d) \(\underline{\phantom{0}}\)

Step 6: Check Verify that \((\text{Quotient})(\text{Divisor}) + \text{Remainder} = \text{Dividend}\)

We leave it to you to verify the solution.

Dividing polynomials using long division can be difficult. Practice dividing an integer by an integer as shown in Example 2 before continuing to divide polynomials using long division. Did you notice that the steps are exactly the same? In Step 3, we subtracted the two polynomials and started the process over in Step 4. It is also possible to subtract only the like terms and then bring down the next before starting the division process again. Try it both ways. Which way do you prefer?
**Objective 3: Divide Polynomials Using Synthetic Division**

2. When using synthetic division to divide two polynomials, it is essential that the dividend be in ______________ form and, if any of the powers of the variable are missing, fill in a _____ coefficient for that term.

3. Synthetic division can be used only when the divisor is of the form _____________ or _____________.

4. Use synthetic division to find the quotient and remainder when \((18 + x^4 - 9x^2 + 3x^3)\) is divided by \((x - 3)\). (See textbook Example 6)

**Step 1:** Write the dividend in descending powers of \(x\). Then copy the coefficients of the dividend. Remember to insert a 0 for any missing power of \(x\).

\[(a)\] ________________

**Step 2:** Insert the division symbol. Rewrite the divisor in the form \(x - c\) and insert the value of \(c\) to the left of the division symbol.

\[(b)\] ________________

**Step 3:** Bring the 1 down 2 rows and enter it in Row 3.

\[(c)\] ________________

**Step 4:** Multiply the latest entry in Row 3 by 3 and place the result in Row 2, one column to the right.

\[(d)\] ________________

**Step 5:** Add the entry in Row 2 to the entry above it in Row 1. Enter the sum in Row 3.

\[(e)\] ________________

**Step 6:** Repeat steps 4 and 5 until no more entries are available in Row 1.

\[(f)\] ________________

**Step 7:** The final entry in Row 3, 99, is the remainder; the other entries in Row 3, 1, 6, 9, and 27, are the coefficients of the quotient, in descending order of degree. The quotient is the polynomial whose degree is one less than the degree of the dividend. State the quotient and the remainder.

\[(g)\] ________________

**Step 8:** Check: \((\text{Quotient})(\text{Divisor}) + \text{Remainder} = \text{Dividend}\). Write your final answer as quotient + \(\frac{\text{remainder}}{\text{divisor}}\)

\[(h)\] ________________

**Objective 5: Use the Remainder and Factor Theorems**

5. **The Remainder Theorem** Let \(f\) be a polynomial function. If \(f(x)\) is divided by \(x - c\), then the remainder is ________________.

6. **The Factor Theorem** Let \(f\) be a polynomial function. Then \(x - c\) is a factor of \(f(x)\) if and only if ________________.
In Problems 1 – 3, divide and simplify.

1. \[ \frac{6z^3 + 9z^2}{3z^2} \]
   1. ___________

2. \[ \frac{3z^4 + 12z^2}{6z^3} \]
   2. ___________

3. \[ \frac{2x^2y^3 - 9xy^3 + 16x^2y}{2x^2y^2} \]
   3. ___________

In Problems 4 – 9, divide using long division.

4. \[ \frac{x^2 - 4x - 21}{x + 3} \]
   4. ___________

5. \[ \frac{4x^2 - 17x - 33}{4x + 7} \]
   5. ___________

6. \[ \frac{x^3 + x^2 - 22x - 40}{x + 2} \]
   6. ___________

7. \[ \frac{a^3 - 49a + 120}{a + 8} \]
   7. ___________

8. \[ \frac{x^3 - 5x^2 - 2x + 10}{x^2 - 2} \]
   8. ___________

9. \[ \frac{2k^3 + 10k^2 - 6k - 8}{2k^2 - 3} \]
   9. ___________
Do the Math Exercises 4.3

In Problems 10 – 12, divide using synthetic division.

10. \( \frac{x^2 + 2x - 17}{x - 4} \)  
11. \( \frac{x^3 - 13x - 17}{x + 3} \)  
12. \( \frac{a^4 - 65a^2 + 55}{a - 8} \)

In Problems 13 and 14, given \( f(x) = 3x^2 - 6x + 5 \) and \( g(x) = 2x + 1 \), find

13. \( \left( \frac{f}{g} \right)(x) \)  
14. \( \left( \frac{f}{g} \right)(2) \)

In Problem 15, use the Remainder Theorem to find the remainder.

15. \( f(x) = 3x^3 + 2x^2 - 5 \) is divided by \( x + 3 \)

In Problem 16, use the Factor Theorem to determine whether \( x - c \) is a factor of the given function for the given value of \( c \).

16. \( f(x) = x^2 + 5x + 6; \ c = 3 \)

17. **Area** The area of a rectangle is \( (15x^2 + x - 2) \) square feet. If the width of the rectangle is \( (3x - 1) \) feet, find the length.

18. **Volume** The volume of a box is \( (4x^3 + 7x^2 - 165x - 126) \) cubic feet. Find the height if the width is \( (4x + 3) \) feet and the length is \( (x - 6) \) feet.
In Problems 1 – 4, write each number as a product of prime factors.

1. 12  
2. 18  
3. 72  
4. 125

5. Use the Distributive Property to remove the parentheses: \(-8(3x - 2)\)

6. Multiply and simplify: \(7u^3(u^2 + 4u - 1)\)

7. List the prime numbers less than 30.

8. Given \(12 \cdot 8 = 96\)
   (a) list the factors
   (b) identify the product

9. Identify the missing factor: \(6x^3 \cdot ? = 18x^5y\)
**Guided Practice 4.4**
Greatest Common Factor; Factoring by Grouping

**Objective 1: Factor Out the Greatest Common Factor**

1. Factor out the greatest common factor: \( 6m^4n^2 + 18m^3n^4 - 22m^2n^5 \)  (See textbook Example 2)

<table>
<thead>
<tr>
<th>Step 1: Find the GCF.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) GCF = __________</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Step 2: Rewrite each term as the product of the GCF and remaining factor.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(b) ( 6m^4n^2 + 18m^3n^4 - 22m^2n^5 = ) ____________________________</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Step 3: Factor out the GCF.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(c) ( 6m^4n^2 + 18m^3n^4 - 22m^2n^5 = ) ____________________________</td>
</tr>
</tbody>
</table>

**Step 4: Check**  
Distribute to verify that the factorization is correct.

2. Factor out the greatest common binomial factor: \( 7(a - 1)^2 - 14(a - 1) \)  (See textbook Example 4)

2. _________________

**Objective 2: Factor by Grouping**

3. Factor by grouping is commonly used when a polynomial has _____ terms.

4. *True or False*  You may need to rearrange the terms in order to be able to identify a common binomial factor.

4. _________________
5. Factor by grouping: \(2x^2 - 4x + 3xy - 6y\)  (See textbook Example 5)

<table>
<thead>
<tr>
<th>Step 1: Group terms with common factors. In this problem the first two terms have a common factor and the last two terms have a common factor.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) common factor of (2x^2 - 4x): ________________________________</td>
</tr>
<tr>
<td>(b) common factor of (3xy - 6y): ________________________________</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Step 2: In each grouping, factor out the common factor.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(c) (2x^2 - 4x + 3xy - 6y = ) ________________________________</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Step 3: Factor out the common factor that remains.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(d) (2x^2 - 4x + 3xy - 6y = ) ________________________________</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Step 4: Check</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiply to verify that the factorization is correct.</td>
</tr>
</tbody>
</table>
Do the Math Exercises 4.4
Greatest Common Factor; Factoring by Grouping

In Problems 1 – 7, factor out the greatest common factor.

1. \(8z + 48\)  
2. \(-4b + 32\)

3. \(12a^2 + 45a\)  
4. \(-6q^3 + 36q^2 - 48q\)

5. \(-18b^3 + 10b^2 + 6b\)  
6. \(6z(5z + 3) + 5(5z + 3)\)

7. \(8a^4b^2 + 12a^3b^3 - 36ab^4\)

In Problems 8 – 13, factor by grouping.

8. \(8x - 8y + bx - by\)  
9. \(3y^3 + 9y^2 - 5y - 15\)

10. \(3a^2 - 15a - 9a + 45\)  
11. \(2y^2 + 14y^2 - 4y^2 - 28y\)

12. \((x + 5)(x - 3) - (x - 1)(x - 3)\)  
13. \(c^3 - c^2 + 5c - 5\)
14. **Surface Area**  The surface area of a cylindrical can whose radius is \( r \) inches and height is 4 inches is given by \( S = 2\pi r^2 + 8\pi r \). Express the surface area in factored form.

15. **Summer Clearance**  Suppose that an electronics store decides to sell last year’s model televisions for a 20% discount.

   (a) Let \( x \) represent the original price of the television. Write an algebraic expression representing the selling price of the television.

   (b) After 1 month, the manager of the store discounts the TVs by another 15%. Write an algebraic expression representing the sale price of the televisions in terms of \( x \), the original selling price.

   (c) Write the algebraic expression in simplified form.

   (d) If the original price of the television was $650, what is the sale price after the second discount?

16. **The Better Deal**  Which is the better deal: (a) receiving a 30% discount or (b) receiving a 15% discount and then another 15% discount after the first 15% discount was applied? Make up an example that demonstrates how you came to this conclusion.
1. Determine the coefficients of $3x^2 - 4x - 7$.

In Problems 2 and 3, find (a) the sum of the numbers and (b) the product of the numbers.

2. $-9$ and $-4$
3. $-12$ and $3$

In Problems 4–7, find two integers with the following properties.

4. sum of 3 and product of $-28$

5. sum of $-15$ and product of 54

6. sum of 16 and product of 48

7. sum of 18 and product of $-63$

8. List the factors of 24 whose sum is 10.

9. List the factors of $-36$ whose sum is $-9$
**Objective 1: Factor Trinomials of the Form \(x^2 + bx + c\)**

**Step 1:** Find the pair of integers whose product is \(c\) and whose sum is \(b\). That is, determine \(m\) and \(n\) such that \(mn = c\) and \(m + n = b\).

**Step 2:** Write \(x^2 + bx + c = (x + m)(x + n)\).

**Step 3:** Check your work by multiplying out the factored form.

1. Factor: \(y^2 + 9y + 18\) (See textbook Example 1)

   **Step 1:** We are looking for factors of \(c = 18\) whose sum is \(b = 9\). We begin by listing all factors of 18 and computing the sum of these factors.

<table>
<thead>
<tr>
<th>Factors whose product is 18</th>
<th>1,18</th>
<th>2,9</th>
<th>3,6</th>
<th>−1, −18</th>
<th>−2, −9</th>
<th>−3, −6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sum of factors</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   Which two factors add to 9 and multiply to 18? **(b)**

   **Step 2:** We write the trinomial in the form \((y + m)(y + n)\).

   **(c)** \(y^2 + 9y + 18 = \) __________

   **Step 3:** Check We multiply to verify our solution.

   We leave it to you to verify the factorization.

2. How can you tell if a polynomial is prime (not factorable)? (See textbook Example 4) ________________

In addition to the textbook example, if the trinomial is of the form \(ax^2 + bx + c\), the polynomial is only factorable when the expression \(b^2 - 4ac\) results in a perfect square. Evaluating \(b^2 - 4ac\) for the trinomial \(y^2 + 9y + 18\) results in 9 so the trinomial is factorable. Evaluating \(b^2 - 4ac\) for the trinomial \(y^2 + 4y + 12\) results in \(-32\) so the trinomial is not factorable.

**Objective 2: Factor Trinomials of the Form \(ax^2 + bx + c, a \neq 1\)**

3. There are two possible methods for factoring trinomials of this type. Name the two methods.

   **(a)** __________________________

   **(b)** __________________________

After Problem 6, we will present a third method that is not in the textbook.
4. Factor by grouping: \(3x^2 - 13x + 12\) (See textbook Example 7)

**Step 1:** Find the value of \(ac\).

(a) Identify the coefficients: \(a = \), \(c = \)

(b) The value of \(a \cdot c = \)

**Step 2:** We want to determine the integers whose product is 36 and whose sum is -13. We know the factors must have the same sign in order to have a positive product. For the sum to be negative, both factors must be negative.

(c) Factors whose product is 36

<table>
<thead>
<tr>
<th>-1, -36</th>
<th>-2, -18</th>
<th>-3, -12</th>
<th>-4, -9</th>
<th>-6, -6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sum of factors</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(d) Which two factors multiply to 36 and add to -13? ________________

**Step 3:** Write

\[ax^2 + bx + c = ax^2 + mx + nx + c\]

(e) \(3x^2 - 13x + 12 = \)

**Step 4:** Factor the expression in Step 3 by grouping.

(f) __________________________  

______________________________________________

**Step 5:** Check

FOIL to verify that the factorization is correct.

5. Factor by trial and error: \(48x^2 - 4xy - 30y^2\) (See textbook Examples 9 – 12)

**Objective 3: Factor Trinomials Using Substitution**

6. Factor: \(12p^4 - p^2 - 1\) (See textbook Example 13)

**Step 1:** Rewrite the trinomial in the form \(au^2 + bu + c\).

(a) Let \(u = \)

Substitute.

(b) \(12p^4 - p^2 - 1 = \)

**Step 2:** Factor the trinomial from part (b).

(c) __________________________

**Step 3:** Rewrite the factored expression from part (c), in \(p\).

(d) __________________________

**Alternative Approach to Factor \(ax^2 + bx + c, a \neq 1\), Using Synthetic Factoring**

Another method for factoring trinomials of the form \(ax^2 + bx + c, a \neq 1\), is synthetic factoring. When using this method, it is very important to first factor out any common factors.
EXAMPLE: How to Factor $ax^2 + bx + c$, $a \neq 1$, by Synthetic Factoring

Factor: $6x^2 + 11x + 3$

First, notice that $6x^2 + 11x + 3$ has no common factors. In this trinomial, $a = 6$, $b = 11$, and $c = 3$.

### Step 1: Find the value of $ac$

The value of $a \cdot c$ is $6 \cdot 3 = 18$.

### Step 2: Find the pair of integers, $m$ and $n$, whose product is $ac$ and whose sum is $b$.

We want to find integers whose product is 18 and whose sum is 11. Because both 18 and 11 are positive, we only list the positive factors of 18.

<table>
<thead>
<tr>
<th>Factors whose product is 18</th>
<th>1, 18</th>
<th>2, 9</th>
<th>3, 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sum of factors</td>
<td>19</td>
<td>11</td>
<td>9</td>
</tr>
</tbody>
</table>

We have $m = 2$, $n = 9$, and $a = 6$, so $\frac{a}{m} = \frac{6}{2}$ and $\frac{a}{n} = \frac{6}{9}$.

### Step 3: Write two fractions in the form $\frac{a}{m}$ and $\frac{a}{n}$.

We have $m = 2$, $n = 9$, and $a = 6$, so $\frac{a}{m} = \frac{6}{2}$ and $\frac{a}{n} = \frac{6}{9}$.

### Step 4: Write each fraction in lowest terms so that $\frac{ap}{mq}$ and $\frac{ar}{ns}$.

We have $m = 2$, $n = 9$, and $a = 6$, so $\frac{a}{m} = \frac{6}{2}$ and $\frac{a}{n} = \frac{6}{9}$.

### Step 5: Write the factors $(px + q)(rx + s)$.

From Step 4, we have $p = 3$, $q = 1$, $r = 2$, and $s = 3$, so

$6x^2 + 11x + 3 = (3x + 1)(2x + 3)$

### Step 6: Check by multiplying the factors.

Multiply $(3x + 1)(2x + 3)$ to verify that the factorization is correct.

---

We summarize below the steps used in the example above.

**Factoring $ax^2 + bx + c$, $a \neq 1$, By Synthetic Factoring, Where $a$, $b$, and $c$ Have No Common Factors**

**Step 1:** Find the value of $ac$

**Step 2:** Find the pair of integers, $m$ and $n$, whose product is $ac$ and whose sum is $b$.

**Step 3:** Write two fractions in the form $\frac{a}{m}$ and $\frac{a}{n}$.

**Step 4:** Write each fraction in lowest terms so that $\frac{a}{m} = \frac{p}{q}$ and $\frac{a}{n} = \frac{r}{s}$.

**Step 5:** Write the factors $(px + q)(rx + s)$.

**Step 6:** Check by multiplying the factors.

7. Factor using synthetic factoring: $-16x^2 - 4x + 6$

(a) Factor out the common factor: ____________________________
(b) Determine each of the following using part (a): \(a = \ldots\), \(b = \ldots\), \(c = \ldots\), \(a \cdot c = \ldots\)

<table>
<thead>
<tr>
<th>Factors of (-24)</th>
<th>()</th>
<th>()</th>
<th>()</th>
<th>()</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sum</td>
<td>()</td>
<td>()</td>
<td>()</td>
<td>()</td>
</tr>
</tbody>
</table>

(c) Which two factors multiply to \(a \cdot c\) and add to \(b\)\? ______

(d) \(\frac{a}{m} = \frac{?}{?}\) \(\frac{p}{q} = \frac{?}{?}\) \(p = \ldots\) and \(q = \ldots\)

(e) \(\frac{a}{n} = \frac{?}{?}\) \(\frac{r}{s} = \frac{?}{?}\) \(r = \ldots\) and \(s = \ldots\)

(f) Write the factors \((px + q)(rx + s)\) __________________________

(g) Write the factored form of \(-16x^2 - 4x + 6\) ________________________

**Work Smart**

Let’s compare factoring by grouping and synthetic factoring by factoring \(3x^2 + 10x + 8\). First, notice there is no GCF in the trinomial.

**Grouping**

**Step 1:** For \(3x^2 + 10x + 8\), \(a = 3\), \(b = 10\), and \(c = 8\), so \(a \cdot c = 3 \cdot 8 = 24\).

**Step 2:** The factors of 24 whose sum is \(b = 10\) are \(6\) and \(4\).

**Step 3:** Write \(3x^2 + 10x + 8\) as \(3x^2 + 10x + 8 = 3x^2 + 6x + 4x + 10\).

**Step 4:** Factor \(3x^2 + 6x + 4x + 8\) by grouping.

\[
3x^2 + 6x + 4x + 8 = (3x^2 + 6x) + (4x + 8) \\
= 3x(x + 2) + 4(x + 2) \\
= (x + 2)(3x + 4)
\]

**Synthetic Factoring**

**Step 1:** For \(3x^2 + 10x + 8\), \(a = 3\), \(b = 10\), and \(c = 8\), so \(a \cdot c = 3 \cdot 8 = 24\).

**Step 2:** The factors of 24 whose sum is \(b = 10\) are \(6\) and \(4\).

**Step 3:** \(\frac{a}{m} = \frac{3}{6}\) and \(\frac{a}{n} = \frac{3}{4}\)

**Step 4:** \(\frac{a}{m} = \frac{3}{6} = \frac{1}{2} = \frac{p}{q}\) and \(\frac{a}{n} = \frac{3}{4} = \frac{r}{s}\)

**Step 5:** \((px + q)(rx + s) = (x + 2)(3x + 4)\), so

\[
3x^2 + 10x + 8 = (x + 2)(3x + 4)
\]

Both methods give the same result. Many people prefer trial and error while others prefer a more directed process such as grouping or synthetic factoring. You might know a process which has not been discussed here. Which method do you prefer and why?

8. Factor: \(21n^2 - 18n^3 + 9n\)  
8. \ldots
Do the Math Exercises 4.5
Factoring Trinomials

In Problems 1 – 22, factor each polynomial completely. If the polynomial cannot be factored, say it is prime.

1. \( z^2 + 3z - 28 \)
2. \( q^2 + 2q - 80 \)

3. \( -p^2 + 3p + 54 \)
4. \( m^2 + 7mn + 10n^2 \)

5. \( -4s^2 - 32s - 48 \)
6. \( 6x^2 - 37x + 6 \)

7. \( 12r^2 + 11r - 15 \)
8. \( 20r^2 + 23r + 6 \)

9. \( 3m^2 + 7mn - 6n^2 \)
10. \( 4x^3 - 52x^2 + 144x \)

11. \( 54x^3y + 33x^2y - 72xy \)
12. \( y^4 + 5y^2 + 6 \)
Do the Math Exercises 4.5

13. \( r^2s^2 + 8rs - 48 \)
14. \( 3(z + 3)^2 + 14(z + 3) + 8 \)

15. \( 24m^2 + 58mn + 9n^2 \)
16. \( r^6 - 6r^3 + 8 \)

17. \( p^2 - 14pq + 45q^2 \)
18. \( 9(a + 2)^2 - 10(a + 2) + 1 \)

19. \( a^2 + a - 6 \)
20. \( 24y^2 + 39y - 18 \)

21. \( -24m^4n - 18m^2n^2 + 27mn^3 \)
22. \( 54x^3y + 33x^2y - 72xy \)
Five-Minute Warm-Up 4.6
Factoring Special Products

1. List the perfect squares that are less than 100. That is, find $1^2$, $2^2$, $3^2$ etc.

____________________________________________________________________________________

2. List the perfect cubes that are less than 100. That is, find $1^3$, $2^3$, $3^3$ etc.

____________________________________________________________________________________

In Problems 3 – 6, evaluate each expression.

3. $\left(\frac{5}{3}\right)^2$

4. $-8^2$

5. $\left(-\frac{4}{3}\right)^2$

6. $\left(-\frac{5}{2}\right)^3$

In Problems 7 and 8, simplify each expression.

7. $(4x^2y)^3$

8. $\left(\frac{3}{2}ab^3\right)^2$

9. Multiply: $(3x - 2)^2$
Guided Practice 4.6
Factoring Special Products

Objective 1: Factor Perfect Square Trinomials

1. Find the product:
   (a) \((A + B)^2 = \) __________________________
   (b) \((A - B)^2 = \) __________________________

2. Factor the perfect square trinomial \(p^2 + 18p + 81\). \((See\ textbook\ Example\ 1)\)
   \[\text{Step 1: Write the trinomial in the form } A^2 + 2AB + B^2.\]
   \[A = \] __________________________
   \[B = \] __________________________
   \[p^2 + 18p + 81 = \] __________________________
   \[\text{Step 2: Factor using } A^2 + 2AB + B^2 = (A + B)^2.\]
   \[\text{(a) Let } A = \] __________________________
   \[\text{(b) Let } B = \] __________________________
   \[\text{(c) } \] __________________________
   \[\text{(d) } \] __________________________

3. What has to be added to \(4x^2 + 36x\) to make it a perfect square trinomial? __________________________

Objective 2: Factor the Difference of Two Squares

4. Find the product: \((A - B)(A + B) = \) __________________________

5. Factor each difference of two squares completely. \((See\ textbook\ Example\ 2)\)
   \[(a) \ x^2 - 64\]
   \[(b) \ 25m^6 - 36n^4\]
   \[5a. \] __________________________
   \[5b. \] __________________________

6. When a polynomial has four or more terms, try factoring by grouping. In Section 4.4, we grouped two terms in each group. Here we will group three terms to form a perfect square trinomial (which ultimately factors into the difference of two perfect squares). Factor: \(x^2 + 4x + 4 - 4y^2\). \((See\ textbook\ Example\ 3)\)
   \[\text{Step 1: The first three terms form a perfect square trinomial.}\]
   \[\text{Group the first three terms.}\]
   \[x^2 + 4x + 4 - 4y^2 = \] __________________________
   \[\text{Step 2: Rewrite the expression as the difference of two squares. Use } A^2 + 2AB + B^2 = (A + B)^2 \text{ to factor the perfect square trinomial.}\]
   \[\text{(b) } \] __________________________
   \[\text{Step 3: Use } A^2 - B^2 = (A + B)(A - B)\]
   \[\text{(c) } \] __________________________
   \[\text{Step 4: Check. Distribute to verify that factorization is correct.}\]
Objective 3: Factor the Sum or Difference of Two Cubes

7. Find the product:
   (a) \((A + B)(A^2 - AB + B^2)\) = ______________________________________________________________________
   (b) \((A - B)(A^2 + AB + B^2)\) = ______________________________________________________________________
   (c) \((A + B)^3\) = ______________________________________________________________________

8. Factor the sum of two cubes: \(p^3 + 64\). (See textbook Example 4)

   **Step 1:** Write \(p^3 + 64\) in the form \(A^3 + B^3\).
   (a) Let \(A = \) ____________________________
   (b) Let \(B = \) ____________________________
   (c) \(A^3 + B^3 = \) ____________________________

   **Step 2:** Factor using \(A^3 + B^3 = (A + B)(A^2 - AB + B^2)\).
   (d) \(p^3 + 64 \) ____________________________

9. Factor completely: \(24x^4 + 375x\). (See textbook Example 5)

   **Step 1:** Factor out the greatest common factor.  \(24x^4 + 375x = \) (a) ____________________________

   **Step 2:** Rewrite the sum of two cubes in the form \(A^3 + B^3\).
   (b) ____________________________

   **Step 3:** Factor completely using \(A^3 + B^3 = (A + B)(A^2 - AB + B^2)\).
   (c) ____________________________
Do the Math Exercises 4.6
Factoring Special Products

In Problems 1 – 4, factor each perfect square trinomial completely.

1. \(9z^2 - 6z + 1\)

2. \(36b^2 + 84b + 49\)

3. \(4a^2 + 20ab + 25b^2\)

4. \(b^4 + 8b^3 + 16\)

In Problems 5 – 7, factor the difference of two squares completely.

5. \(81 - a^2\)

6. \(x^4 - 9y^2\)

7. \(36x^2z - 64y^2z\)

In Problems 8 – 12, factor the sum or difference of two cubes completely.

8. \(z^3 + 64\)

9. \(216 - n^3\)

10. \(16m^3 + 54n^3\)

11. \((2z + 3)^3 + 27z^3\)

12. \(m^9 + n^{12}\)
Do the Math Exercises 4.6

In Problems 13 – 24, factor each polynomial completely.

13. \(9a^2 - b^2\) \hspace{1cm} 14. \(64x^3 - 125\)

15. \(3m^4 - 81mn^3\) \hspace{1cm} 16. \(p^4 - 18p^2 + 81\)

17. \(9m^2n^2 - 30mn + 25\) \hspace{1cm} 18. \(p^2 + 8p + 16 - q^2\)

19. \(-5a^3 - 40\) \hspace{1cm} 20. \(36m^2 + 12mn + n^2 - 81\)

21. \(y^2 - 3y + 9\) \hspace{1cm} 22. \(p^2 - 4p + 4 - q^2\)

23. \(x^2 + 0.6x + 0.09\) \hspace{1cm} 24. \(\frac{a^2}{36} - \frac{b^2}{49}\)
In Problems 1 – 4, find each product.

1. \((-4x^2y^4)(2xy^3)\)  
2. \(\frac{4}{3}ab\left(\frac{9}{2}a^2b - \frac{3}{4}a^3 + \frac{15}{8}b\right)\)

3. \((4x + 3y)(5x - 2y)\)  
4. \((a + 4b)^2\)

In Problems 5 and 6, factor completely.

5. \(-27a^3 + 9a^2 - 18a\)  
6. \(4p^2 - 8pq + 3p - 6q\)
Objective 1: Factor Polynomials Completely

1. Review the Steps for Factoring listed at the beginning of Section 4.7. Step 1 should always be __________________________, if possible.

2. Factor: $-12x^3 - 20x^2y + 8xy^2$ (See textbook Example 1)

Step 1: Factor out the greatest common factor (GCF), if any exists.
   - Determine the GCF: 
     
   - Factor out the GCF:

Step 2: Count the number of terms.
   - How many terms are in the polynomial in parentheses?

Step 3: We concentrate on the trinomial in parentheses. It is not a perfect square trinomial. Use either factoring by grouping or the trial and error method to factor the trinomial whose leading coefficient is not 1.
   - Factor the trinomial:

Step 4: Check
   - We leave it to you to multiply and then distribute to verify the factors are correct.

3. Factor: $64a^2 - 25b^4$ (See textbook Example 2)

Step 1: Factor out the greatest common factor (GCF), if any exists.
   - There is no GCF.

Step 2: Count the number of terms.
   - How many terms are in the polynomial?

Step 3: Because the first term $64a^2 = (8a)^2$, and the second term, $25b^4 = (5b^2)^2$, are both perfect squares, we have the difference of two squares.
   - Factor the binomial:

Step 4: Check
   - We leave it to you to multiply to verify the factors are correct.
Guided Practice 4.7

Objective 2: Write Polynomial Functions in Factored Form

4. Write the polynomial function \( f(x) = -x^3 - 2x^2 + 5x + 10 \) in factored form. *(See textbook Example 8)*

Step 1: Factor out the greatest common factor (GCF), if any exists. (a)___________________________

Step 2: Attempt to factor by grouping. Group the first two terms and the last two terms. 
\[ f(x) = -x^3 - 2x^2 + 5x + 10 = \text{(b)___________________________} \]

Step 3: In each grouping, factor out the common factor. (c)___________________________

Step 4: Factor out the common binomial factor. (d)___________________________

Step 5: Check. We leave it to you to verify that the factorization is complete.

5. What does factored completely mean?
Do the Math Exercises 4.7
Factoring: A General Strategy

In Problems 1 – 20, factor each polynomial completely.

1. \( 3x^2 + 6x - 105 \)
2. \(-5a^2 + 80 \)

3. \( 8m^2 - 42m + 49 \)
4. \( 54p^6 - 2q^3 \)

5. \(-4c^3 + 16c^2 - 28c \)
6. \(18t^2 - 9t - 20 \)

7. \(12p^2 + 50q^2 \)
8. \(16w^4 - 1 \)

9. \(4w^2 - 3w - 6 \)
10. \(20p^4q - 2p^2q^2 - 4pq \)

11. \(54p^5 + 16p^2q^3 \)
12. \(4z^4 - 25 \)
Do the Math Exercises 4.7

13. \(4b^4 + 4b^2 - 15\)

14. \((3x + 5)^2 + 4(3x + 5) - 21\)

15. \(a^2 + 12a + 36 - 4b^2\)

16. \(w^6 + 4w^3 - 5\)

17. \(q^6 + 1\)

18. \(-2y^3 - 4y^2 + 32y + 64\)

19. \(-5z - 20z^3\)

20. \(18h^5 + 154h^3 - 72h\)

In Problems 21 – 24, factor each polynomial function.

21. \(f(x) = x^2 + 3x - 40\)

22. \(H(p) = 5p^2 + 28p - 12\)

23. \(g(x) = -100x^2 + 36\)

24. \(G(x) = 2x^3 - x^2 - 18x + 9\)
In Problems 1 and 2, solve each equation.

1. \(2x - 3 = 0\)

2. \(5(2x - 1) + 7 = 0\)

3. Evaluate \(3x^2 - 5x + 6\) when (a) \(x = 9\) and (b) \(x = -\frac{1}{2}\).

4. If \(f(x) = -2x + 3\), solve \(f(x) = -3\). What point is on the graph of \(f\)?

5. If \(f(x) = \frac{7}{5}(2x + 5)\), find \(f(-5)\). What point is on the graph of \(f\)?

6. Find the zero of \(f(x) = \frac{8}{3}x + 4\).
Guided Practice 4.8
Polynomial Equations

Objective 1: Solve Polynomial Equations Using the Zero-Product Property

1. State the Zero-Product Property. _______________________________________________________
   ____________________________________________________________________________________
   ____________________________________________________________________________________

2. A second-degree equation is also called a ________________________________________________

3. What does it mean to write a quadratic equation in standard form? _______________________
   ____________________________________________________________________________________

4. Solve: \(2x^2 - 3x = 2\) (See textbook Example 2)

   **Step 1:** Write the quadratic equation in standard form, \(ax^2 + bx + c = 0\).
   \[2x^2 - 3x = 2\]
   Subtract 2 from both sides: (a) ____________________________

   **Step 2:** Factor the expression on the left side of the equation.
   (b) ____________________________

   **Step 3:** Set each factor to 0.
   (c) ____________________________ (d) ____________________________

   **Step 4:** Solve each first-degree equation.
   (e) ____________________________ (f) ____________________________

   **Step 5:** Check Substitute your values into the original equation.
   We leave it to you to verify the solutions.
   Write the solution set: (g) ____________________________

5. Solve: \((x - 2)(x - 3) = 56\) (See textbook Example 3)

   (a) To solve this equation, the first step is ____________________________

   (b) Next, write the quadratic equation in ____________________________

   (c) Solve and state the solution set. ____________________________
6. Solve: \( p^3 + 2p^2 - 9p = 18 \)  \( \text{(See textbook Example 4)} \)

**Step 1:** We put the equation in standard form by subtracting 18 from both sides of the equation.

\[ p^3 + 2p^2 - 9p = 18 \]

(a) \[ \]  

**Step 2:** Factor the expression on the left side of the equation. Because there are four terms, we factor by grouping.

(b) \[ \]

**Step 3:** Set each factor to 0.

(c) \[ \]

(d) \[ \]

(e) \[ \]

**Step 4:** Solve each first-degree equation.

(f) \[ \]

(g) \[ \]

(h) \[ \]

**Step 5:** Check Substitute your values into the original equation. We leave it to you to verify the solutions.

(i) \[ \]

**Objective 2: Solve Equations Involving Polynomial Functions**

7. Suppose \( f(x) = 3x^2 - 13x - 10 \). \( \text{(See textbook Example 6)} \)

(a) Write an equation to find the zeros of \( f \).

(a) \[ \]

(b) Use the equation from part (a) to find the zeros of \( f \).

(b) \[ \]

(c) What are the \( x \)-intercepts of the graph of the function?

(c) \[ \]

**Objective 3: Model and Solve Problems Involving Polynomials**

8. The length and width of two sides of a rectangle are consecutive odd integers. The area of the rectangle is 255 square centimeters. Find the dimensions of the rectangle.

**Step 1:** Identify This is a geometry problem involving the area of a rectangle. It also involves consecutive odd integers.

(a) If \( n \) represents one of the odd integers, express the next consecutive odd integer: \[ \]

(b) In general, what formula do we use to calculate the area of a rectangle?

(b) \[ \]

(c) **Step 2:** Name Let \[ \] represent the width of the rectangle and \[ \] represent the length.

(d) **Step 3:** Translate Write an equation that will model the area of the rectangle.

(d) \[ \]

**Step 4:** Solve the equation from step 3.

(e) \[ \]

**Step 5:** Check Is your answer reasonable? \[ \] Does it meet the necessary conditions? \[ \]

**Step 6:** Answer the question.

(g) \[ \]
Do the Math Exercises 4.8
Polynomial Equations

In Problems 1 – 14, solve each equation.

1. \(2x(3x + 4) = 0\)
2. \(3a(a - 9)(a + 11) = 0\)

3. \(5c^2 + 15c = 0\)
4. \(x^2 + 3x - 40 = 0\)

5. \(a^2 + 12a + 36 = 0\)
6. \(4c^2 + 6 = 25c\)

7. \(6z^2 + 17z = -5\)
8. \(-6n^2 - 9n + 60 = 0\)

9. \(\frac{2}{3}x^2 + \frac{7}{3}x = 5\)
10. \(y(y + 4) = 45\)

11. \(w^3 + 5w^2 - 16w - 80 = 0\)
12. \(-24a^3 + 27a = 18a^2\)
13. \((x + 7)(x - 3) = 11\)

14. \(-7z^2 + 42z = 0\)

15. \(f(x) = 3\)

16. \(f(x) = 17\)

17. Find the zeros of the function \(h(x) = 8x^2 - 18x - 35\).

18. **Area** The base of a triangle is 4 meters shorter than its height. What are the height and base of the triangle if its area is 48 square meters?

19. **Landscape Design** Robert Boehm just designed a cloister (a rectangular garden surrounded by a covered walkway on all four sides). The outside dimensions of the garden are 12 feet by 8 feet, and the area of the garden and the walkway together are 252 square feet. What is the width of the walkway?

20. **Making a Box** A box is to be made from a rectangular piece of corrugated cardboard, where the length is 8 inches more than the width, by cutting a square piece 3 inches on side from each corner. The volume of the box is to be 315 cubic inches. Find the dimensions of the rectangular piece of cardboard.
Five-Minute Warm-Up 5.1
Multiplying and Dividing Rational Expressions

In Problems 1 – 2, factor completely.

1. \( x^2 - 3x - 10 \)
   
   1. ___________

2. \( 6z^2 + z - 2 \)
   
   2. ___________

3. Solve: \( x^2 + 9x + 20 = 0 \)
   
   3. ___________

4. Determine the reciprocal of \( \frac{9}{2} \).
   
   4. ___________

5. Determine which of the following are in the domain of the variable \( x \) for the expression \( \frac{2x}{x^3 - x - 6} \).
   
   (a) \( x = -3 \)  (b) \( x = 0 \)  (c) \( x = -2 \)

   5. ___________

In Problems 6 – 7, perform the indicated operation. Be sure to express the answer in lowest terms.

6. \( \frac{9}{25} \cdot \frac{10}{9} \)
   
   6. ___________

7. \( \frac{9}{4} \div \frac{75}{15} \)
   
   7. ___________
Guided Practice 5.1
Multiplying and Dividing Rational Expressions

**Objective 1: Determine the Domain of a Rational Expression**

1. To determine the domain of a rational expression we exclude values that cause ________________.

2. For each of the following rational expressions, determine the domain. *(See textbook Example 1)*

   (a) \( \frac{x - 4}{x + 2} \)
   
   (b) \( \frac{a^2 - 2a - 3}{a^2 - 9a + 18} \)

   2a. ________________
   
   2b. ________________

**Objective 2: Simplify Rational Expressions**

3. To simplify a rational expression, we use the Reduction Property. Remember to always factor first and then divide out common factors. *(See textbook Examples 2 and 3)*

   Simplify the rational expression \( \frac{x^2 - 8x - 20}{2x^2 + 3x - 2} \).

   \( \frac{x^2 - 8x - 20}{2x^2 + 3x - 2} = \)

   **Step 1:** Factor the numerator and denominator.  
   
   (a) __________________________

   **Step 2:** Divide out common factors using the Reduction Property.  
   
   (b) __________________________

**Objective 3: Multiply Rational Expressions** *(See textbook Example 4)*

4. Multiply \( \frac{x^2 - 4x}{x^2 - 4} \cdot \frac{x^2 - x - 6}{x - 4} \). Simplify the product.

   **Step 1:** Completely factor each polynomial in the numerator and denominator.

   \( \frac{x^2 - 4x}{x^2 - 4} \cdot \frac{x^2 - x - 6}{x - 4} = \)  
   
   (a) ________________ . ________________

   **Step 2:** Multiply using  
   
   \( \frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd} \)

   \( = \)  
   
   (b) ________________ . ________________

   **Step 3:** Divide out common factors in the numerator and denominator.

   \( = \)  
   
   (c) __________________________

   Express the answer as a simplified rational expression in factored form.  
   
   (d) __________________________
Guided Practice 5.1

**Objective 4: Divide Rational Expressions**

5. To divide two rational expressions, we rewrite the division problem as an equivalent multiplication problem and then follow the steps for multiplying rational expressions. If \( x, \ y, \ p \) and \( q \) are rational expressions and \( y \neq 0, \ p \neq 0, \ q \neq 0 \), then

\[
\frac{x}{y} \div \frac{p}{q} = \frac{xy}{pq}
\]

6. Divide the following rational expression. Simplify the quotient, if possible. *(See textbook Example 6)*

\[
\frac{27xy^3}{60x^2y^4} \div \frac{36x}{24y^5}
\]

**Step 1:** Rewrite the division problem as a multiplication problem.

(a) __________________________

**Step 2:** Multiply.

(b) __________________________

**Step 3:** Divide out common factors and simplify.

(c) __________________________

**Objective 5: Work with Rational Functions**

7. A **rational function** is a function of the form \( R(x) = \frac{p(x)}{q(x)} \) where \( p \) and \( q \) are polynomial functions and \( q \) is not the zero polynomial. The domain consists of all real numbers except those for which ______________.

8. Find the domain of \( R(x) = \frac{4x}{2x^2 + 6x - 80} \). *(See textbook Example 7)*

8. ___________
Do the Math Exercises 5.1
Multiplying and Dividing Rational Expressions

In Problems 1 – 3, state the domain of each rational function.

1. \( \frac{4}{x - 7} \)  
2. \( \frac{x - 2}{x^2 + 4} \)  
3. \( \frac{x + 5}{x^2 + 8x + 16} \)

In Problems 4 – 7, simplify each rational expression.

4. \( \frac{x^2 - 3x}{x^2 - 9} \)  
5. \( \frac{w^2 + 5w - 14}{w^2 + 6w - 16} \)  
6. \( \frac{x^2 - xy - 6y^2}{x^2 - 4y^2} \)  
7. \( \frac{v^3 + 3v^2 - 5v - 15}{v^2 + 6v + 9} \)

In Problems 8 – 13, multiply each rational expression. Simplify the product, if possible.

8. \( \frac{5x^2}{x + 3} \cdot \frac{x^2 + 7x + 12}{20x} \)  
9. \( \frac{3x^2 + 14x - 5}{x^2 + x - 30} \cdot \frac{x^2 - 2x - 15}{3x^2 + 8x - 3} \)  
10. \( \frac{2y^2 - 5y - 12}{2y^2 - y - 6} \cdot \frac{4y^2 - 5y - 6}{4 - y} \)  
11. \( \frac{a^2 + 2ab + b^2}{3a + 3b} \cdot \frac{b - a}{a^2 - b^2} \)
Do the Math Exercises 5.1

12. \( \frac{x^3 - 27}{2x^2 + 5x - 25} \cdot \frac{x^2 + 2x - 15}{x^3 + 3x^2 + 9x} \)

13. \( \frac{5m - 5}{m^2 + 6m} \cdot \frac{m^2 + 2m - 24}{m^2 + 3m - 4} \)

In Problems 14 – 17, divide each rational expression. Simplify the quotient, if possible.

14. \( \frac{x - 2}{3x} \div \frac{5x - 10}{x} \)

15. \( \frac{9m^3}{2n^2} \div \frac{3m}{8n^4} \)

16. \( \frac{y^2 - 9}{2y^2 - y - 15} \div \frac{3y^2 + 10y + 3}{2y^2 + y - 10} \)

17. \( \frac{x^2 + 2xy + y^2}{x^2 + 3xy + 2y^2} \div \frac{x^2 - y^2}{x + 2y} \)

In Problems 18 and 19, determine the domain of each rational function.

18. \( R(x) = \frac{3x + 2}{(4x - 1)(x + 5)} \)

19. \( R(x) = \frac{4x}{4x^2 + 1} \)

20. If \( f(x) = \frac{x^2 - 7x - 8}{2x - 5} \), \( g(x) = \frac{2x^2 + 3x - 20}{x^2 - 10x + 16} \), and \( h(x) = \frac{x^2 - 3x - 40}{x + 9} \), find

(a) \( R(x) = f(x) \cdot g(x) \)

(b) \( R(x) = \frac{f(x)}{h(x)} \)

20a. _________

20b. _________
Five-Minute Warm-Up 5.2
Adding and Subtracting Rational Expressions

1. Determine the additive inverse of $-21$.

2. Determine the least common denominator: \( \frac{7}{24} \) and \( \frac{14}{75} \).

3. Using the rational numbers in problem 3, rewrite each number with LCD.

4. \( \frac{9}{4} + \frac{15}{4} \)

5. \( \frac{17}{30} + \frac{19}{45} \)

6. \( \frac{9}{28} - \frac{15}{28} \)

7. \( \frac{5}{16} - \left( -\frac{5}{24} \right) \)

In Problems 4 – 7, perform the indicated operation. Be sure to express the result in lowest terms.
## Guided Practice 5.2
### Adding and Subtracting Rational Expressions

**Objective 1: Add or Subtract Rational Expressions with a Common Denominator**

1. To add or subtract rational expressions with a common denominator we use the following properties:

\[
\frac{a}{c} + \frac{b}{c} = \frac{a + b}{c} \quad \text{and} \quad \frac{a}{c} - \frac{b}{c} = \frac{a - b}{c}
\]

where \(\frac{a}{c}\) and \(\frac{b}{c}\) are two rational expressions, \(c \neq 0\), and then simplify the result.

Perform the indicated operation: \(\frac{x^2 - 3}{2x + 3} + \frac{x^2 + x}{2x + 3}, x \neq -\frac{3}{2}\) (See textbook Example 1)

**Step 1:** Add the numerators and write the result over the common denominator

\[
\frac{x^2 - 3}{2x + 3} + \frac{x^2 + x}{2x + 3} = \frac{x^2 - 3 + x^2 + x}{2x + 3}
\]

(a) ________________________

Combine like terms in the numerator:

(b) ________________________

**Step 2:** Simplify the rational expression.

Factor the numerator:

(c) ________________________

Divide out common factors and express the answer as a simplified rational expression in factored form.

(d) ________________________

2. Perform the indicated operation: \(\frac{x^2 - 11}{x^2 - 25} + \frac{-3x - 1}{25 - x^2}, x \neq 5, x \neq -5\) (See textbook Examples 1 and 2)

**Step 1:** Subtract the numerators and write the result over the common denominator

\[
\frac{x^2 - 11}{x^2 - 25} + \frac{-3x - 1}{25 - x^2} = \frac{x^2 - 11 + (-3x - 1)}{x^2 - 25}
\]

(a) ________________________

Factor -1 from \(25 - x^2\) and use

\[
\frac{a}{-b} = \frac{-a}{b}
\]

to find the LCD.

Write the sum on the LCD.

(b) ________________________

Distribute the -1:

(c) ________________________

**Step 2:** Simplify the rational expression.

Factor the numerator and the denominator:

(d) ________________________

Divide out common factors and express the answer as a simplified rational expression in factored form.

(e) ________________________
Objective 2: Find the Least Common Denominator of Two or More Rational Expressions

3. List the steps for finding the least common denominator (LCD) of two or more rational expressions.

Objective 3: Add or Subtract Rational Expressions with Different Denominators

4. List the steps for adding or subtracting rational expressions with unlike denominators.

5. Add \( \frac{3}{x + 2} + \frac{8 - 2x}{x^2 - 4} \). Simplify the result, if possible. (See textbook Example 5)

(a) What is the LCD?

(b) Express each rational expression as an equivalent expression written on the common denominator.

(c) Add and simplify the result, if possible.

6. Subtract \( \frac{a}{a^2 + 12a + 20} - \frac{1}{a^2 + 8a - 20} \). Simplify the result, if possible. (See textbook Example 6)

(a) What is the LCD?

(b) Express each rational expression as an equivalent expression written on the common denominator.

(c) Subtract and simplify the result, if possible.

7. Perform the indicated operations and simplify the result, if possible. (See textbook Example 7)

\[ \frac{x}{x^2 - 2x + 1} - \frac{2}{x} - \frac{x + 1}{x^2 - x^3} \]

(a) What is the LCD?

(b) Express each rational expression as an equivalent expression written on the common denominator.

(c) Perform the indicated operations and simplify the result, if possible.
**Do the Math Exercises 5.2**

Adding and Subtracting Rational Expressions

In Problems 1 – 4, perform the indicated operation and simplify the result.

1. \( \frac{5x}{x-3} + \frac{2}{x-3} \)

2. \( \frac{9x}{6x-5} - \frac{2}{6x-5} \)

3. \( \frac{3x}{x-6} + \frac{2}{6-x} \)

4. \( \frac{x^2 + 2x - 5}{x-4} - \frac{x^2 - 5x - 15}{4-x} \)

In Problems 5 – 7, find the least common denominator.

5. \( \frac{1}{8a^3b} \) and \( \frac{5}{12ab^2} \)

6. \( \frac{2m - 7}{m^2 + 3m - 18} \) and \( \frac{5m + 1}{m^2 - 7m + 12} \)

7. \( \frac{x - 6}{x^2 - 9} \) and \( \frac{3x}{x^3 - 3x^2} \)

In Problems 8 – 17, add or subtract, as indicated, and simplify the result.

8. \( \frac{2}{9x} + \frac{5}{3x^2} \)

9. \( \frac{x + 2}{x - 3} - \frac{x + 2}{x + 1} \)

10. \( \frac{z + 1}{z + 3} - \frac{z + 17}{z^2 - z - 12} \)

11. \( \frac{x - 5}{x^2 + 4x + 3} + \frac{x - 2}{x^2 - 1} \)

12. \( \frac{m - 2n}{m^2 + 4mn + 4n^2} + \frac{m - n}{m^2 - mn - 6n^2} \)

13. \( \frac{3}{x^2 + 7x + 10} - \frac{4}{x^2 + 6x + 5} \)
14. \[ \frac{y^2 + 4y + 4}{y^2 - 9} + \frac{y^2 + 4y + 4}{9 - y^2} \] 15. \[ \frac{7}{m - 3} - \frac{5}{m} - \frac{2m + 6}{m^2 - 9} \]

16. \[ \frac{2}{x} - \frac{2}{x + 2} + \frac{2}{(x + 2)^2} \] 17. \[ 3 + \frac{x + 4}{x - 4} \]

18. Given that \( f(x) = \frac{5}{x + 2} \) and \( g(x) = \frac{3}{x - 1} \),

(a) find \( R(x) = f(x) + g(x) \)
(b) state the domain of \( R(x) \)

18a. ________  
18b. ________

19. **Surface Area of a Can** The volume of a cylindrical can is 200 cubic centimeters. Its surface area \( S \) as a function of the radius \( r \) of the can is given by the function:

\[ S(r) = 2\pi r^2 + \frac{400}{r} \]

(a) Write \( S \) over a common denominator. That is, write \( S \) so that the rule is a single rational expression.  
(b) Find and interpret \( S(4) \). Round your answer to two decimal places.

19a. ________  
19b. ________
1. Factor $8x^2 + 2x - 3$.

In Problem 2 – 5, simplify each expression.

2. $\frac{24pq^2}{30pq^3}$

3. $\frac{36x^3y^3}{80x^{12}y^5}$

4. $(2r^3)^4$

5. $\frac{y^4z^{-2}}{(y^4z^3)}$

In Problems 6 and 7, perform the indicated operation. Be sure to express the result in lowest terms.

6. $\frac{4}{27} \div \frac{18}{54}$

7. $\frac{18}{42} \div \left( -\frac{24}{49} \right)$
Guided Practice 5.3
Complex Rational Expressions

Objective 1: Simplify a Complex Rational Expression by Simplifying the Numerator and Denominator Separately (Method I)

1. In your own words, define a complex rational expression.

2. Simplify \( \frac{x}{x + \frac{1}{1 + \frac{1}{x - 1}}} \), \( x \neq 1, x \neq -1 \) using Method I. (See textbook Examples 1 and 2)

   **Step 1:** Write the denominator of the complex rational expression as a single rational expression.

   Determine the LCD of 1 and \( x - 1 \):

   (a) ____________________

   Write the equivalent rational expressions on the LCD and then add using \( \frac{a + b}{c} = \frac{a}{c} + \frac{b}{c} \):

   (b) ____________________

   **Step 2:** Write the numerator of the complex rational expression as a single rational expression.

   This is already done.

   **Step 3:** Rewrite the complex rational expression using the rational expressions determined in Steps 1 and 2.

   (c) ____________________

   **Step 4:** Simplify the rational expression using the techniques for dividing rational expressions from Section 5.1

   Rewrite the division problem as a multiplication problem:

   (d) ____________________

   Divide out common factors and express the answer as a simplified rational expression in factored form.

   (e) ____________________

Objective 2: Simplify a Complex Rational Expression Using the Least Common Denominator (Method II)

3. Method II uses the LCD to simplify complex rational expressions. We use several of the properties of real numbers to simplify the complex rational expression; that is, to find an equivalent rational expression which has a single fraction bar. State the property of real numbers that is illustrated below.

   (a) \( \frac{x - 9}{x - 9} = 1 \)

   3a. __________

   (b) \( \frac{2}{x} \cdot \frac{x - 9}{x - 9} = \frac{2}{x} \)

   3b. __________

   (c) \( \frac{2}{x} \cdot \frac{x - 9}{x - 9} = \frac{2x - 18}{x^2 - 9x} \)

   3c. __________
4. Simplify \( \frac{x^2}{x^2 - 16} - \frac{x}{x + 4} - \frac{1}{x - 4} \), \( x \neq 4, x \neq -4 \) using Method II. (See textbook Examples 3 and 4)

<table>
<thead>
<tr>
<th>Step 1: Find the least common denominator among all the denominators in the complex rational expression.</th>
<th>Determine the LCD of ( x^2 - 16, x + 4 ) and ( x - 4 ):</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) ( \frac{x^2}{x^2 - 16}, \frac{x}{x + 4}, \frac{1}{x - 4} )</td>
<td>(b) ( \frac{x^2 - 16}{x^2 - 16}, \frac{x + 4}{x + 4}, \frac{1}{x - 4} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Step 2: Multiply both the numerator and denominator of the complex rational expression by the LCD found in Step 1.</th>
<th>Distribute the LCD to each term:</th>
</tr>
</thead>
<tbody>
<tr>
<td>(c) ( \frac{x^2}{x^2 - 16} \cdot \frac{x^2 - 16}{x^2 - 16}, \frac{x}{x + 4} \cdot \frac{x + 4}{x + 4}, \frac{1}{x - 4} \cdot \frac{1}{x - 4} )</td>
<td>(d) ( \frac{x^2 - 16}{x^2 - 16}, \frac{x + 4}{x + 4}, \frac{1}{x - 4} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Step 3: Simplify the rational expression.</th>
<th>Simplify the expression:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Divide out the common factors:</td>
<td>(e) ( \frac{x^2 - 16}{x^2 - 16} - \frac{x + 4}{x + 4} - \frac{1}{x - 4} )</td>
</tr>
</tbody>
</table>

5. Simplify \( \frac{x^{-1} + 3^{-1}}{x^{-2} - 9^{-1}} \) as a rational expression that contains no negative exponents. (See textbook Example 5)

<table>
<thead>
<tr>
<th>(a) Rewrite the expression as a complex rational expression which does not contain negative exponents.</th>
<th>(b) Simplify by either Method I or Method II.</th>
</tr>
</thead>
<tbody>
<tr>
<td>5a. ( \frac{x^{-1} + 3^{-1}}{x^{-2} - 9^{-1}} )</td>
<td>5b. ( \frac{x^{-1} + 3^{-1}}{x^{-2} - 9^{-1}} )</td>
</tr>
</tbody>
</table>
Do the Math Exercises 5.3
Complex Rational Expressions

In Problems 1 – 14, simplify the complex rational expression using either Method I or Method II.

1. \[
\frac{1 + \frac{1}{x^2}}{1 - \frac{1}{x^2}}
\]

2. \[
\frac{\frac{7}{w} + \frac{9}{x}}{\frac{9}{w} - \frac{7}{x}}
\]

3. \[
\frac{\frac{a}{a+1} - 1}{\frac{a}{a+3} - \frac{1}{a-2}}
\]

4. \[
\frac{\frac{x+5}{x-2} - \frac{x+3}{x-1}}{\frac{3x+1}{3}}
\]

5. \[
\frac{x-4}{x-1} - \frac{x}{x-3}
\]

6. \[
\frac{\frac{1}{x} - \frac{4}{3}}{\frac{16}{z} - \frac{1}{z}}
\]

7. \[
\frac{n^2 - m^2}{m - n}
\]

8. \[
\frac{1 + \frac{5}{x}}{1 + \frac{1}{x} + 4}
\]
9. \( \frac{5 - x}{x} \) \( \frac{1 - \frac{5}{x^2}}{5} \)

10. \( \frac{x - 3}{x + 3} \) \( \frac{x - 3}{x - 4} \)

9. __________

10. __________

11. \( \frac{-6}{x^2 + 5x + 6} \) \( \frac{2 - \frac{3}{x + 2}}{x + 3} \)

12. \( \frac{2x^{-1} + 2y^{-1}}{xy^{-1} - x^{-1}y} \)

11. __________

12. __________

13. \( \frac{(x - y)^{-1}}{x^{-1} - y^{-1}} \)

14. \( \frac{a^{-3} + 8b^{-3}}{a^{-2} - 4b^{-2}} \)

13. __________

14. __________

15. Electric Circuits An electric circuit contains three resistors connected in parallel. If the resistance of each is \( R_1 \), \( R_2 \), and \( R_3 \) ohms, respectively, then their combined resistance is given by the formula

\[
R = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}
\]

(a) Express \( R \) as a simplified rational expression.

(b) Evaluate the rational expression if \( R_1 = 4 \) ohms, \( R_2 = 6 \) ohms and \( R_3 = 10 \) ohms.

15a. __________

15b. __________
Five-Minute Warm-Up 5.4
Rational Equations

1. Solve: \( \frac{3x + 5}{2} = 4 + \frac{4x + 2}{4} \)
   1. __________

2. Solve: \(-3p^2 + 6p = -72\)
   2. __________

3. Factor: \(3z^2 - 14z + 8\)
   3. __________

4. Determine which of the following are in the domain of the variable \(x\) for the expression \(\frac{x + 4}{x^2 + 2x - 8}\).
   (a) \(x = -4\)  (b) \(x = -2\)  (c) \(x = 2\)
   4. __________

5. Given \(f(x) = x^2 - 12x - 45\) solve \(f(x) = 0\).
   5. __________

6. If \(g(-2) = 12\), what is the point on the graph of \(g\)?
   6. __________
Guided Practice 5.4
Rational Equations

Objective 1: Solve Equations Containing Rational Expressions

1. When solving rational equations it is important to identify the domain of the variable. We exclude all values of the variable that result in __________________________________________________________.

2. Solve: \( \frac{x + 5}{x - 7} = \frac{x - 3}{x + 7} \) (See textbook Example 1)

   **Step 1:** Determine the domain of the variable in the rational equation.
   For what values of \( x \) will either denominator be equal to zero? (a) ____________________
   Express the domain of \( x \): (b) ____________________

   **Step 2:** Determine the least common denominator (LCD) of all the denominators.
   LCD: (c) ____________________

   **Step 3:** Multiply both sides of the equation by the LCD and simplify the expression on each side of the equation.
   Multiply both sides by the LCD: (d) ____________________
   Divide out like factors: (e) ____________________
   Multiply: (f) ____________________

   **Step 4:** Solve the resulting equation.
   Solve for \( x \): (g) ____________________

   **Step 5:** Check Verify your solution using the original equation.
   We leave it to you to verify your solution.
   Write the solution set: (h) ____________________

3. Solve: \( \frac{x}{12} + \frac{1}{2} = \frac{1}{3x} + \frac{2}{x^2} \) (See textbook Example 2)

   (a) Determine the domain of the variable. 3a. __________
   (b) Determine the LCD of all of the denominators. 3b. __________
   (c) Multiply both sides of the equation by the LCD. What is the resulting equation? 3c. __________
   (d) Solve the resulting equation. 3d. __________
Guided Practice 5.4

4. Solve: \( \frac{6}{x^2 - 1} - \frac{5}{x - 1} - \frac{3}{x + 1} \) (See textbook Example 4)

(a) Determine the domain of the variable. 4a. __________

(b) Determine the LCD of all of the denominators. 4b. __________

(c) Multiply both sides of the equation by the LCD and solve the resulting equation. What is the solution to this equation? 4c. __________

(d) What is the solution set? 4d. __________

5. Solve: \( \frac{5}{x - 4} + \frac{3}{x - 2} = \frac{x^2 - x - 2}{x^3 - 6x + 8} \) (See textbook Example 5)

(a) Determine the domain of the variable. 5a. __________

(b) Determine the LCD of all of the denominators. 5b. __________

(c) Multiply both sides of the equation by the LCD and solve the resulting equation. What is the solution to this equation? 5c. __________

(d) What is the solution set? 5d. __________

Objective 2: Solve Equations Involving Rational Functions

6. For the function \( f(x) = x + \frac{7}{x} \), \( f(x) = 8 \), what point(s) are on the graph of \( f \)? (See textbook Example 6)

(a) To begin, we write the equation: 6a. __________

(b) Determine the domain of the variable. 6b. __________

(c) Determine the LCD of all of the denominators. 6c. __________

(d) Multiply both sides of the equation by the LCD and solve the resulting equation. What is the solution to this equation? 6d. __________

(e) What is the solution set? 6e. __________

(f) Write the ordered pair(s) that are on the graph of \( f \). 6f. __________
Do the Math Exercises 5.4
Rational Equations

In Problems 1 – 12, solve each equation. Be sure to verify your results.

1. \[\frac{8}{p} + \frac{1}{4p} = \frac{11}{8}\]
2. \[\frac{2x + 1}{x + 3} = \frac{4(x - 1)}{2x + 3}\]

3. \[m + \frac{8}{m} = 6\]
4. \[8b - \frac{3}{b} = 2\]

5. \[2 - \frac{3}{p} + 2 = \frac{6}{p}\]
6. \[\frac{5}{x + 2} = 1 - \frac{3}{x - 2}\]

7. \[\frac{4}{x + 3} + \frac{5}{x - 6} = \frac{4x + 1}{x^2 - 3x - 18}\]
8. \[\frac{3}{x - 4} = \frac{5x + 4}{x^2 - 16} - \frac{4}{x + 4}\]

9. \[p + \frac{25}{p} = 10\]
10. \[\frac{3}{a^2 + 3a - 10} + \frac{2}{a^2 + 7a + 10} = \frac{4}{a^2 - 4}\]
11. \( \frac{9}{b} + \frac{4}{5b} = \frac{7}{10} \)  
12. \( \frac{x + 3}{x - 2} + 4 = \frac{x + 2}{x + 1} \)  

11. \underline{}  
12. \underline{}  

13. Solve: \( 2 + 11a^{-1} = -12a^{-2} \)  

13. \underline{}  

14. For the function \( f(x) = 2x + \frac{8}{x} \), solve \( f(x) = -10 \). What point(s) are on the graph of \( f \)?  

14. \underline{}  

15. Let \( f(x) = \frac{4x + 1}{8x + 5} \) and \( g(x) = \frac{x - 4}{2x - 7} \). For what value(s) of \( x \) does \( f(x) = g(x) \)? What are the point(s) of intersection of the graphs of \( f \) and \( g \)?  

15. \underline{}  

In Problems 16 – 19, simplify or solve.

16. \( \left( \frac{z^{-2}}{2z^{-3}} \right)^{-1} + 3(z - 1)^{-1} \)  
17. \( \frac{5}{x - 6} + \frac{2}{x + 2} = \frac{1}{x^2 - 4x - 12} \)  

16. \underline{}  
17. \underline{}  

18. \( \frac{2a^6}{a^3} - \frac{5a^2}{a^3} = \frac{3a}{a^3} \)  
19. \( \frac{3}{x - 2} - \frac{2x + 1}{x + 1} \)  

18. \underline{}  
19. \underline{}
Five-Minute Warm-Up 5.5
Rational Inequalities

1. Write in interval notation: \(-3 \leq x < 2\)

2. Solve and graph the solution set: 
   \(-5x + 1 > 9 + 3(2x + 1)\)

3. Solve and write the solution set in interval notation:
   \(\frac{3z - 1}{4} - 1 \leq \frac{6z + 5}{2}\)

4. Determine whether \(x = -2\) satisfies the inequality \(5x + 17 \geq 7\).
Guided Practice 5.5
Rational Inequalities

Objective 1: Solve a Rational Inequality

1. Solving rational inequalities depends on finding intervals where the polynomials in the rational expression are positive or negative. This means that the inequality must be of the form:
   \[
   \frac{p(x)}{q(x)} > \quad \text{or} \quad \frac{p(x)}{q(x)} \geq \quad \text{when the quotient is positive and}
   \]
   \[
   \frac{p(x)}{q(x)} < \quad \text{or} \quad \frac{p(x)}{q(x)} \leq \quad \text{when the quotient is negative}.
   \]

2. The quotient is positive when both \( p(x) \) and \( q(x) \) are ______________ or when both \( p(x) \) and \( q(x) \) are ______________.

3. The quotient is negative when \( p(x) \) is ______________ and \( q(x) \) is ______________ or when \( p(x) \) is ______________ and \( q(x) \) is ______________.

4. We always exclude the values that cause _________________.

5. Solve \( \frac{x + 2}{x - 4} \leq 0 \). Graph the solution set. (See textbook Example 1)

   **Step 1:** Write the inequality so that a rational expression is on one side of the inequality and zero is on the other. Be sure to write the rational expression as a single quotient.

   \( \frac{x + 2}{x - 4} \leq 0 \) is already in the required form.

   **Step 2:** Determine the numbers for which the rational expression equals 0 or is undefined.

   Value(s) when the rational expression equals 0:
   (a) ________________

   Value(s) when the rational expression is undefined:
   (b) ________________

   **Step 3:** Use the numbers found in Step 2 to separate the real number line into intervals.

   Select a test point in the interval \((-\infty, -2)\):
   (c) ________________

   Sign of \( x + 2 \):
   (d) ________________

   Sign of \( x - 4 \):
   (e) ________________

   Sign of the quotient: (f) ________________

   *continued next page*
Select a test point in the interval \((-2, 4)\):

\[(g)\] ____________________________

Sign of \(x + 2\):

\[(h)\] ____________________________

Sign of \(x - 4\):

\[(i)\] ____________________________

Sign of the quotient:

\[(j)\] ____________________________

Select a test point in the interval \((4, \infty)\):

\[(k)\] ____________________________

Sign of \(x + 2\):

\[(l)\] ____________________________

Sign of \(x - 4\):

\[(m)\] ____________________________

Sign of the quotient:

\[(n)\] ____________________________

In this problem, are we looking for the quotient to be positive or negative? \[(o)\] ____________________________

What value must be excluded? \[(p)\] ____________________________

Graph the solution set:

\[(q)\] ____________________________

6. Rewrite the rational expression \(\frac{5}{x - 1} - \frac{7}{x + 1} > 0\) as single rational expression which is positive.

6. __________

7. Rewrite the rational expression \(\frac{3x - 7}{x + 2} < 2\) as single rational expression which is negative.

7. __________

8. Complete the following chart used to solve \(\frac{(x + 4)(x - 3)}{x - 2} \geq 0\). (See textbook Example 2)

<table>
<thead>
<tr>
<th>Interval</th>
<th>((-\infty, -4))</th>
<th>(-4)</th>
<th>((-4, 2))</th>
<th>(2)</th>
<th>((2, 3))</th>
<th>(3)</th>
<th>((3, \infty))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test Point</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sign of (x + 4)</td>
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<tr>
<td>Sign of (x - 3)</td>
<td></td>
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<tr>
<td>Sign of (x - 2)</td>
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<td></td>
</tr>
<tr>
<td>Sign of quotient</td>
<td></td>
<td></td>
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<td></td>
<td></td>
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</tr>
<tr>
<td>Conclusion</td>
<td></td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

9. Graph the solution set to Problem 8.
In Problems 1 – 10, solve each rational inequality. Express your answer in interval notation.

1. \( \frac{x+8}{x+2} < 0 \)

2. \( \frac{x+12}{x-2} \geq 0 \)

3. \( \frac{x-10}{x+5} \leq 0 \)

4. \( \frac{(5x-2)(x+4)}{x-5} < 0 \)

5. \( \frac{(3x-2)(x-6)}{x+1} \geq 0 \)

6. \( \frac{x+3}{x-4} > 1 \)

7. \( \frac{3x-7}{x+2} \leq 2 \)

8. \( \frac{3x+20}{x+6} < 5 \)

9. \( \frac{2}{x+3} + \frac{2}{x} \leq 0 \)

10. \( \frac{1}{x-4} \geq \frac{3}{2x+1} \)
Do the Math Exercises 5.5

In Problems 11 and 12, for each function find the values of $x$ that satisfy the given condition.

11. Solve $R(x) \geq 0$ if $R(x) = \frac{x + 3}{x - 8}$

12. Solve $R(x) < 0$ if $R(x) = \frac{3x + 2}{x - 4}$

In Problems 13 and 14, find the $x$-intercept of the graph of each function.

13. $h(x) = -3x^2 - 7x + 20$

14. $R(x) = \frac{x^2 + 5x + 6}{x + 2}$

15. **Average Cost** Suppose that the daily cost $C$ of manufacturing $x$ bicycles is given by $C(x) = 90x + 5000$. Then the average daily cost $\bar{C}$ is given by $\bar{C}(x) = \frac{90x + 5000}{x}$.

How many bicycles must be produced each day in order for the average cost to be no more than $130?
1. Solve for $x$: $4x + 3y = -24$  

   \[ 1. \quad \text{___________} \]

2. Solve for $y$: $\frac{9x + 2y}{3} = 12$  

   \[ 2. \quad \text{___________} \]

Train Station  Two trains leave a station at the same time. One train is traveling east at 10 mph faster than the other train, which is traveling west. After 6 hours, the two trains are 720 miles apart. At what speed did the faster train travel?

\[ 3. \quad \text{___________} \]
Guided Practice 5.6
Models Involving Rational Expressions

Objective 1: Solve for a Variable in a Rational Expression

1. The expression “to solve for the variable” means to get the variable by itself on one side of the equation with all other variables and constants, if any, on the other side. Solve for the indicated variable.
   (See textbook Example 1)
   \[
   \frac{2}{x} - \frac{1}{y} = \frac{6}{z} \quad \text{for } y
   \]
   1. __________

Objective 2: Model and Solve Ratio and Proportion Problems

2. Write an example of a ratio. 2. ___________

3. Write an example of a proportion. 3. ___________

4. Solve: \[
\frac{5}{k + 4} = \frac{2}{k - 1}
\]
4. ___________

5. In your own words, what does it mean if two geometric figures are similar?

6. Suppose that a 6-foot tall man casts a show of 3.2 feet. At the same time of day, a tree casts a shadow of 8 feet. How tall is the tree? (See textbook Example 2)
   (a) Write a proportion that can be used to solve this problem. 6a. __________
   (b) Solve the proportion. How tall is the tree? 6b. __________

Objective 3: Model and Solve Work Problems

7. Problems of this type involve completing a job or a task when working at a constant rate. We convert the time \( t \) that it takes to complete the job into a unit rate. That is, if it takes \( t \) hours to complete a job, then \( \frac{1}{t} \) of the job is completed per hour.
   (a) If it takes 12 minutes to complete the job, what part of the job is completed per minute? 7a. __________
   (b) If it takes \( x \) hours to complete the job, what part of the job is completed per hour? 7b. __________
   (c) If it takes \( t + 2 \) hours to complete the job, what part of the job is completed per hour? 7c. __________
8. Josh can clean the math building on his campus in 3 hours. Ken takes 5 hours to clean the same building. If they work together, how long will it take for Josh and Ken to clean the math building? (See textbook Example 4)

**Step 1: Identify** We want to know how long it will take Josh and Ken working together to clean the building.

**Step 2: Name** We let \( t \) represent the time (in hours) that it takes to clean the building when working together.

**Step 3: Translate** What fraction of the job is completed in one hour when working individually and when working together? Write the following ratios:

(a) Part of the job completed by Josh in one hour: 8a. __________

(b) Part of the job completed by Ken in one hour: 8b. __________

(c) Part of the job completed when working together in one hour: 8c. __________

(d) Write the model for this problem: 8d. __________

**Step 4: Solve** the equation from Step 3.

**Step 5: Check** Is your answer reasonable?

(e) **Step 6: Answer** the question. 8e. __________

---

**Objective 4: Model and Solve Uniform Motion Problems**

9. A small plane can travel 1000 miles with the wind in the same time it can go 600 miles against the wind. If the speed of the plane in still air is 180 mph, what is the speed of the wind? (See textbook Example 6)

(a) Rate (in mph) when traveling with the wind: 9a. __________

(b) Rate (in mph) when traveling against the wind: 9b. __________

(c) Use \( t = \frac{d}{r} \) to write a rational expression for the time traveled with the wind: 9c. __________

(d) Use \( t = \frac{d}{r} \) to write a rational expression for the time traveled against the wind: 9d. __________

(e) Write an equation that can be used to solve this problem: 9e. __________

(f) Solve your equation and answer the question. 9f. __________
Do the Math Exercises 5.6
Models Involving Rational Expressions

In Problems 1 – 4, solve each formula for the indicated variable.

1. Solve \( \frac{V_1}{V_2} = \frac{P_2}{P_1} \) for \( V_2 \).

2. Solve \( P = \frac{A}{1 + r} \) for \( r \).

3. Solve \( m = \frac{y - y_1}{x - x_1} \) for \( x \).

4. Solve \( v_2 = \frac{2m_1v_1}{m_1 + m_2} \) for \( m_1 \).

5. Flight Accidents  According to the Statistical Abstract of the United States, in 2010, there were 1.27 fatal airplane accidents per 100,000 flight hours. Also, in 2010, there were a total of 267 fatal accidents. How many flight hours were flown in 2010?

6. Painting a Room  Latoya can paint five 10 foot by 14 foot rooms by herself in 14 hours. Lisa can paint five 10 foot by 14 foot rooms by herself in 10 hours. Working together, how long would it take to paint five 10 foot by 14 foot rooms?

7. Assembling a Swing Set  Alexandra and Frank can assemble a King Kong swing set working together in 6 hours. One day, when Frank called in sick, Alexandra was able to assemble a King Kong swing set in 10 hours. How long would it take Frank to assemble a King Kong swing set if he worked by himself?
8. **Running a Race** Roger can run one mile in 8 minutes. Jeff can run one mile in 6 minutes. If Jeff gives Roger a 1 minute head start, how long will it take before Jeff catches up to Roger? How far will each have run?

9. **Draining a Pond** A pond can be emptied in \(3.75 \left(\frac{15}{4}\right)\) hours using a 10 horsepower pump along with a 4 horsepower pump. The 4 horsepower pump requires 4 hours more than the 10 horsepower pump to empty the pond when working by itself. How long would it take to empty the pond using just the 10 horsepower pump?

10. **Round trip** A plane flies 600 miles west (into the wind) and makes the return trip following the same flight path. The effect of the jet stream on the plane is 15 miles per hour. The roundtrip takes 9 hours. What is the speed of the plane in still air?

11. **Riding Your Bicycle** Every weekend, you ride your bicycle on a forest preserve path. The path is 20 miles long and ends at a waterfall, at which point you relax and then make the trip back to the starting point. One weekend, you find that in the same time it takes you to travel to the waterfall you are only able to return 12 miles. Your average speed going to the waterfall is 4 miles per hour faster than your average speed on the return trip. What was your average speed going to the waterfall?
1. Solve each equation.

(a) \(-75 = 15x\)

(b) \(-\frac{2}{3}k = \frac{8}{27}\)

1a. __________

1b. __________

2. Solve: \(-12 = \frac{k}{4}\)

2. __________

3. Graph the equation \(y = \frac{1}{2}x\).

4. Graph the equation \(y = 2x\).
Objective 1: Model and Solve Direct Variation Problems

1. We say that \( y \) varies directly with \( x \), or \( y \) is directly proportional to \( x \), if there is a nonzero number \( k \) such that __________.

2. The number \( k \) is called the ________________.

3. If \( y \) varies directly with \( x \), then \( y \) is a ______________ function of \( x \) and has a \( y \)-intercept of ________.

4. Suppose that \( y \) is directly proportional to \( x \) and when \( x = -12, y = 5 \). Find \( y \) when \( x = 20 \). Write the direct variation equation, calculate the constant \( k \), and then use the given values to solve the unknown.

   4. __________

Objective 2: Model and Solve Inverse Variation Problems

5. We say that \( y \) varies inversely with \( x \), or \( y \) is inversely proportional to \( x \), if there is a nonzero number \( k \) such that __________.

6. Suppose that \( y \) varies inversely with \( x \). When \( x = 4, y = 12 \). Find \( y \) when \( x = 18 \). Write the inverse variation equation, calculate the constant, \( k \), and then use the given values to solve for the unknown. (See textbook Example 3)

   6. __________
**Objective 3: Model and Solve Joint Variation and Combined Variation Problems**

7. Suppose that $r$ varies jointly with $s$ and $t$. When $r = 12$, $s = 8$ and $t = 3$. Find $r$ when $s = 14$ and $t = 6$. Write the joint variation equation, calculate the constant, $k$, and then use the given values to solve for the unknown. *(See textbook Example 4)*

8. **Gas Laws** The volume $V$ of an ideal gas varies directly with the temperature $T$ and inversely with the pressure $P$. If a cylinder contains oxygen at a temperature of 300 kelvin (K) and a pressure of 15 atmospheres (Atm) in a volume of 100 liters, what is the constant of proportionality $k$? If a piston is lowered into the cylinder, decreasing the volume occupied by the gas to 70 liters and raising the temperature to 315 K, what is the pressure?

(a) Write the equation that shows the relationship between the variables, $V$, $T$, and $P$.  

(b) To calculate the constant of proportionality, substitute each of the following variables into your equation from 8a.

(c) Solve your equation to determine the constant of proportionality, $k$.

(d) Substitute your value of $k$ into 8a to determine the function relating the variables.

(e) Substitute the values of $V$ and $T$ in the last sentence into your equation from 8d to answer the question.
In Problems 1 and 2, (a) find the constant of proportionality, \( k \), (b) write the linear function relating the two variables, and (c) find the quantity indicated.

1. Suppose that \( y \) varies directly with \( x \).  
   When \( x = 3 \), then \( y = 15 \).  Find \( y \) when \( x = 5 \).  

2. Suppose that \( y \) is directly proportional to \( x \).  
   When \( x = 20 \), then \( y = 4 \).  Find \( y \) when \( x = 35 \).  

   1a. _______  
   1b. _______  
   1c. _______  
   2a. _______  
   2b. _______  
   2c. _______

In Problems 3 and 4, find the quantity indicated.

3. \( A \) is directly proportional to \( B \).  If \( A \) is 360 when \( B \) is 72, find \( B \) when \( A \) is 400.  

4. \( m \) varies directly with \( r \).  If \( r \) is 24 when \( m \) is 9, find \( r \) when \( m \) is 24.  

   3. _______  
   4. _______

5. **Mortgage Payments**  
   The monthly payment \( p \) on a mortgage varies directly with the amount borrowed \( b \).  Suppose that you decide to borrow $120,000 using a 15-year mortgage at 5.5% interest.  You are told that your payment is $980.50.  Assume that you have decided to buy a more expensive home that requires you borrow $150,000.  What will your monthly payment be?  

   5. ________

6. **Buying Gasoline**  
   The cost to purchase a tank of gasoline varies directly with the number of gallons purchased.  You notice that the person in front of you spent $34.50 on 15 gallons of gas.  If your SUV needs 35 gallons of gas, how much will you spend?  

   6. ________
7. **Falling Objects** The velocity of a falling object (ignoring air resistance) \( v \) is directly proportional to the time \( t \) of the fall. If, after 2 seconds, the velocity of the object is 64 feet per second, what will its velocity be after 3 seconds?

7. __________

In Problems 8 – 10, (a) find the constant, \( k \); (b) write the function relating the two variables; and (c) find the quantity indicated.

8. Suppose that \( y \) is inversely proportional to \( x \). When \( x = 20 \), then \( y = 4 \). Find \( y \) when \( x = 35 \).

8a. __________

8b. __________

8c. __________

9. Suppose that \( y \) varies jointly with \( x \) and \( z \). When \( y = 20 \), \( x = 6 \) and \( z = 10 \). Find \( y \) when \( x = 8 \) and \( z = 15 \).

9a. __________

9b. __________

9c. __________

10. Suppose that \( Q \) varies directly with \( x \) and inversely with \( y \). When \( Q = \frac{14}{5} \), \( x = 4 \) and \( y = 3 \). Find \( Q \) when \( x = 8 \) and \( y = 3 \).

10a. __________

10b. __________

10c. __________

11. **Resistance** The current, \( i \), in a circuit is inversely proportional to its resistance, \( R \), measured in ohms. Suppose that when the current in a circuit is 30 amperes, the resistance is 8 ohms. Find the current in the same circuit when the resistance is 10 ohms.

11. __________

12. **Intensity of Light** The intensity, \( I \), of light (measured in foot-candles) varies inversely with the square of the distance from the bulb. Suppose the intensity of a 100-watt light bulb at a distance of 2 meters is 0.075 foot-candles. Determine the intensity of the bulb at a distance of 3 meters.

12. __________
In Problems 1 – 4, simplify each expression.

1. $\sqrt{0.25}$

2. $\left(\frac{4}{\sqrt{9}}\right)^2$

3. $\sqrt{(4x + 3)^2}$

4. $\sqrt{x^2 - 2xy + y^2}$

In Problems 5 – 8, simplify each expression.

5. $7^{-2}$

6. $\frac{1}{x^2}$

7. $\left(\frac{3x}{y^2}\right)^2$

8. $\left(\frac{4a^2b^{-1}}{8a^2b^3}\right)^{-3}$
Objective 1: Evaluate nth Roots

1. In the notation $\sqrt[3]{8} = 2$, 3 is called the _______, 8 is called the _______ and 2 is the _______.

2. If the index is even, then the radicand must be _________ in order for the radical to simplify to a real number.

If the index is odd, then the radicand can be __________________________ and the expression will simplify to any real number.

3. Since $(-4)^2 = 16$ and $4^2 = 16$, it could be interpreted that $\sqrt[4]{16} = -4$ or 4. In fact, $\sqrt[4]{16} = 4$ only, because when the index even, we use the _________ _________, which must be $\geq 0$.

4. Evaluate each root without using a calculator. (See textbook Example 1)
   (a) $\sqrt[3]{-27}$  (b) $\sqrt[3]{-36}$  (c) $\sqrt[4]{\frac{16}{81}}$

   4a. _________  
   4b. _________  
   4c. _________

5. Write $\sqrt[4]{64}$ as a decimal rounded to two decimal places. (See textbook Example 2)

   5. _________

Objective 2: Simplify Expressions of the form $\sqrt[n]{a^n}$

6. Simplify: $\sqrt[4]{(2x - 1)^4}$ (See textbook Example 3)

   6. _________

Objective 3: Evaluate Expressions of the Form $a^{\frac{1}{n}}$

7. Write each of the following expressions as a radical and simplify, if possible. (See textbook Example 4)
   (a) $144^\frac{1}{2}$  (b) $(-64)^\frac{1}{3}$  (c) $2x^\frac{1}{3}$

   7a. _________  
   7b. _________  
   7c. _________
Guided Practice 6.1

Objective 4: Evaluate Expressions of the Form $a^{\frac{m}{n}}$

8. If $a$ is a real number and $\frac{m}{n}$ is a rational number in lowest terms with $n \geq 2$, then

$$a^{\frac{m}{n}} = \text{__________ or __________},$$

provided that $\sqrt[n]{a}$ exists.

9. Evaluate each of the following expressions, if possible. (See textbook Example 6)

(a) $36^{\frac{3}{2}}$  
(b) $-16^{\frac{3}{2}}$  
(c) $(-64)^{\frac{2}{3}}$

9a. __________  
9b. __________  
9c. __________

10. Write $25^{\frac{2}{3}}$ as a decimal rounded to two decimal places. (See textbook Example 7)

10. __________

11. Write each radical expression with a rational exponent. (See textbook Example 8)

(a) $\sqrt[3]{(2x)^3}$  
(b) $\left(\sqrt[3]{2x^2y}\right)^2$

11a. _________  
11b. _________

12. Rewrite each of the following with positive exponents, and completely simplify, if possible. (See textbook Example 9)

(a) $64^{\frac{1}{2}}$  
(b) $\frac{2}{16^{\frac{1}{2}}}$  
(c) $(4x)^{\frac{5}{2}}$

12a. _________  
12b. _________  
12c. _________
Do the Math Exercises 6.1

\textit{nth Roots and Rational Exponents}

\textit{In Problems 1 – 7, simplify each radical.}

1. \( \sqrt[3]{216} \) \\
2. \( \sqrt[3]{-64} \)

3. \( -\sqrt[4]{256} \) \\
4. \( \sqrt[3]{\frac{8}{125}} \)

5. \( \sqrt[4]{6^4} \) \\
6. \( \sqrt[n]{n^5} \)

7. \( \sqrt[6]{(2x - 3)^6} \)

\textit{In Problems 8 – 16, evaluate each expression, if possible.}

8. \( 16^{\frac{1}{2}} \) \\
9. \( -25^{\frac{1}{3}} \)

10. \( -81^{\frac{1}{2}} \) \\
11. \( (-81)^{\frac{1}{2}} \)

12. \( -100^{\frac{5}{2}} \) \\
13. \( -(-32)^{\frac{3}{5}} \)

14. \( 121^{\frac{1}{2}} \) \\
15. \( \frac{1}{49^{\frac{3}{2}}} \)

16. \( 27^{\frac{4}{3}} \)
Do the Math Exercises 6.1

In Problems 17 – 19, rewrite each of the following radicals with a rational exponent.
17. \( \sqrt[3]{x^3} \) 
18. \( \left( \sqrt[3]{3x} \right)^2 \)
19. \( \sqrt[3]{(3pq)^3} \)

In Problems 20 – 21, use a calculator to write each expression as a decimal rounded to two decimal places.
20. \( \sqrt[3]{585} \)
21. \( 100^{0.25} \)

In Problems 22 – 26, evaluate each expression, if possible.
22. \( 100^{\frac{3}{2}} \)
23. \( 4^{-1} \)
24. \( 125^{-\frac{1}{3}} \)
25. \( 100^{\frac{1}{2}} - 4^{\frac{3}{2}} \)
26. \( (-125)^{\frac{1}{3}} \)

27. Explain why \((-9)^{\frac{1}{3}}\) is not a real number, but \(-9^{\frac{1}{3}}\) is a real number.
In Problems 1 – 8, simplify each expression.

1. $9z^6 \cdot 6z$

2. $\frac{18u^5}{12u^3}$

3. $25^{-2}$

4. $\left(\frac{4}{3}\right)^{-3}$

5. $\frac{9}{7}x^3y^{-2} \cdot \frac{28}{12}xy$

6. $(-5p^4)^{-2}$

7. $\left(\frac{8a^{-1}}{b^{-5}}\right)^{-2}$

8. $\left(\frac{2x^2y}{6x^{-3}y^2}\right)^{-1} \left(\frac{xy^{-2}}{9xy^2}\right)^2$

9. Evaluate: $\sqrt[4]{\frac{81}{49}}$
2. Simplify each of the following expressions involving rational exponents. Express the answer with positive rational exponents, if necessary. (See textbook Examples 1 and 2)

(a) \( \frac{12}{5^2} \cdot 6^{-2/5} \)  
(b) \( \frac{(-8a^4)^{1/3}}{a^{5/6}} \)  
   
   2a. __________  
   
   2b. __________

3. Simplify the following expression involving rational exponents. Express the answer with positive rational exponents, if necessary. (See textbook Example 3)

\( \left( xy^{3/8} \right) \cdot \left( \frac{1}{x^2 y^{-2}} \right)^{3/2} \)  

3. __________
Objective 2: Simplify Radical Expressions

4. Rewrite the radical as an expression involving a rational exponent and simplify. Express the answer with a simplified radical, if necessary. (See textbook Example 4)

(a) \( \sqrt[3]{81^3} \)  
(b) \( \sqrt[4]{128x^8y^4} \)  
(c) \( \sqrt[4]{x^2} \)  
(d) \( \sqrt[3]{p^2} \)  

4a. 
4b. 
4c. 
4d. 

Objective 3: Factor Expressions Containing Rational Exponents

5. Simplify \( 6x^{\frac{2}{3}} + 2x^{\frac{1}{3}}(5x - 2) \) by factoring out \( 2x^{\frac{1}{3}} \). Express the answer with positive rational exponents, if necessary. (See textbook Example 5)

5. 

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### Do the Math Exercises 6.2

**Simplifying Expressions Using the Laws of Exponents**

*In Problems 1 – 9, simplify each of the following expressions.*

1. \( \frac{1}{3^3} \cdot \frac{5}{3^3} \)

2. \( \frac{10^5}{2} \cdot \frac{10^5}{2} \)

3. \( \frac{y^3}{9} \cdot \frac{1}{y^{10}} \)

4. \( \left( \frac{36}{\frac{1}{4} \cdot 9^4} \right)^{-2} \)

5. \( \left( \frac{5}{a^3} \cdot b^{\frac{2}{3}} \right)^{\frac{2}{3}} \)

6. \( \left( a^\frac{4}{3} \cdot b^{-\frac{1}{2}} \right) \cdot \left( a^{-2} \cdot b^{\frac{5}{2}} \right) \)

7. \( \left( 25p^\frac{2}{5} q^{-1} \right)^{\frac{1}{2}} \)

8. \( \left( \frac{64m^2n^4}{m^{-2}n^{-3}} \right)^{\frac{1}{2}} \)

9. \( \left( \frac{27x^{\frac{1}{2}} y^{-1}}{y^{-\frac{1}{3}} x^{\frac{1}{2}}} \right)^{\frac{1}{3}} - \left( \frac{4x^{\frac{1}{2}} y^{\frac{4}{3}}}{x^{-\frac{1}{2}} y^{\frac{2}{3}}} \right)^{\frac{1}{2}} \)

*In Problems 10 – 12, distribute and simplify.*

10. \( x^3 \left( \frac{5}{x^3} + 4 \right) \)

11. \( 3a \cdot \frac{1}{2} (2 - a) \)

12. \( 8p^3 \left( p^\frac{4}{3} - 4p^{-\frac{2}{3}} \right) \)
Do the Math Exercises 6.2

In Problems 13 – 17, use rational exponents to simplify each radical. Assume all variables are positive.

13. \(\sqrt[3]{x^6}\)  

14. \(\sqrt[3]{125^6}\)  

15. \(\sqrt{25x^4y^6}\)  

16. \(\sqrt[3]{p^3} \cdot \sqrt[3]{p}\)  

17. \(\sqrt[5]{ \sqrt{25}}\)

In Problems 18 – 21, simplify by factoring out the given expression.

18. \(3(x - 5)^{\frac{1}{2}}(3x + 1) + 6(x - 5)^{\frac{3}{2}}(x - 5)^{\frac{1}{2}}\)

19. \(x^{\frac{-2}{3}}(3x + 2) + 9x^{\frac{1}{3}}; x^{\frac{-2}{3}}\)

20. \(4(x + 3)^{\frac{1}{2}} + (x + 3)^{\frac{1}{2}}(2x + 1); (x + 3)^{\frac{1}{2}}\)

21. \(24x(x^2 - 1)^{\frac{1}{3}} + 9(x^2 - 1)^{\frac{4}{3}}; (x^2 - 1)^{\frac{1}{3}}\)

In Problems 22 – 25, simplify each expression.

22. \(\sqrt[3]{27^2}\)  

23. \(\frac{3}{25^4} \cdot \frac{3}{25^3}\)  

24. \((8^4)^{\frac{5}{2}}\)  

25. \(\sqrt[4]{a^6} - \sqrt[5]{a^5}\)
Five-Minute Warm-Up 6.3
Simplifying Radical Expressions Using Properties of Radicals

1. List the perfect squares that are less than 150.
   __________________________________________

2. List the perfect cubes that are less than 150.
   __________________________________________

3. List the perfect fourths that are less than 150.
   __________________________________________

In Problems 4 – 9, simplify each radical.

4. \(\sqrt{-4}\)  
   5. \(\sqrt{10^2}\)  
   4. ________  
   5. ________

6. \(\sqrt{x^2}\)  
   7. \(\sqrt{(4x + 1)^2}\)  
   6. ________  
   7. ________

8. \(\sqrt{4p^2 - 4p + 1}\)  
   9. \(\sqrt{144 + 25}\)  
   8. ________  
   9. ________
Guided Practice 6.3
Simplifying Radical Expressions Using Properties of Radicals

**Objective 1: Use the Product Property to Multiply Radical Expressions**

1. The Product Property of Radicals states that if $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers and $n \geq 2$ is an integer, then $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$. Use this property to multiply each of the following. (See textbook Example 1)

<table>
<thead>
<tr>
<th>Expression</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) $\sqrt{5} \cdot \sqrt{7}$</td>
<td>1a. ________</td>
</tr>
<tr>
<td>(b) $\sqrt{x-5} \cdot \sqrt{x+5}$</td>
<td>1b. ________</td>
</tr>
<tr>
<td>(c) $\sqrt[3]{7x} \cdot \sqrt[3]{2x}$</td>
<td>1c. ________</td>
</tr>
</tbody>
</table>

**Objective 2: Use the Product Property to Simplify Radical Expressions**

2. A radical expression is simplified when _____________________________________________________.

3. Simplify: $\sqrt{75}$ (See textbook Example 2)

   **Step 1:** Since the index is 2, we write each factor of the radicand as the product of two factors, one of which is a perfect square.
   
<table>
<thead>
<tr>
<th>What perfect square is a factor of 75?</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) ________________________________</td>
</tr>
</tbody>
</table>

   **Step 2:** Write the radical as the product of two radicals, one of which contains the perfect square.
   
   | $\sqrt{75} = $ (b) __________________________ |

   **Step 3:** Take the square root of each perfect power.
   
   | (c) ________________________________ |

4. Simplify: $\frac{-6 + \sqrt{48}}{2}$ (See textbook Example 4)

   **Step 1:** Since the index is 2, write each factor of the radicand as the product of the factors, one of which is a perfect square.
   
   | (a) ________________________________ |

   **Step 2:** Use $\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}$.
   
   | (b) ________________________________ |

   **Step 3:** Take the square root of each perfect power.
   
   | (c) ________________________________ |

   **Step 4:** Factor out the 2 in the numerator.
   
   | (d) ________________________________ |

   **Step 5:** Divide out the common factor.
   
   | (e) ________________________________ |
5. Explain how to simplify an expression with an index that does not divide evenly into the exponent on the variable in the radicand.

Objective 3: Use the Quotient Property to Simplify Radical Expressions

6. Simplify $\sqrt[12]{\frac{12x^2}{121}}$. Assume $x \geq 0$. (See textbook Example 8)

7. Simplify $\frac{\sqrt{72a}}{\sqrt[2]{2a^5}}$. Assume $a > 0$. (See textbook Example 9)

Objective 4: Multiply Radicals with Unlike Indices

8. Multiply: $\sqrt{6} \cdot \sqrt[4]{48}$ (See textbook Example 10)

(a) What is the first step to multiply $\sqrt{6} \cdot \sqrt[4]{48}$?

(b) Determine the LCD of 2 and 4.

(c) Use $a^n = \left(\frac{1}{a^m}\right)^n$ to rewrite each factor:

(d) Use $a^r \cdot b^s = (a \cdot b)^{r+s}$:

(e) Multiply:

(f) Write the expression using radicals and simplify:
In Problems 1 – 3, use the Product Rule to multiply. Assume that all variables can be any real number.

1. \( \sqrt[4]{6a^2} \cdot \sqrt[4]{7b^2} \)  
2. \( \sqrt{p-5} \cdot \sqrt{p+5} \)  
3. \( \sqrt[6]{9x^2} \cdot \sqrt[6]{4} \cdot \sqrt[6]{3x} \)

In Problems 4 – 11, simplify each radical using the Product Property. Assume that all variables can be any real number.

4. \( \sqrt[3]{162} \)  
5. \( \sqrt[2]{20a^2} \)  
6. \( \sqrt[4]{-64p^3} \)  
7. \( 4\sqrt{27b} \)  
8. \( \sqrt{s^9} \)  
9. \( \sqrt[5]{x^{12}} \)  
10. \( \sqrt[3]{-54q^{12}} \)  
11. \( \sqrt{75x^6y} \)

In Problems 12 and 13, simplify each expression.

12. \( \frac{5 - \sqrt{100}}{5} \)  
13. \( \frac{-6 + \sqrt{48}}{8} \)
Do the Math Exercises 6.3

In Problems 14 – 17, multiply and simplify. Assume that all variables are greater than or equal to zero.

14. \( \sqrt{3} \cdot \sqrt{12} \)

15. \( \sqrt{6x} \cdot \sqrt{30x} \)

16. \( \sqrt[3]{9a} \cdot \sqrt[3]{6a^2} \)

17. \( \sqrt[3]{16m^2n} \cdot \sqrt[3]{27m^2n} \)

In Problems 18 – 23, simplify. Assume that all variables are greater than zero.

18. \( \sqrt[3]{\frac{5}{36}} \)

19. \( \sqrt[3]{\frac{5x^4}{16}} \)

20. \( \sqrt[3]{\frac{-27x^9}{64y^{12}}} \)

21. \( \sqrt[4]{\frac{64}{4^4}} \)

22. \( \sqrt[3]{\frac{54y^5}{\sqrt{3y}}} \)

23. \( \sqrt[3]{\frac{-128x^8}{\sqrt{2x^{-1}}}} \)

In Problems 24 and 25, multiply and simplify.

24. \( \sqrt{2} \cdot \sqrt{7} \)

25. \( \sqrt[3]{5} \cdot \sqrt[3]{5} \)
Five-Minute Warm-Up 6.4
Adding, Subtracting, and Multiplying Radical Expressions

1. Add: \((2z^3 - 7z^2 + 1) + (z^3 + 2z^2 - 4z - 1)\)

2. Subtract: \((-2a^2b^2 + ab - 3b^2) - (a^2b^2 + a^2 - 5ab - 2b^2)\)

3. Multiply: \(-4x^2(2x^2 + 3xy - 5y^2)\)

4. Multiply: \(\frac{3}{2}x^2\left(\frac{4}{27}x^3 - \frac{8}{21}x^2 + \frac{2}{3}\right)\)

5. Multiply: \((9c + 2)(2c - 3)\)

6. Multiply: \((ab - 2)(ab + 2)\)

7. Multiply: \((7n - 3)^2\)
**Guided Practice 6.4**

**Adding, Subtracting, and Multiplying Radical Expressions**

**Objective 1: Add or Subtract Radical Expressions**

1. Describe the characteristics of *like* radicals.

2. Add or subtract, as indicated. Assume all variables are greater than or equal to zero. (*See textbook Example 1*)
   
   (a) \(8\sqrt{x} + \sqrt{x}\)  
   (b) \(13\sqrt[3]{p} - 6\sqrt[3]{p} + \sqrt[3]{p}\)  

   2a. _________  
   2b. _________

3. Simplify the radicals and then perform the indicated operations. Assume all variables are greater than or equal to zero. (*See textbook Examples 2 and 3*)

   (a) \(3\sqrt{8} - 4\sqrt{32}\)  
   (b) \(3n^2 \sqrt{54n^4} - 2n\sqrt{150n^6} + 2\sqrt{24n^9}\)  

   3a. _________  
   3b. _________
Objective 2: Multiply Radical Expressions

4. Multiply and simplify: \((9 + 2\sqrt{6})(2 - 3\sqrt{2})\)  
(See textbook Example 4)

5. Multiply and simplify: \((7 - \sqrt{3})(7 + \sqrt{3})\)  
(See textbook Example 5)

Step 1: What special products formula can be used to multiply \((7 - \sqrt{3})(7 + \sqrt{3})\)?

5a. _________

Step 2: Use the formula from part (a) to multiply and simplify \((7 - \sqrt{3})(7 + \sqrt{3})\).

5b. _________

6. Use Heron’s Formula for finding the area of triangle whose sides are known. Heron’s Formula states that the area \(A\) of a triangle with sides \(a\), \(b\), and \(c\), is

\[ A = \sqrt{s(s-a)(s-b)(s-c)} \]

where

\[ s = \frac{1}{2}(a+b+c) \]

Find the area of the shaded region by computing the difference in the areas of the triangles. That is, compute “area of larger triangle minus area of smaller triangle.” Write your answer as a radical in simplified form.

6. _________
### Do the Math Exercises 6.4
Adding, Subtracting, and Multiplying Radical Expressions

**In Problems 1 – 9, add or subtract as indicated. Assume all variables are positive or zero.**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>$6\sqrt{3} + 8\sqrt{3}$</td>
</tr>
<tr>
<td>2.</td>
<td>$12\sqrt{z} - 5\sqrt{z}$</td>
</tr>
<tr>
<td>3.</td>
<td>$4\sqrt{5} - 3\sqrt{5} + 7\sqrt{5} - 8\sqrt{5}$</td>
</tr>
<tr>
<td>4.</td>
<td>$6\sqrt{3} + \sqrt{12}$</td>
</tr>
<tr>
<td>5.</td>
<td>$7\sqrt{48} - 4\sqrt{243}$</td>
</tr>
<tr>
<td>6.</td>
<td>$2\sqrt{48z} - \sqrt{75z}$</td>
</tr>
<tr>
<td>7.</td>
<td>$3\sqrt{63z^3} + 2z\sqrt{28z}$</td>
</tr>
<tr>
<td>8.</td>
<td>$\sqrt{48y^2} - 4y\sqrt{12} + \sqrt{108y^2}$</td>
</tr>
<tr>
<td>9.</td>
<td>$-2\sqrt{5x^3} + 4x\sqrt{40} - \sqrt{135}$</td>
</tr>
</tbody>
</table>

**In Problems 10 – 18, multiply and simplify. Assume all variables are positive or zero.**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>10.</td>
<td>$\sqrt{5}(5 + 3\sqrt{3})$</td>
</tr>
<tr>
<td>11.</td>
<td>$\sqrt[3]{6}(\sqrt[3]{2} + \sqrt[3]{12})$</td>
</tr>
<tr>
<td>12.</td>
<td>$(5 + \sqrt{5})(3 + \sqrt{6})$</td>
</tr>
<tr>
<td>13.</td>
<td>$(9 + 5\sqrt{10})(1 - 3\sqrt{10})$</td>
</tr>
</tbody>
</table>
Do the Math Exercises 6.4

14. \((\sqrt{6} - 2\sqrt{2})(2\sqrt{6} + 3\sqrt{2})\)

15. \((2 - \sqrt{3})^2\)

16. \((\sqrt{3} - 1)(\sqrt{3} + 1)\)

17. \((6 + 3\sqrt{2})(6 - 3\sqrt{2})\)

18. \((\sqrt[3]{y} - 6)(\sqrt[3]{y} + 3)\)

In Problems 19 – 25, perform the indicated operation and simplify. Assume all variables are positive or zero.

19. \((\sqrt{6} - 2\sqrt{2})(2\sqrt{6} + 3\sqrt{2})\)

20. \((\sqrt{7} - \sqrt{3})^2\)

21. \((\sqrt{5} - \sqrt{3})^2 - \sqrt{60}\)

22. \((4 + \sqrt{2x + 3})^2\)

23. \((\sqrt{3a} - \sqrt{4b})(\sqrt{3a} + \sqrt{4b}) + 4\sqrt{b^2}\)

24. \((\sqrt{2} - \sqrt{7})^2 - \sqrt{56}\)

25. \(\frac{4}{5}\left(-\frac{\sqrt{5}}{5}\right) + \left(-\frac{3}{5}\right)\left(-\frac{2\sqrt{5}}{5}\right)\)
Five-Minute Warm-Up 6.5
Rationalizing Radical Expressions

1. By what would you multiply 75 so that the product is a perfect square? There is more than one right answer so choose the smallest possible factor that yields a perfect square.

2. Simplify: \( \sqrt{121a^2}, a > 0 \)

3. Multiply: \( \frac{\sqrt{5}}{\sqrt{2}} \cdot \frac{\sqrt{6}}{\sqrt{2}} \)

4. Multiply: \( (2 + \sqrt{3})(2 - \sqrt{3}) \)

5. Multiply: \( (2\sqrt{5} + \sqrt{3})(3\sqrt{5} - 2\sqrt{3}) \)
Objective 1: Rationalize a Denominator Containing One Term

1. In your own words, what does it mean to rationalize the denominator of a rational expression?

2. To rationalize a denominator containing a single square root, we multiply the numerator and denominator of the quotient by a square root so that the radicand in the denominator becomes _________________.

3. Determine what to multiply each quotient by so that the denominator contains a radicand which is a perfect square. (See textbook Example 1)
   (a) $\frac{3}{\sqrt{3}}$  
   (b) $\frac{4}{\sqrt{20}}$  
   (c) $\frac{1}{\sqrt{8a}}$  
   3a. __________  
   3b. __________  
   3c. __________

4. Determine what to multiply each quotient by so that the denominator contains a radicand which is a perfect power. (See textbook Example 2)
   (a) $\frac{6}{\sqrt{5}}$  
   (b) $\frac{\sqrt{2}}{\sqrt{12}}$  
   (c) $\frac{4x}{\sqrt[4]{27x^2y}}$  
   4a. __________  
   4b. __________  
   4c. __________
Objective 2: Rationalize a Denominator Containing Two Terms

5. To rationalize a denominator containing two terms, we multiply both numerator and denominator by the ________________ of the denominator.

6. Identify the conjugate of each expression. Then multiply the expression by its conjugate. (See textbook Example 3)

(a) $3 + \sqrt{5}$  
(b) $2\sqrt{3} - 5\sqrt{2}$

6a. __________  
6b. __________

7. Determine what to multiply the quotient by to rationalize the denominator $\frac{\sqrt{8}}{\sqrt{2} - 3}$. (See textbook Example 3)

7. __________

8. Rationalize the denominator: $\frac{12}{\sqrt{2} - \sqrt{5}}$  
8. __________
<table>
<thead>
<tr>
<th>Exercise</th>
<th>Expression</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>$\frac{2}{\sqrt{3}}$</td>
<td>$\frac{2\sqrt{3}}{3}$</td>
</tr>
<tr>
<td>2.</td>
<td>$-\frac{3}{2\sqrt{3}}$</td>
<td>$-\frac{3\sqrt{3}}{6}$</td>
</tr>
<tr>
<td>3.</td>
<td>$\frac{5}{\sqrt{20}}$</td>
<td>$\frac{\sqrt{5}}{2}$</td>
</tr>
<tr>
<td>4.</td>
<td>$\frac{\sqrt{3}}{\sqrt{11}}$</td>
<td>$\sqrt{\frac{3}{11}}$</td>
</tr>
<tr>
<td>5.</td>
<td>$\frac{\sqrt{5}}{\sqrt{z}}$</td>
<td>$\frac{\sqrt{5z}}{z}$</td>
</tr>
<tr>
<td>6.</td>
<td>$\frac{\sqrt{32}}{\sqrt{a^5}}$</td>
<td>$\frac{4}{a^{\frac{3}{2}}}$</td>
</tr>
<tr>
<td>7.</td>
<td>$-\frac{\sqrt{4}}{\sqrt{p}}$</td>
<td>$-\frac{2}{\sqrt{p}}$</td>
</tr>
<tr>
<td>8.</td>
<td>$\frac{\sqrt{-5}}{\sqrt{72}}$</td>
<td>$\frac{\sqrt{-5}}{6\sqrt{2}}$</td>
</tr>
<tr>
<td>9.</td>
<td>$\frac{8}{\sqrt[3]{36z^2}}$</td>
<td>$\frac{2}{z\sqrt{z}}$</td>
</tr>
<tr>
<td>10.</td>
<td>$\frac{6}{\sqrt[4]{9b^5}}$</td>
<td>$\frac{2b^{\frac{1}{2}}}{\sqrt[4]{3b}}$</td>
</tr>
<tr>
<td>11.</td>
<td>$\frac{6}{\sqrt{7} - 2}$</td>
<td>$\frac{6(\sqrt{7} + 2)}{3}$</td>
</tr>
<tr>
<td>12.</td>
<td>$\frac{10}{\sqrt{10} + 3}$</td>
<td>$\frac{10(\sqrt{10} - 3)}{7}$</td>
</tr>
<tr>
<td>13.</td>
<td>$\frac{\sqrt{3}}{\sqrt{15} - \sqrt{6}}$</td>
<td>$\frac{\sqrt{3}(\sqrt{15} + \sqrt{6})}{3}$</td>
</tr>
<tr>
<td>14.</td>
<td>$\frac{2\sqrt{3} + 3}{\sqrt{12} - \sqrt{3}}$</td>
<td>$\frac{2\sqrt{3} + 3}{3\sqrt{3} - 3}$</td>
</tr>
</tbody>
</table>
Do the Math Exercises 6.5

In Problems 15 – 18, perform the indicated operation and simplify.

15. \( \sqrt{5} - \frac{1}{\sqrt{5}} \)  
16. \( \frac{\sqrt{5}}{2} + \frac{3}{\sqrt{5}} \)

17. \( \sqrt{\frac{2}{5}} + \sqrt{20} - \sqrt{45} \)  
18. \( \sqrt{\frac{4}{3}} + \frac{4}{\sqrt{48}} \)

In Problems 19 – 23, simplify each expression so that the denominator does not contain a radical.

19. \( \frac{\sqrt{2}}{\sqrt{18}} \)  
20. \( \frac{\sqrt{9}}{\sqrt{5}} \)

21. \( \frac{\sqrt{2} - 5}{\sqrt{2} + 5} \)  
22. \( \frac{5}{\sqrt{6} + 4} \)

23. \( \frac{\sqrt{75}}{\sqrt{3}} \)

In Problems 24 and 25, rationalize the numerator.

24. \( \frac{\sqrt{3} + 2}{2} \)  
25. \( \frac{\sqrt{a} - \sqrt{b}}{\sqrt{2}} \)

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In Problems 1 and 2, simplify each expression.

1. $\sqrt{144}$
2. $\sqrt{n^2}, n > 0$

3. Solve: $-3x - 9 \geq 0$

4. Given $f(x) = -2x^2 + 16x$, find $f(-2)$.

5. Graph $f(x) = x^2 + 2$ using point plotting.
Guided Practice 6.6
Functions Involving Radicals

**Objective 1: Evaluate Functions Involving Radicals**

1. For the functions \( f(x) = 2\sqrt{3x - 1} \), \( g(x) = \frac{3x}{\sqrt{x + 4}} \), and \( h(x) = \sqrt{\frac{x + 2}{x - 2}} \), find each of the following. 

   (See textbook Example 1)

   (a) \( f(7) \)
   (b) \( g(-3) \)
   (c) \( h(6) \)

   1a. __________
   1b. __________
   1c. __________

**Objective 2: Find the Domain of a Function Involving a Radical**

2. If the index on a radical is even, then the radicand must be ________________________________.

3. If the index on a radical is odd, then the radicand can be ________________________________.

4. Find the domain of each of the following functions. (See textbook Example 2)

   (a) \( f(x) = \sqrt{2x - 3} \)
   (b) \( g(x) = \sqrt[3]{6x - 9} \)
   (c) \( h(t) = \sqrt[4]{14 - 7t} \)

   4a. __________
   4b. __________
   4c. __________

**Objective 3: Graph Functions Involving Square Roots**

5. Given the function \( f(x) = \sqrt{x + 3} \), (See textbook Example 3)

   (a) find the domain.

   5a. __________

   (b) graph the function using point-plotting.

   (c) Based on the graph, determine the range.

   5c. __________
Guided Practice 6.6

Objective 4: Graph Functions Involving Cube Roots

6. Given the function \( g(x) = \sqrt[3]{x} - 3 \) \text{(See textbook Example 4)}

(a) find the domain. 6a. __________

(b) graph the function using point-plotting.

(c) Based on the graph, determine the range. 6c. __________
Do the Math Exercises 6.6  
Functions Involving Radicals

In Problems 1 – 3, evaluate each radical function at the indicated values.

1. \( f(x) = \sqrt{x + 10} \)

(a) \( f(6) \)

(b) \( f(2) \)

(c) \( f(-6) \)

In Problems 4 – 8, find the domain of the radical function.

4. \( f(x) = \sqrt{x + 4} \)

5. \( G(x) = \sqrt{5 - 2x} \)

6. \( G(z) = \sqrt[3]{5z - 3} \)

7. \( C(y) = \sqrt[3]{3y - 2} \)

8. \( f(x) = \sqrt[3]{\frac{3}{x - 3}} \)
In Problems 9 – 12, (a) determine the domain of the function; (b) determine the range of the function; (c) graph the function using point-plotting.

9. \( f(x) = \sqrt{x} - 1 \)

10. \( F(x) = \sqrt{4 - x} \)

9a. _________

9b. _________

10a. _________

10b. _________

11. \( f(x) = \sqrt{x} + 1 \)

12. \( g(x) = \sqrt[3]{x} - 4 \)

11a. _________

11b. _________

12a. _________

12b. _________
1. Solve: $4x + 12 = 0$

2. Solve: $-4x^2 + 12x - 8 = 0$

3. Simplify: $\left(\sqrt{2x}\right)^2$, $x > 0$

4. Multiply: $\left(\sqrt{2x} + 5\right)^2$, $x > 0$

5. Simplify: $\left(3x + 8\right)^{\frac{2}{3}}$, $x > 0$

6. Evaluate: $\sqrt{2(-3) + (-3)(-18)}$
Guided Practice 6.7
Radical Equations and Their Applications

Objective 1: Solve Radical Equations Containing One Radical

1. Solve: \( \sqrt{4x + 1} - 2 = 3 \)  (See textbook Example 1)

Step 1: Isolate the radical.

Add 2 to both sides:

\( \sqrt{4x + 1} = 5 \)  (a)

Step 2: Raise both sides to the power of the index.

The index is 2, so we square both sides:

\( 4x + 1 = 25 \)  (b)

Simplify:

\( 4x = 24 \)  (c)

Step 3: Solve the equation that results.

\( x = 6 \)  (d)

Step 4: Check

Substitute the value you found in (d) into the original equation. Does this yield a true statement? (e)

State the solution set:  (f)

2. What is meant by extraneous solutions?

3. When solving radical equations, what is the key first step?

4. Solve: \( \sqrt[3]{7x + 2} + 5 = 2 \)  (See textbook Example 2)

   (a) What value did you find for \( x \)?  4a.

   (b) Does this satisfy the original equation?  4b.

   (c) State the solution set.  4c.

   (d) Without solving, how can you tell that this equation has no real solution?  4d.
5. Solve: \( \sqrt{x+5} + 1 = x \)  
(See textbook Example 3)

(a) What value(s) did you find for \( x \)?

(b) Does this value (or values) satisfy the original equation?

(c) State the solution set.

6. Solve: \( (3x + 1)^{\frac{3}{2}} - 2 = 6 \)  
(See textbook Example 5)

(a) Isolate the expression containing the rational exponent.

(b) To what power do we need to raise both sides in order to eliminate the exponent on the variable expression?

(c) Solve the resulting equation. What value(s) did you find for \( x \)?

(d) Check and then state the solution set.

Objective 2: Solve Radical Equations Containing Two Radicals

7. Solve: \( \sqrt{2x^2 - 5x - 20} = \sqrt{x^2 - 3x + 15} \)  
(See textbook Example 6)

<table>
<thead>
<tr>
<th>Step 1:</th>
<th>The radical on the left side of the equation is isolated:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Isolate one of the radicals.</td>
<td>(a) ( \sqrt{2x^2 - 5x - 20} = \sqrt{x^2 - 3x + 15} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Step 2:</th>
<th>The index is 2, so we square both sides:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Raise both sides to the power of the index.</td>
<td>(b)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Step 3:</th>
<th>Solve:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Because there is no radical, we solve the equation that results.</td>
<td>(c)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Step 4:</th>
<th>Substitute the value(s) you found in (b) into the original equation. Does this yield a true statement?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Check.</td>
<td>(d)</td>
</tr>
</tbody>
</table>

| State the solution set: | (e) |
Do the Math Exercises 6.7
Radical Equations and Their Applications

In Problems 1 – 18, solve each equation.

1. \( \sqrt{y-5} = 3 \)  
2. \( \sqrt{6p-5} = -5 \) 

3. \( \sqrt[3]{9w} = 3 \)  
4. \( \sqrt{q-5} = 2 \) 

5. \( \sqrt{x-4} + 4 = 7 \)  
6. \( 4\sqrt{t} - 2 = 10 \) 

7. \( \sqrt{w} = 6 - w \)  
8. \( \sqrt{1-4x} - 5 = x \) 

9. \( \sqrt{3x+1} = \sqrt{2x+7} \)  
10. \( \sqrt{2x^2 + 7x - 10} = \sqrt{x^2 + 4x + 8} \) 

11. \( \sqrt{2x-1} - \sqrt{x-1} = 1 \)  
12. \( \sqrt{4x+1} - \sqrt{2x+1} = 2 \) 

13. \( (6p+3)^{\frac{1}{5}} = (4p-9)^{\frac{1}{5}} \)  
14. \( (2x+3)^{\frac{1}{2}} = 3 \)
Do the Math Exercises 6.7

15. \( \sqrt{x + 20} = x \) 16. \( \sqrt{x + 23} = 2 \)

17. \( \sqrt{x - 3} + \sqrt{x + 4} = 7 \) 18. \( \sqrt{a - 5} = -2 \)

In Problems 19 – 21, solve for the indicated variable.

19. Solve \( v = \sqrt{ar} \) for \( a \). 20. Solve \( r = \sqrt{\frac{S}{4\pi}} \) for \( S \).

21. Solve \( V = \sqrt{\frac{2U}{C}} \) for \( U \).

22. Money The annual rate of interest \( r \) (expressed as a decimal) required to have \( A \) dollars after \( t \) years from an initial deposit of \( P \) dollars can be calculated with the following formula:

\[
r = \sqrt{\frac{A}{P}} - 1
\]

Suppose that you deposit $1,000 in an account that pays 5% annual interest so that \( r = 0.05 \). How much will you have after \( t = 2 \) years?
1. List the numbers in the set \( \left\{ 8, \frac{-6}{3}, -5, 0.\overline{3}, \frac{15}{0}, \frac{-2}{5}, \pi, \frac{0}{-12} \right\} \) which are:

(a) Natural numbers

(b) Whole numbers

(c) Integers

(d) Rational numbers

(e) Irrational numbers

(f) Real numbers

In Problems 2 – 5, perform the indicated operations and simplify.

2. \((x^2 - 1) + (2x^2 - 1)\) 

3. \(4p^5(-3p + 9)\)

4. \((2x - 7)(-3x - 4)\)

5. \((x^2 + 4)(x^2 - 4)\)
1. The *imaginary unit*, denoted by *i*, is the number whose square is −1. \( i^2 = ____ \) or \( i = ____ \).

2. *Complex numbers* are numbers of the form \( ____ \) where \( a \) and \( b \) are \( ____ \). When a number is in a form such as \( 6 − 2i \), we say that the number is in \( ____ \) form. The *real part* of the complex number is \( ____ \) and the *imaginary part* is \( ____ \).

**Objective 1: Evaluate the Square Root of Negative Real Numbers**

3. Write \( \sqrt{-81} \) as a pure imaginary number. *(See textbook Example 1)*

   3. ________

4. Write \( 6 + \sqrt{4} \) in standard form. *(See textbook Example 2)*

   4. ________

**Objective 2: Add or Subtract Complex Numbers**

5. Before beginning any operations with complex numbers, you must write the number in \( ____ \) form.

6. In your own words, explain how to add complex numbers ______________________________________
   _______________________________________________________________________________________

**Objective 3: Multiply Complex Numbers**

7. Use the Distributive Property to multiply \( \frac{3}{2}i(6 − 8i) \). *(See textbook Example 5)*

   7. ________

8. Multiply \( \sqrt{-4} \cdot \sqrt{-9} \). *(See textbook Example 6)*
   (a) Explain why the Product Property of Radicals cannot be used to multiply these radicals.
      
      (a) ___________________________________________
   
   (b) Express the radicals as pure imaginary numbers.
      
      8b. ________
   
   (c) Multiply.
      
      8c. ________
Objective 4: Divide Complex Numbers

9. Divide: \( \frac{3 + 6i}{4 + 4i} \). (See textbook Example 8)

Step 1: Write the numerator and denominator in standard form, \( a + bi \).

The numerator and denominator are already in standard form.

Step 2: Multiply the numerator and denominator by the complex conjugate of the denominator.

Identify the conjugate of the denominator: (a) ________________

Multiply the quotient by 1, written with the conjugate: (b) ________________

Step 3: Simplify by writing the quotient in standard form, \( a + bi \).

Multiply the numerator; the denominator is of the form \((a + bi)(a - bi) = a^2 + b^2\): (c) ________________

Combine like terms; \( i^2 = -1 \): (d) ________________

Divide the denominator into each term of the numerator to write in standard form: (e) ________________

Write each fraction in lowest terms: (f) ________________

Objective 5: Evaluate the Powers of \( i \)

10. The powers of \( i \) are a cyclic function, meaning that the values cycle through a set list. Complete the table to see the only values for the powers of \( i \).

\[ i^1 = i \quad \text{(d)} \quad i^5 = \______ \]

(a) \( i^2 = \______ \quad \text{(e)} \quad i^6 = \______ \)

(b) \( i^3 = \______ \quad \text{(f)} \quad i^7 = \______ \)

(c) \( i^4 = \______ \quad \text{(g)} \quad i^8 = \______ \)
Do the Math Exercises 6.8
The Complex Number System

In Problems 1 and 2, write each expression as a pure imaginary number.

1. \(-\sqrt{-100}\) 
2. \(\sqrt{-162}\)

In Problems 3 – 5, write each expression as a complex number in standard form.

3. \(10 + \sqrt{-32}\) 
4. \(10 - \sqrt{-25} \div 5\)
5. \(\frac{15 - \sqrt{-50}}{5}\)

In Problems 6 – 9, add or subtract as indicated.

6. \((-6 + 2i) + (3 + 12i)\) 
7. \((-7 + 3i) - (-3 + 2i)\)
8. \((-4 + \sqrt{-25}) + (1 - \sqrt{-16})\)
9. \((-10 + \sqrt{-20}) - (-6 + \sqrt{-45})\)

In Problems 10 – 17, multiply.

10. \(3i(-2 - 6i)\) 
11. \((3 - i)(1 + 2i)\)
12. \((2 + 8i)(-3 - i)\)
13. \(\left(\frac{2}{3} + \frac{4}{3}i\right)\left(\frac{1}{2} - \frac{3}{2}i\right)\)
14. \((2 + 5i)^2\)  
15. \((2 - 7i)^2\)

16. \(\sqrt{-36} \cdot \sqrt{-4}\)  
17. \((1 - \sqrt{-64})(-2 + \sqrt{-49})\)

In Problems 18 – 21, divide.

18. \(\frac{2 - i}{2i}\)  
19. \(\frac{2}{4 + i}\)

20. \(\frac{-4}{-5 - 3i}\)  
21. \(\frac{2 + 5i}{5 - 2i}\)

In Problems 22 – 24, simplify.

22. \(i^{72}\)  
23. \(i^{110}\)

24. \(i^{131}\)
Five-Minute Warm-Up 7.1
Solving Quadratic Equations by Completing the Square

1. Multiply: \((3x - 1)^2\)  
   1. __________

2. Factor: \(x^2 - 4x + 4\)  
   2. __________

In Problems 3 and 4, solve each polynomial equation.

3. \(x^2 - \frac{2}{3}x + \frac{1}{9} = 0\)  
   3. __________

4. \(25n^2 - 49 = 0\)  
   4. __________

In Problems 5 and 6, simplify each expression.

5. \(\sqrt{\frac{81}{25}}\)  
   5. __________

6. \(\sqrt{(8x - 3)^2}\)  
   6. __________

In Problems 7 and 8, simplify each expression using complex numbers.

7. \(\sqrt{-12}\)  
   7. __________

8. \(\frac{8 + \sqrt{-4}}{4}\)  
   8. __________

9. Find the complex conjugate of \(-15 + 7i\).  
   9. __________
Guided Practice 7.1
Solving Quadratic Equations by Completing the Square

Objective 1: Solve Quadratic Equations Using the Square Root Property

1. State the Square Root Property: If \( x^2 = p \), where \( x \) is any variable expression and \( p \) is a real number, then \[ \text{__________________________________________________________}. \]

2. If the solution to a quadratic equation is \( n = -1 \pm 2\sqrt{3} \), write the solution set. \[ \text{____________________________} \]

3. True or False \( \sqrt{x^2 - 16} = \sqrt{81} \) simplifies to \( x - 4 = \pm 9 \). \[ \text{____________________________} \]

4. True or False \( y^2 = -9 \) has no solution. \[ \text{____________________________} \]

5. Solve: \( n^2 - 144 = 0 \) (See textbook Example 1)

   Step 1: Isolate the expression containing the square term. \( n^2 - 144 = 0 \)
   
   Add 144 to both sides: \[ (a) \text{____________________________} \]

   Step 2: Use the Square Root Property. Don’t forget the ± symbol.
   
   Take the square root of both sides of the equation: \[ (b) \text{____________________________} \]
   
   Simplify the radical: \[ (c) \text{____________________________} \]

   Step 3: Isolate the variable, if necessary.
   
   The variable is already isolated.

   Step 4: Verify your solution(s).
   
   State the solution set: \[ (d) \text{____________________________} \]

6. There is no reason that the solution to a quadratic equation must be real. List the possible nature of the solution(s) of a quadratic equation.

   (a) Real, which includes: \[ \text{_______________, _______________, _______________} \] and

   (b) \[ \text{_______________} \] in form \[ \text{_______________} \]

Objective 2: Complete the Square in One Variable

7. If a polynomial is of the form \( x^2 + bx + c \), \( c \) must be equal to \[ \text{____________________________} \] in order to be a perfect square trinomial.
Guided Practice 7.1

8. Determine the number that must be added to the expression to make it a perfect square trinomial. Then factor the expression. (See textbook Example 5)

(a) \( p^2 - 14p \)  
(b) \( n^2 + 9n \)  
(c) \( z^2 + \frac{4}{3}z \)

8a. ________________  
8b. ________________  
8c. ________________  

Objective 3: Solve Quadratic Equations by Completing the Square

9. Solve: \( p^2 - 6p - 18 = 0 \) (See textbook Example 6)

Step 1: Rewrite \( x^2 + bx + c = 0 \) as \( x^2 + bx = -c \) by adding or subtracting the constant from both sides of the equation.

Add 18 to both sides:

(a) ________________

Step 2: Complete the square in the expression \( x^2 + bx \) by making it a perfect square trinomial.

What value must be added to both sides to make the expression on the left a perfect square trinomial?

(b) ________________

Add this number to both sides of the equation and simplify:

(c) ________________

Step 3: Factor the perfect square trinomial on the left side of the equation.

Use: \( A^2 - 2AB + B^2 = (A - B)^2 \)

(d) ________________

Step 4: Solve the equation using the Square Root Property.

Take the square root of both sides of the equation:

(e) ________________

Simplify the square root:

(f) ________________

Add 3 to both sides:

(g) ________________

Step 5: Verify your solutions(s).

State the solution set:

(h) ________________

10. To solve the equation \( 3x^2 - 9x + 12 = 0 \) by the completing the square, the first step is ________________.

Objective 4: Solve Problems Using the Pythagorean Theorem

11. State the Pythagorean Theorem in words. If \( x \) and \( y \) are the lengths of the legs and \( z \) is the length of the hypotenuse, write an equation which uses these variables to state the Pythagorean Theorem.
Do the Math Exercises 7.1
Solving Quadratic Equations by Completing the Square

In Problems 1 – 6, solve each equation using the Square Root Property.

1. \( z^2 = 48 \)  
2. \( w^2 - 6 = 14 \)

3. \( (y - 2)^2 = 9 \)  
4. \( (2p + 3)^2 = 16 \)

5. \( \left( y + \frac{3}{2} \right)^2 = \frac{3}{4} \)  
6. \( q^2 - 6q + 9 = 16 \)

In Problems 7 – 9, find the value to complete the square in each expression. Then factor the perfect square trinomial.

7. \( p^2 - 4p \)  
8. \( z^2 - \frac{1}{3}z \)

9. \( m^2 + \frac{5}{2}m \)

In Problems 10 – 15, solve each quadratic equation by completing the square.

10. \( y^2 + 3y - 18 = 0 \)  
11. \( q^2 + 7q + 7 = 0 \)

12. \( x^2 - 5x - 3 = 0 \)  
13. \( n^2 = 10n + 5 \)

14. \( 3a^2 - 4a - 4 = 0 \)  
15. \( 2z^2 + 6z + 5 = 0 \)
Do the Math Exercises 7.1

In Problems 16 and 17, the lengths of the legs of a right triangle are given. Find the length of the hypotenuse. Give the exact answers and decimal approximations rounded to two decimal places.

16. $a = 7, b = 24$

17. $a = 2, b = \sqrt{5}$

16. __________

17. __________

18. **Right Triangle** A right triangle has a leg of length 2 and hypotenuse of length 10.

Find the length of the missing leg. Give the exact answer and a decimal approximation, rounded to 2 decimal places.

18. __________

19. Given that $h(x) = (x + 1)^2$, find all values of $x$ such that $h(x) = 32$.

19. __________

20. **Fire Truck Ladder** A fire truck has a 75-foot ladder. If the truck can safely park 20 feet from a building, how far up the building can the ladder reach assuming that the top of the base of the ladder is resting on top of the truck and the truck is 10 feet tall? Give the decimal approximation, rounded to 3 decimal places.

20. __________

21. **The converse of the Pythagorean Theorem is also true.** That is, in a triangle, if the square of the length of one side equals the sum of the squares of the lengths of the other two sides, then the triangle is a right triangle. The $90^\circ$ angle is opposite the longest side.

The lengths of the sides of a triangle are: 20, 48, and 52. Determine whether the triangle is a right triangle. If it is, identify the hypotenuse.

21. __________
In Problems 1 and 2, simplify each expression.

1. $\sqrt{125}$  
2. $\frac{15 + \sqrt{72}}{6}$

In Problems 3 and 4, simplify each expression using complex numbers.

3. $\sqrt{-28}$  
4. $\frac{6 - 2\sqrt{-4}}{6}$

5. Divide: $\frac{9x^2 - 27x + 3}{3}$

6. Evaluate the expression $\sqrt{b^2 - 4ac}$ if $a = 2$, $b = -4$, $c = -3$.
Guided Practice 7.2
Solving Quadratic Equations by the Quadratic Formula

Objective 1: Solve Quadratic Equations Using the Quadratic Formula

1. If $ax^2 + bx + c = 0$, then $x =$ ____________________.

2. When using the quadratic formula, the first step is to write the quadratic equation in ________________.

3. Write the quadratic equation in standard form and then identify the values assigned to $a$, $b$, and $c$. Do not solve the equation.
   (a) $x - 2x^2 = -4$       (b) $2 - 3x^2 = 8$       (c) $3x^2 = 6x$
   3a. _____________________
   3b. _____________________
   3c. _____________________

4. Solve: $8n^2 - 2n - 3 = 0$ (See textbook Example 1)

   **Step 1:** Write the equation in standard form $ax^2 + bx + c = 0$ and identify the values of $a$, $b$, and $c.$
   Since the equation is already in standard form, identify the values for $a$, $b$, and $c.$
   $8n^2 - 2n - 3 = 0$
   (a) $a = _____; b = _____; c = _____$

   **Step 2:** Substitute the values of $a$, $b$, and $c$ into the quadratic formula.
   Write the quadratic formula:
   (b) __________________________________
   Substitute the values for $a$, $b$, and $c.$
   (c) __________________________________

   **Step 3:** Simplify the expression found in Step 2.
   What is the value of the radicand?
   (d) __________________________________
   Simplify the expression in (c):
   (e) __________________________________
   Write the two expressions using $a \pm b$ means $a - b$ or $a + b$:
   (f) __________________________________

   **Step 4:** Check
   State the solution set:
   (g) __________________________________
Objective 2: Use the Discriminant to Determine the Nature of Solutions of a Quadratic Equation

5. In the quadratic equation \( ax^2 + bx + c = 0 \), the discriminant is used to determine the nature and number of solutions. To find the discriminant, substitute the identified values for \( a \), \( b \), and \( c \) into part of the quadratic formula, _____________________.

6. Determine the discriminant of each quadratic equation. Use the value of the discriminant to determine whether the quadratic equation has two rational solutions, two irrational solutions, one repeated real solution, or two complex solutions that are not real. (See textbook Example 5)

\[(a) \ 4x^2 + 5x - 9 = 0 \quad (b) \ x^2 + 4x + 9 = 0 \quad (c) \ 4x^2 = 4x - 1 \]

6a. ________________  
6b. ________________  
6c. ________________

Objective 3: Model and Solve Problems Involving Quadratic Equations

7. Projectile Motion  The height \( s \) of a toy rocket after \( t \) seconds, when fired straight up with an initial speed of 150 feet per second from an initial height of 2 feet, can be modeled by the function
\[ s(t) = -16t^2 + 150t + 2 \]

When will the height of the rocket be 200 feet? Round your answer to the nearest tenth of a second. (See textbook Examples 6 and 7)

(a) Step 1: Identify  Here we want to know the value \( t \) when \( s = ? \) ______________________________

Step 2: Name  The variables are named in the problem. \( t \) is the time the rocket has traveled and \( s \) is height of the rocket.

(b) Step 3: Translate  Use the information from (a) and the given formula to write a model for this problem.

(c) Step 4: Solve  Solve the equation in (b). What values did you find for \( t \)? ______________________________

Step 5: Check

(d) Step 6: Answer the Question ______________________________

(e) Will the rocket ever reach a height of 500 feet? ______________________________

(f) When will the rocket hit the ground? ______________________________
Do the Math Exercises 7.2
Solving Quadratic Equations by the Quadratic Formula

In Problems 1 – 6, solve each equation using the quadratic formula.

1. \(p^2 - 4p - 32 = 0\)  
2. \(10x^2 + x - 2 = 0\)  
3. \(2q^2 - 4q + 1 = 0\)  
4. \(x + \frac{1}{x} = 3\)  
5. \(2z^2 + 7 = 4z\)  
6. \(1 = 5w^2 + 6w\)

In Problems 7 – 9, determine the discriminant of each quadratic equation. Use the value of the discriminant to determine whether the quadratic equation has two rational solutions, two irrational solutions, one repeated real solution, or two complex solutions that are not real.

7. \(p^2 + 4p - 2 = 0\)  
8. \(16x^2 + 24x + 9 = 0\)  
9. \(6x^2 - x = -4\)

In Problems 10 – 15, solve each equation using any method you wish.

10. \(q^2 - 7q + 7 = 0\)  
11. \(3x^2 + 5x = 2\)
Do the Math Exercises 7.2

12. \( 5m - 4 = \frac{5}{m} \)  
13. \( 8p^2 - 40p + 50 = 0 \)  
12. __________  
13. __________

14. \( (a - 3)(a + 1) = 2 \)  
15. \( \frac{x-1}{x^2+4} = 1 \)  
14. __________  
15. __________

In Problem 16, suppose that \( f(x) = x^2 + 2x - 8 \).
16a. Solve for \( x \), if \( f(x) = 0 \)  
16b. Solve for \( x \), if \( f(x) = -8 \)  
16a. __________  
16b. __________

17. Area The area of a rectangle is 60 square inches. The length of the rectangle is 6 inches more than the width. What are the dimensions of the rectangle?  
17. __________

18. Area The area of a triangle is 35 square inches. The height of the rectangle is 2 inches less than the base. What are the base and height of the triangle?  
18. __________

19. Roundtrip A Cessna aircraft flies 200 miles due west into the jet stream and flies back home on the same route. The total time of the trip (excluding the time on the ground) takes 4 hours. The Cessna aircraft can fly 120 miles per hour in still air. What is the net effect of the jet stream on the aircraft?  
19. __________

20. Given the quadratic equation \( ax^2 + bx + c = 0 \), show that the sum of the solutions to any quadratic equation in this form is \( -\frac{b}{a} \). Show that the product of the solutions to this quadratic equation is \( \frac{c}{a} \).
In Problems 1 and 2, factor completely.

1. \(a^4 - 7a^2 - 18\)
2. \(6(x - 1)^2 - 13(x - 1) + 6\)

In Problems 3 and 4, simplify each expression.

3. \((5x^{-1})^2\)
4. \((-\frac{2}{3}x^3)^2\)

5. Solve: \(2x^2 + x - 6 = 0\)
Guided Practice 7.3
Solving Equations Quadratic in Form

Objective 1: Solve Equations That Are Quadratic in Form

1. In general, if a substitution \( u \) transforms an equation into one of the form \( au^2 + bu + c = 0 \), then the original equation is called an equation quadratic in form. Here \( u \) represents any variable expression and we solve the equation by substituting \( u \) for the variable expression written with the coefficient \( b \). For these equations quadratic in form, identify the substitution for \( u \) and then write a new equation using your substitution.

(a) \( 2y - 11\sqrt{y} + 15 = 0 \)
\[ u = \text{______________} \]

(b) \( v^4 + 10v^2 + 1 = 0 \)
\[ u = \text{______________} \]

(c) \( (x - 1)^2 - 5(x - 1) + 6 = 0 \)
\[ u = \text{______________} \]

(d) \( x^2 - \frac{1}{3} - 3 = 0 \)
\[ u = \text{______________} \]

(e) \( 4x^{-2} + x^{-1} - 3 = 0 \)
\[ u = \text{______________} \]

2. Solve: \( x^4 - 6x^2 - 16 = 0 \) (See textbook Example 1)

Step 1: Determine the appropriate substitution and write the equation in the form \( au^2 + bu + c = 0 \).

Here we let \( u = ? \) 
\[ \text{(a)} \quad x^4 - 6x^2 - 16 = 0 \]

Rewrite the equation using your substitution from (a):
\[ \text{(b)} \]

Step 2: Solve the equation \( au^2 + bu + c = 0 \).

Factor:
\[ \text{(c)} \]

Set each factor to 0 and solve:
\[ \text{(d)} \]

Step 3: Solve for the variable in the original equation using the value of \( u \) from (a).

Substitute from (a) into your solution from (d):
\[ \text{(e)} \]

Solve the equations. In this case we use the Square Root Property:
\[ \text{(f)} \]

Simplify the radicals:
\[ \text{(g)} \]

Step 4: Verify your solution(s).

State the solution set:
\[ \text{(h)} \]
3. Solve: \( (x^2 + 3)^2 - 6(x^2 + 3) + 8 = 0 \) \( (See \ textbook \ Example \ 2) \)

(a) Here we let \( u = \) _______________.

(b) In this problem, \( u = 4 \) or \( u = 2 \). Replace \( u \) and solve for \( x \). State the solution set. _______________

4. Solve: \( 2x + \sqrt{x} - 10 = 0 \) \( (See \ textbook \ Example \ 3) \)

(a) Here we let \( u = \) _______________.

(b) In this problem, \( u = \frac{5}{2} \) or \( u = 2 \). Replace \( u \) and solve for \( x \). State the solution set. _______________

5. Solve: \( \frac{2}{n^3} - 2n^{\frac{1}{3}} - 3 = 0 \) \( (See \ textbook \ Example \ 5) \)

(a) Here we let \( u = \) _______________.

(b) In this problem, \( u = 3 \) or \( u = -1 \). Replace \( u \) and solve for \( x \). State the solution set. _______________

6. As always, it is important to check for extraneous solutions. List two cases when extraneous solutions might appear.

(a) ___________________________ (b) ___________________________
In Problems 1 – 14, solve each equation.

1. \[ x^4 - 10x^2 + 9 = 0 \]
2. \[ 4b^4 - 5b^2 + 1 = 0 \]

3. \[ (x + 2)^2 - 3(x + 2) - 10 = 0 \]
4. \[ x - 5\sqrt{x} - 6 = 0 \]

5. \[ z + 7\sqrt{z} + 6 = 0 \]
6. \[ q^2 + 2q^{-1} = 15 \]

7. \[ 10a^2 + 23a^{-1} = 5 \]
8. \[ y^2 - 2y^{\frac{2}{3}} - 3 = 0 \]

9. \[ \frac{1}{x^3} - \frac{7}{x} + 12 = 0 \]
10. \[ y^6 - 7y^3 - 8 = 0 \]
Do the Math Exercises 7.3

11. \(6b^2 - b^{-1} = 1\)  
12. \(x^4 + 3x^2 = 4\)

13. \(\frac{1}{a^2} + \frac{1}{b^4} - 12 = 0\)  
14. \(\left(\frac{1}{x-1}\right)^2 + \frac{7}{x-1} = 8\)

In Problems 15 and 16, suppose that \(f(x) = x^4 + 5x^2 + 3\). Find the values of \(x\) such that

15. \(f(x) = 3\)  
16. \(f(x) = 17\)

In Problems 17 and 18, suppose that \(h(x) = 3x^4 - 9x^2 - 8\). Find the values of \(x\) such that

17. \(h(x) = -8\)  
18. \(h(x) = 22\)

In Problems 19 and 20, find the zeros of the function.

19. \(f(x) = x^4 - 13x^2 + 42\)  
20. \(h(p) = 8p - 18\sqrt{p} - 35\)
In Problems 1 and 2, graph using the point-plotting method.

1. \( y = -x^2 \)

2. \( y = x^2 + 2 \)

In Problems 3 and 4, find the function value.

3. \( f(x) = -x^2 + 6x - 2; f(-3) \)

4. \( f(x) = 2x^2 + x + 5; f(-4) \)

5. What is the domain of \( f(x) = \frac{1}{2}x^2 + \frac{4}{3}x - 6 \)?
Guided Practice 7.4
Graphing Quadratic Functions Using Transformations

Objective 1: Graph Quadratic Functions of the Form \( f(x) = x^2 + k \)

In Problems 1 and 2, match the function to either graph (a) or graph (b).

1. \( f(x) = x^2 \)  
   (a) \hspace{1cm} (b) \hspace{1cm} 1. __________

2. \( f(x) = -x^2 \)  
   \hspace{1cm} \hspace{1cm} 2. __________

3. To obtain the graph of \( f(x) = x^2 - 2 \), shift the graph of \( f(x) = x^2 \)
   (a) horizontally or vertically? (See textbook Examples 1 and 2)  
      3a. __________
   (b) How many units will the graph shift?  
      3b. __________
   (c) Does the graph shift right, left, up, or down?  
      3c. __________

Objective 2: Graph Quadratic Functions of the Form \( f(x) = (x - h)^2 \)

4. To obtain the graph of \( f(x) = -(x - 1)^2 \), shift the graph of \( f(x) = -x^2 \)
   (a) horizontally or vertically? (See textbook Examples 3 and 4)  
      4a. __________
   (b) How many units will the graph shift?  
      4b. __________
   (c) Does the graph shift right, left, up, or down?  
      4c. __________

Objective 3: Graph Quadratic Functions of the Form \( f(x) = ax^2 \)

5. The graph of \( f(x) = ax^2 + bx + c \) is a parabola that opens either up or down. This is determined by the coefficient \( a \). (See textbook Example 6)
   (a) If \( a > 0 \), the parabola opens  
      5a. __________
   (b) If \( a < 0 \), the parabola opens  
      5b. __________

6. The value of \( a \) will also determine the breadth of the parabola.
   (a) If \( |a| \) ______ we say the graph is vertically stretched (taller, thinner, steeper).
   (b) If _______ \( |a| \) _______ we say the graph is vertically compressed (shorter, fatter, flatter).
### Guided Practice 7.4

**Objective 4: Graph Quadratic Functions of the Form** \( f(x) = ax^2 + bx + c \)

7. Graph \( f(x) = 3x^2 - 12x + 7 \). (See textbook Examples 7 and 8)

#### Step 1: Write the function

\( f(x) = ax^2 + bx + c \) as

\( f(x) = a(x - h)^2 + k \) by completing the square.

<table>
<thead>
<tr>
<th>Group the terms involving ( x ):</th>
<th>(a) ________________</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factor out the coefficient of the square term, 3, from the parentheses:</td>
<td>(b) ________________</td>
</tr>
<tr>
<td>Identify the number required to complete the square:</td>
<td>(c) ________________</td>
</tr>
<tr>
<td>When this added to the right side of the equation, what must be done to maintain the equality?</td>
<td>(d) ________________</td>
</tr>
<tr>
<td>Write the amended equation:</td>
<td>(e) ________________</td>
</tr>
<tr>
<td>Factor the perfect square trinomial:</td>
<td>(f) ________________</td>
</tr>
</tbody>
</table>

#### Step 2: Graph the function

\( f(x) = a(x - h)^2 + k \) using transformations.

| Identify the vertex: | (g) ________________ |
| Opening direction: | (h) ________________ |
| Axis of symmetry: | (i) ________________ |

---

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In Problems 1 – 6, use the graph of $y = x^2$ to graph the quadratic function.

1. $f(x) = x^2 - 1$

2. $f(x) = (x + 4)^2$

3. $G(x) = 5x^2$

4. $P(x) = -3x^2$

5. $g(x) = (x + 2)^2 - 1$

6. $G(x) = (x - 4)^2 + 2$
In Problems 7 – 10, write each function in the form \( f(x) = a(x - h)^2 + k \). Then determine the vertex and the axis of symmetry.

7. \( h(x) = x^2 - 7x + 10 \)  
8. \( f(x) = 3x^2 + 18x + 25 \)

9. \( g(x) = -x^2 - 8x - 14 \)  
10. \( h(x) = -4x^2 + 4x \)

11. Write a quadratic function in the form \( f(x) = a(x - h)^2 + k \) with the properties:

   - opens up;
   - vertically compressed by a factor of \( \frac{1}{2} \);
   - vertex at \(( -5, 0 )\).

12. Determine the quadratic function whose graph is shown below. Each tick mark represents one unit of length.
Five-Minute Warm-Up 7.5
Graphing Quadratic Functions Using Properties

1. Find the intercepts of the graph of $3x - 4y = -12$.

2. Solve: $2x^2 - 13x - 24 = 0$

3. Find the zeros of $f(x) = -2x^2 + 10x + 12$

4. If $f(x) = -x^2 - 3x - 2$, find $f(-5)$.
Guided Practice 7.5
Graphing Quadratic Functions Using Properties

Objective 1: Graph Quadratic Functions of the Form \( f(x) = ax^2 + bx + c \)

1. The vertex is the turning point of the parabola. If \( f(x) = ax^2 + bx + c \), the \( x \)-coordinate of the vertex is at \( x = \) __________.

2. The \( x \)-intercepts of the parabola can be found by ____________________________________________.

3. Graph \( f(x) = x^2 + 2x - 8 \) using its properties. (See textbook Example 1)

   **Step 1:** Determine whether the parabola opens up or down.

   **Step 2:** Determine the vertex and axis of symmetry.

   **Step 3:** Determine the \( y \)-intercept.

   **Step 4:** Find the discriminant, \( b^2 - 4ac \), to determine the number of the \( x \)-intercepts. Then determine the \( x \)-intercepts, if any.
Guided Practice 7.5

**Step 5:** Plot the vertex, y-intercept, and x-intercepts. Use the axis of symmetry to find an additional point. Draw the graph of the quadratic function.

---

**Objective 2: Find the Maximum or Minimum Value of a Quadratic Function**

4. The maximum or minimum value of a quadratic function is the ______________ of the vertex.

In Problems 5 and 6, determine whether the function has a maximum or a minimum value. (See textbook Example 5)

5. \( f(x) = -2x^2 + 4x + 3 \)
6. \( f(x) = 3x^2 + 12x - 1 \)

5. __________
6. __________

---

**Objective 3: Model and Solve Optimization Problem Involving Quadratic Functions**

7. Martin is making a dog run in which he will keep his show dogs and decides to enclose an area of his backyard. He uses one side of his house for the pen and encloses the other three sides with fencing. If he has 50 feet of fencing he plans to use, what are the dimensions of the pen that encloses the most area? (See textbook Example 7)

**Step 1: Identify** We wish to determine the dimensions of the rectangle that maximize the area.

**Step 2: Name** We let \( x \) represent one side of the rectangle and \( y \) represent the other side.

**Step 3: Translate** Since we are looking for the maximum area, we use the formula \( A = lw \). Our goal is to have \( A \) written in terms of one variable. In this case we will choose \( x \), but \( y \) will work as well.

(a) Our rectangle has sides \( x \) and \( y \). Write an equation for the area using \( x \) and \( y \). _____________________

(b) There is 50 feet of fencing for three sides. Write this equation: _____________________

(c) Solve for \( y \): _____________________

(d) Substitute this expression into your equation from (a): _____________________

(e) **Step 4: Solve**

(f) Substitute your value for \( x \) into (b) to find the value for \( y \):

**Step 5: Check**

(g) **Step 6: Answer the Question**

(h) What is the maximum area Martin can enclose? _____________________
Do the Math Exercises 7.5
Graphing Quadratic Functions Using Properties

In Problems 1 – 3, use the discriminant to determine the number of x-intercepts the graph of each function will have. Then determine the x-intercepts.

1. \( g(x) = 2x^2 - 7x - 4 \)

2. \( f(x) = x^2 - 6x + 9 \)

3. \( P(x) = -2x^2 + 3x + 1 \)

In Problems 4 – 9, graph each quadratic function.

4. \( f(x) = x^2 - 2x - 8 \)

5. \( g(x) = -x^2 + 2x + 15 \)

6. \( f(x) = x^2 - 4x + 7 \)

7. \( P(x) = -x^2 - 12x - 36 \)
8. \( g(x) = (x + 2)^2 - 1 \)

9. \( G(x) = (x - 4)^2 + 2 \)

In Problems 10 and 11, determine whether the quadratic function has a maximum or minimum. Then find the maximum or minimum value.

10. \( H(x) = -3x^2 + 12x - 1 \)

11. \( G(x) = 5x^2 + 10x - 1 \)

12. **Fun with Numbers** The sum of two numbers is 50. Find the numbers such that their product is a maximum.

13. **Fun with Numbers** The difference of two numbers is 10. Find the numbers such that their product is a minimum.

14. **Punkin Chunkin** Suppose that catapult in the Punkin Chunkin contest releases a pumpkin 8 feet above the ground at an angle of 45° to the horizontal with an initial speed 220 feet per second. The model \( s(t) = -16t^2 + 155t + 8 \) can be used to estimate the height \( s \) of an object after \( t \) seconds.

   (a) Determine the time at which the pumpkin is at a maximum height.

   (b) Determine the maximum height of the pumpkin.

   (c) After how long will the pumpkin strike the ground?
Five-Minute Warm-Up 7.6
Polynomial Inequalities

*In Problems 1 – 4, write in interval notation.*

1. $-4 \leq x < -2$  
2. $-3 < x \leq 1$  
   1. __________
   2. __________

3. $x > -2$  
4. $x \leq 5$  
   3. __________
   4. __________

5. Solve: $6x + 3 \leq 11x - 7$  
   5. __________

6. Solve: $(x - 3)(x + 1) = 2$  
   6. __________
Guided Practice 7.6
Polynomial Inequalities

Objective 1: Solve Quadratic Inequalities

1. A quadratic inequality is an inequality written in one of the following forms:
   \[ ax^2 + bx + c > 0; \ ax^2 + bx + c \geq 0; \ ax^2 + bx + c < 0; \ ax^2 + bx + c \leq 0. \]

   (a) If \( f(x) \) is a quadratic function and \( f(x) > 0 \), we are interested in finding \( x \) values for which the function is _____________( above or below) the \( x \)-axis.  

   1a. __________

   (b) If \( f(x) \) is a quadratic function and \( f(x) < 0 \), we are interested in finding \( x \) values for which the function is _____________( above or below) the \( x \)-axis.

   1b. __________

2. We will present two methods for solving quadratic inequalities. These methods are:

   2a. __________

   2b. __________

3. Solve \( x^2 + 2x - 3 \leq 0 \) using the graphical method. (See textbook Example 1)

   **Step 1:** Write the inequality so that \( ax^2 + bx + c \) is on one side of the inequality and 0 is on the other.

   The inequality is already in this form:  
   \[ x^2 + 2x - 3 \leq 0 \]

   **Step 2:** Graph the function \( f(x) = ax^2 + bx + c \). Be sure to label the \( x \)-intercepts on the graph.

   Function to be graphed: (a) __________________________

   \( x \)-intercepts: (b) __________________________

   Vertex: (c) __________________________

   (d) Graph:

   **Step 3:** From the graph, determine where the function is positive and where the function is negative. Use the graph to determine the solution set to the inequality.

   Are we looking for positive or negative function values in this problem? (e) __________________________

   State the solution set: (f) __________________________
4a. If \( f(x) \) is a quadratic function in factored form and \( f(x) > 0 \), we are interested in finding the product of the factors for which the function is ________ (positive or negative).

4b. If \( f(x) \) is a quadratic function in factored form and \( f(x) < 0 \), we are interested in finding the product of the factors for which the function is ________ (positive or negative).

5. Solve \( x^2 + 2x - 35 > 0 \) using the algebraic method. \((See \ textbook \ Example \ 2)\)

\textbf{Step 1:} Write the inequality so that \( ax^2 + bx + c \) is on one side of the inequality and 0 is on the other.

The inequality is already in this form: \( x^2 + 2x - 35 > 0 \)

\textbf{Step 2:} Determine the solutions to the equation \( ax^2 + bx + c = 0 \). Factor:

(a) __________

Use Zero-Product Property:

(b) __________

\textbf{Step 3:} Use the solutions to the equation solved in Step 2 to separate the real number line into intervals.

List the intervals:

(c) __________

\textbf{Step 4:} Write the expression in factored form. Within each interval formed in Step 3, choose a test point and determine the sign of each factor. Then determine the sign of the product. Also determine the value of the expression at each solution found in Step 2.

(d) Complete the chart below:

<table>
<thead>
<tr>
<th>Interval</th>
<th>((-\infty, _____))</th>
<th>(x = _____)</th>
<th>((_____, _____))</th>
<th>(x = _____)</th>
<th>((_____, \infty))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test Point</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sign of</td>
<td></td>
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<tr>
<td>Sign of</td>
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</tr>
<tr>
<td>Product</td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Conclusion</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

To find the solution, select the intervals where the product is (positive or negative)?

(e) __________

Is 0 included in the solution set?

(f) __________

State the solution set:

(g) __________
Do the Math Exercises 7.6

Polynomial Inequalities

In Problems 1 – 10, solve each inequality. Write the solution set in interval notation.

1. \((x – 8)(x + 1) \leq 0\)
2. \((x – 4)(x – 10) > 0\)

3. \(p^2 + 5p + 4 < 0\)
4. \(2b^2 + 5b < 7\)

5. \(x + 6 < x^2\)
6. \(x^2 – 3x – 5 \geq 0\)

7. \(-3p^2 < 3p – 5\)
8. \(y^2 + 3y + 5 \geq 0\)

9. \(3w^2 + w < -2\)
10. \(p^2 – 8p + 16 \leq 0\)
Do the Math Exercises 7.6

In Problems 11 and 12, for each function find the values of $x$ that satisfy the given condition.
11. Solve $f(x) > 0$ if $f(x) = x^2 + 4x$
12. Solve $f(x) \leq 0$ if $f(x) = x^2 + 2x - 48$

11. __________
12. __________

In Problems 13 and 14, find the domain of the given function.
13. $f(x) = \sqrt{x^2 - 5x}$
14. $G(x) = \sqrt{x^2 + 2x - 63}$

13. __________
14. __________

15. Revenue Function Suppose that the marketing department of Samsung has found that, when a certain model of cellular telephone is sold a price of $p$ dollars, the daily revenue $R$ (in dollars) as a function of the price $p$ is $R(p) = -5p^2 + 600p$. Determine the prices for which revenue will exceed $17,500$. That is, solve $R(p) > 17,500$.

15. __________

In Problems 16 and 17, solve each polynomial inequality.
16. $(3x + 4)(x - 2)(x - 6) \geq 0$
17. $3x^3 + 5x^2 - 20x < 0$

16. __________
17. __________
Five-Minute Warm-Up 8.1
Composite Functions and Inverse Functions

1. Determine the domain: \( R(x) = \frac{4}{x^2 - 3x + 2} \).
   1. ______________

2. Use the function \( f(x) = -3x^2 + 1 \) to find the following.
   (a) \( f(-4) \)
   2a. _____________
   (b) \( f(a - 2) \)
   2b. _____________
   (c) \( f(x + h) \)
   2c. _____________

In Problem 3, use the Vertical Line Test to determine whether the following is the graph of a function.

3a. _____________
3b. _____________
Guided Practice 8.1
Composite Functions and Inverse Functions

Objective 1: Form the Composite Function

1. Function composition is the process of evaluating one function and then carrying the result forward to be evaluated in a second function. The notation $f(g(x))$ means evaluate the inner function, $g(x)$, and then evaluate the result in the outer function, $f(x)$. This function composition is written with the notation $f(g(x))$. This notation is read "$f$ composed with $g". The mathematical notation “$\circ$” is called small circle, so we can also say "$f$ small circle $g$ of $x$".

In Problems 2 and 3, suppose that $f(x) = 2x^2 + 7$ and $g(x) = x - 1$. (See textbook Example 1)

2. Find $(f \circ g)(-2)$.
   (a) Evaluate $g(-2)$.
   (b) Use this result to find $f(g(-2))$.
   (c) What is $(f \circ g)(-2)$?

3. Find $(g \circ f)(4)$.
   (a) Evaluate $f(4)$.
   (b) Use this result to find $g(f(4))$.
   (c) What is $(g \circ f)(4)$?

Objective 2: Determine Whether a Function is One-to-One

4. In your own words, what does it mean for a function to be one-to-one?

5. Determine whether or not the function is one-to-one. (See textbook Example 3)
   (a) $\{(1, -3), (-3, 1), (2, -4), (-2, 4)\}$
   (b) $\{(0, 1), (1, 2), (2, 3), (3, 1)\}$

6. Graphically, how can you test whether a function is one-to-one?

Objective 3: Find the Inverse of a Function Defined by a Map or Set of Ordered Pairs

7. In order for the inverse of a function to also be a function, the function must be __________________.
8. If \( f(x) \) is a one-to-one function, then its inverse function is written with the notation \( ________ \). This means that if the ordered pair \((a, b)\) satisfies \( f(x) \), then the ordered pair \( ________ \) satisfies the inverse function.

9. Given the function \( \{(−3, 15), (−1, −5), (0, 0), (2, 10)\} \), find
   (a) the inverse function (See textbook Example 6)
   (b) the domain of the function
   (c) the range of the function
   (d) the domain of the inverse function
   (e) the range of the inverse function

   9a. ________  
   9b. ________  
   9c. ________  
   9d. ________  
   9e. ________

Objective 4: Obtain the Graph of the Inverse Function from the Graph of the Function

10. Graphs can be transformed in several different ways: translation (slide), reflection (flip), and rotation (turn). A graph is symmetric about a line if one part of the graph is a mirror image of the other. The line on which the graph is flipped (or folded) is called the axis of symmetry. For instance, \( y = x^2 \) is symmetric about the \( y \)-axis. The graph of a function and its inverse are symmetric about the line ________.

Objective 5: Find the Inverse of a Function Defined by an Equation

11. Find the inverse of \( f(x) = 4x − 8 \). (See textbook Example 8)

   **Step 1:** Replace \( f(x) \) with \( y \) in the equation for \( f(x) \).
   
   Replace \( f(x) \) with \( y \).  
   \( f(x) = 4x − 8 \)
   (a) ______________________

   **Step 2:** Interchange the variables to write in inverse.
   
   Rewrite equation (a) exchanging the variables \( x \) and \( y \).
   (b) ______________________

   **Step 3:** Solve the equation found in Step 2 for \( y \) in terms of \( x \).
   
   Add 8 to both sides:  
   (c) ______________________
   Divide both sides by 4:  
   (d) ______________________

   **Step 4:** Replace \( y \) with \( f^{-1}(x) \).
   (e) ______________________

   **Step 5:** Verify your result by showing that \( f^{-1}(f(x)) = x \) and \( f(f^{-1}(x)) = x \).
Do the Math Exercises 8.1
Composite Functions and Inverse Functions

In Problems 1 – 4, use the functions \( f(x) = x^2 - 3 \) and \( g(x) = 5x + 1 \) to find each of the following.

1. \((f \circ g)(3)\)  
2. \((g \circ f)(-2)\)  
3. \((f \circ f)(1)\)  
4. \((g \circ g)(-4)\)  

5. Given the functions \( f(x) = x - 3 \) and \( g(x) = 4x \), find \((f \circ g)(x)\).

6. Given the functions \( f(x) = \sqrt{x + 2} \) and \( g(x) = x - 2 \), find \((g \circ f)(x)\).

7. Given the functions \( f(x) = \frac{2}{x - 1} \) and \( g(x) = \frac{4}{x} \), find \((f \circ f)(x)\).

In Problems 8 – 10, determine whether the function is one-to-one.

8. \(\{(−2, −8), (−1, −1), (0, 0), (1, 1), (2, 8)\}\)  
9. \(\{(−3, 0), (−2, 3), (−1, 0), (0, −3)\}\)  
10. [Graph of a function]
11. Find the inverse of the function \( \{(-10, 1), (-5, 4), (0, 3), (-5, 2)\} \).

12. \( f(x) = 10x; g(x) = \frac{x}{10} \)

13. \( f(x) = \frac{2}{x + 4}; g(x) = \frac{2}{x} - 4 \)

14. \( g(x) = x + 6 \)

15. \( H(x) = 3x + 8 \)

16. \( f(x) = x^3 - 2 \)

17. \( G(x) = \frac{2}{3 - x} \)

18. \( R(x) = \frac{2x}{x + 4} \)

19. \( g(x) = \sqrt[3]{x + 2} - 3 \)

20. **Volume of a Balloon** The volume \( V \) of a hot-air balloon (in cubic meters) as a function of its radius \( r \) is given by \( V(r) = \frac{4}{3} \pi r^3 \). If the radius \( r \) of the balloon is increasing as a function of time \( t \) (in minutes) according to \( r(t) = 3\sqrt[3]{t} \), for \( t \geq 0 \), find the volume of the balloon as a function of time \( t \). What will be the volume of the balloon after 30 minutes?
In Problems 1 – 4, evaluate each expression.

1. \(2^4\)  
2. \(2^{-2}\)  
3. \(3^0\)  
4. \(10^{-1}\)

5. Write 6.023455 as a decimal  
   (a) rounded to 4 decimal places  
   (b) truncated to 4 decimal places.

6. \(5x^3 \cdot 2x^{-5}\)  
7. \(\frac{4a^2}{10a^{-5}}\)  
8. \((4p^5)^3\)

9. Solve: \(6x^2 = 7x + 5\)
Guided Practice 8.2
Exponential Functions

Objective 1: Evaluate Exponential Expressions

1. An exponential function is a function of the form $f(x) = a^x$ where $a$ is _______________ and $a \neq ____$. 

In Problems 2 and 3, evaluate each expression to 5 decimal places. (See textbook Example 1)

2. $2^{1.4}$
3. $2^{\sqrt{2}}$

Objective 2: Graph Exponential Functions

In Problems 4 and 5, match the function to the graph. (See textbook Examples 2 and 3)

4. $f(x) = a^x; a > 1$  
   (a) 

5. $f(x) = a^x; 0 < a < 1$  
   (b)

Objective 3: Define the Number e

6. What is an approximate value of $e$ to the nearest thousandth?

7. Evaluate each of the following to two decimal places.
   (a) $e^3$
   (b) $e^{-2}$

Objective 4: Solve Exponential Equations

8. The Property for Solving Exponential Equations states that if two exponential functions have the same base and the exponential functions are equal, then it must be true that ____________________________.

Name:  Date:  Instructor:  Section:
9. Solve: \(3^{2x+1} = 27\)  
(See textbook Example 5)

**Step 1:** Use the Laws of Exponents to write both sides of the equation with the same base.

Prime factor 27 and write in exponential form:

\[ (a) \quad 3^{2x+1} = 27 \]

**Step 2:** Set the exponents on each side of the equation equal to each other.

\[ (b) \quad 3^{2x+1} = 27 \]

**Step 3:** Solve the equation resulting from Step 2.

Subtract 1 from both sides:

\[ (c) \quad 3^{2x+1} = 27 \]

Divide both sides by 2:

\[ (d) \quad 3^{2x+1} = 27 \]

**Step 4:** Verify your solution(s).

Substitute your value into the original equation.

We leave it to you to verify your solution.

State the solution set:

\[ (e) \quad 3^{2x+1} = 27 \]

**Objective 5: Use Exponential Models that Describe Our World**

10. Strontium 90 is a radioactive material that decays according to the function \(A(t) = A_0e^{-0.087t}\), where \(A_0\) is the initial amount present and \(A\) is the amount present at time \(t\) (in days). If you begin with 100 grams of Strontium 90, how much will be present after 5 days?

11. What is the compound interest formula? Identify each of the variables in the formula.

12. Suppose that you deposit $100 into a savings account that earns 7% compounded quarterly for a period of 5 years.

(a) Identify the value of each variable in the compound interest formula.

12a. _______

(b) How much is in the account 5 years after the $100 deposit?

12b. _______

(c) You also deposit $100 in another bank because you believe you are getting a better deal when you receive 7% compounded daily. Assuming there are 360 days per business year, identify the value of each variable in the compound interest formula.

12c. _______

(d) How much is in the account after 5 years?

12d. _______
**Do the Math Exercises 8.2**

**Exponential Functions**

*In Problem 1 and 2, approximate each number using a calculator. Round your answer to three decimal places.*

1a. \(5^{1.4}\) 1b. \(5^{1.41}\)

1c. \(5^{1.414}\) 1d. \(5^{1.4142}\)

1e. \(5^{\sqrt{2}}\)

2a. \(e^{3}\) 2b. \(e^{1.5}\)

*In Problems 3 – 5, graph each function.*

3. \(g(x) = 10^x\)

4. \(H(x) = 2^{x-2}\)

5. \(f(x) = e^x - 1\)
In Problems 6 – 11, solve each equation.

6. \(3^x = 3^{-2}\)  
7. \(2^{x+3} = 128\)  
6. \(\)  
7. \(\)

8. \(3^{x+4} = 27^x\)  
9. \(9^{2x} \cdot 27^{x^2} = 3^{-1}\)  
8. \(\)  
9. \(\)

10. \(\left(\frac{1}{6}\right)^x - 36 = 0\)  
11. \(e^{3x} = e^2\)  
10. \(\)  
11. \(\)

In Problem 12 and 13, suppose that \(f(x) = 3^x\).

12. What is \(f(2)\)? What point is on the graph of \(f\)?  
13. If \(f(x) = \frac{1}{81}\), what is \(x\)? What point is on the graph of \(f\)?  
12. \(\)  
13. \(\)

14. A Population Model  According to the U.S. Census Bureau, the population of the world in 2012 was 7018 million people. In addition, the population of the world was growing at a rate of 1.26% per year. Assuming that this growth rate continues, the model \(P(t) = 7018(1.0126)^t - 2012\) represents the population \(P\) (in millions of people) in year \(t\).

(a) According to this model, what will be the population of the world in 2015?  
(b) According to this model, what will be the population of the world in 2025?
Five-Minute Warm-Up 8.3
Logarithmic Functions

1. Solve: \(4x + 6 > 0\)  
   \[1. \text{__________}

2. Solve: \(\sqrt{2x + 3} = x\)  
   \[2. \text{__________}

3. Solve: \(x^2 = -7x - 12\)  
   \[3. \text{__________}

In Problems 4 and 5, evaluate each expression.

4. \(4^{-2}\)  
   \[4. \text{__________}

5. \(\left(\frac{1}{2}\right)^{-3}\)  
   \[5. \text{__________}

In Problems 6 and 7, find the function value.

6. \(f(x) = x^2; f(-2)\)  
   \[6. \text{__________}

7. \(f(x) = 2^x; f(-2)\)  
   \[7. \text{__________}
1. The logarithmic function to the base $a$, where $0 < a \neq 1$, is denoted by $y = \log_a x$ and is read as “$y$ is the logarithm to the base $a$ of $x$”. This is defined as

$$y = \log_a x$$

is equivalent to _____________________

---

**Objective 1: Change Exponential Equations to Logarithmic Equations**

In Problems 2 and 3, rewrite each exponential expression as an equivalent expression involving a logarithm.

*(See textbook Example 1)*

2. $2^{-3} = \frac{1}{8}$

3. $a^2 = 3$

2. __________

3. __________

---

**Objective 2: Change Logarithmic Equations to Exponential Equations**

In Problems 4 and 5, rewrite each logarithmic expression as an equivalent expression involving an exponent.

*(See textbook Example 2)*

4. $p = \log_3 30$

5. $\log_a 3 = -5$

4. __________

5. __________

---

**Objective 4: Determine the Domain of a Logarithmic Function**

*(See textbook Example 5)*

6a. The domain of the logarithmic function is

6b. The range of the logarithmic function is

6c. Determine the domain of the function $f(x) = \log_3 (10 - 2x)$. 
Objective 5: Graph Logarithmic Functions

7. Because exponential functions and logarithmic functions are inverses of each other, we know that the graph of \( y = \log_5 x \) is a reflection of \( \underline{\text{____________}} \) across the line \( y = x \). Graph both of these functions in the same coordinate plane. \( \text{(See textbook Example 6)} \)

Objective 6: Work with Natural and Common Logarithms

8. The natural logarithm: \( y = \ln x \) is written in exponential form as \( \underline{\text{____________}} \)

9. The common logarithm: \( y = \log x \) is written in exponential form as: \( \underline{\text{____________}} \)

Objective 7: Solve Logarithmic Equations

In Problems 10 and 11, solve the logarithmic equation. Be sure to check your answers as extraneous solutions may appear. \( \text{(See textbook Examples 9 and 10)} \)

10. \( \log_3 (5 - 2x) = 2 \)

11. \( \ln x = -2 \)

10. \( \underline{\text{____________}} \)

11. \( \underline{\text{____________}} \)

Objective 8: Use Logarithmic Models That Describe Our World

12. List 3 applications of logarithmic functions that are used in the world today.

12a. \( \underline{\text{____________}} \)

12b. \( \underline{\text{____________}} \)

12c. \( \underline{\text{____________}} \)
In Problems 1 – 3, change each exponential expression to an equivalent expression involving a logarithm.

1. \(64 = 4^3\)  
2. \(b^4 = 23\)  
3. \(10^{-3} = z\)

In Problems 4 – 6, change each logarithmic expression to an equivalent expression involving an exponent.

4. \(\log_3 81 = 4\)  
5. \(\log_6 x = -4\)  
6. \(\log_a 16 = 2\)

In Problems 7 – 8, find the exact value of each logarithm without using a calculator.

7. \(\log_4 16\)  
8. \(\log_{\sqrt{3}} 3\)

In Problems 9 – 10, find the domain of each function.

9. \(f(x) = \log_3(x - 2)\)  
10. \(G(x) = \log_4(3 - 5x)\)
Do the Math Exercises 8.3

In Problems 11 and 12, use a calculator to evaluate each expression. Round your answer to three decimal places.

11. \( \log 0.78 \)    12. \( \ln \frac{1}{2} \)

11. ________

12. ________

In Problems 13 – 18, solve each logarithmic equation.

13. \( \log_3(5x - 3) = 3 \)    14. \( \log_4(8x + 10) = 3 \)

13. ________

14. ________

15. \( \log_{.81} 2 \)    16. \( \log (2x + 3) = 1 \)

15. ________

16. ________

17. \( \ln e^{2x} = 8 \)    18. \( \log_3(x^2 + 1) = 2 \)

17. ________

18. ________

19. Alaska, 1964  According to the United States Geological Survey, an earthquake on March 28, 1964 in Prince William Sound, Alaska resulted in a seismographic reading of 1,584,893 millimeters 100 kilometers from its epicenter. What was the magnitude of this earthquake? This earthquake was the second largest ever recorded, with the largest being the Great Chilean Earthquake of 1960, whose magnitude was 9.5 on the Richter scale.
Five-Minute Warm-Up 8.4
Properties of Logarithms

1. Write 1.13985 as a decimal (a) rounded to 3 decimals places (b) truncated to 3 decimal places.  
   1a. __________  
   1b. __________

In Problems 2 and 3, write each expression with a rational exponent.

2. \( \sqrt{x} \)  
   2. __________

3. \( \sqrt[3]{a^2} \)  
   3. __________

In Problems 4 and 5, write in exponential form using prime numbers in the base.

4. \( \frac{1}{16} \)  
   4. __________

5. \( \frac{27}{8} \)  
   5. __________

In Problems 6 and 7, simplify each expression.

6. \( x^0, x \neq 0 \)  
   6. __________

7. \( 6a^0, a \neq 0 \)  
   7. __________

8. Find the inverse function of \( y = \log_3 x \). Write the equation in exponential form.  
   8. __________
Objective 1: Understand the Properties of Logarithms

1. There are four useful properties that can be quickly derived from the definition of a logarithm. While it is not essential, it will make your work with logarithms faster and easier if you memorize them.

For any real number \( a \), for which the logarithm is defined:

\[
\log_a 1 = 0 \quad \log_a a = 1 \quad \log_a a^r = r \quad a^{\log_a M} = M
\]

Use these properties to simplify the following. (See textbook Examples 1 and 2)

(a) \( 3^{\log_3 \sqrt{2}} \)  
(b) \( \log_5 1 \)  
(c) \( \ln e \)  
(d) \( \log_e x^7 \)

1a. _________  
1b. _________  
1c. _________  
1d. _________

Objective 2: Write a Logarithmic Expression as a Sum or Difference of Logarithms

2. There are three important rules that you will need as well. These should be easy to remember because they are very similar to rules you already know, Laws of Exponents. For each of these, \( M, N, \) and \( a \) are positive real numbers with \( a \neq 1 \).

(a) The Product Rule: \( \log_a (M \cdot N) = \) __________________________

(b) The Quotient Rule: \( \log_a \left(\frac{M}{N}\right) = \) __________________________

(c) The Power Rule: \( \log_a M^r = \) __________________________

3. Use the Product Rule of Logarithms to write \( \log(9x) \) as the sum of logarithms. (See textbook Example 4)

3. __________

4. Use the Quotient Rule of Logarithms to write \( \ln \left(\frac{2}{x}\right) \) as the difference of logarithms. (See textbook Example 5)

4. __________
**Objective 3: Write a Logarithmic Expression as a Single Logarithm**

5. Now we will reverse the process and write an expanded logarithmic expression as a single logarithm. This will be an important skill in the next section.

Use the Quotient Rule of Logarithms to write \( \log_4 (x + 1) - \log_4 (x^2 - 1) \) as a single logarithm.

(See textbook Example 9)

5. __________

**Objective 4: Evaluate Logarithms Whose Base is Neither 10 Nor e**

6. Calculators only have keys for the common logarithm, \( \log \), and the natural logarithm, ln. When we need to find the logarithmic value of an expression that uses a base other than 10 or \( e \), we use the Change-of-Base Formula. If \( M, a \) and \( b \) are positive real numbers with \( a \neq 1, b \neq 1 \), then

\[
\text{Change-of-Base Formula: } \log_a M = \frac{\log_b M}{\log_b a}
\]

7. Approximate \( \log_2 9 \) using the Change-of-Base Formula. Round your answer to three decimal places.

(See textbook Example 12)

7. __________

8. Write an examples to illustrate: \( \log_2 (x + y) \neq \log_2 x + \log_2 y \)
In Problem 1, use properties of logarithms to find the exact value of each expression. Do not use a calculator.

1a. \( \log_5 5^{-3} \) 

1b. \( \sqrt{2}^{\log_2 \sqrt{2}} \)

1c. \( e^{\ln 10} \)

In Problem 2, suppose that \( \ln 2 = a \) and \( \ln 3 = b \). Use properties of logarithms to write each logarithm in terms of \( a \) and \( b \).

2a. \( \ln 4 \)

2b. \( \ln 18 \)

In Problems 3 – 10, write each expression as a sum and/or difference of logarithms. Express exponents as factors.

3. \( \log_4 \left( \frac{a}{b} \right) \)

4. \( \log_3 (a^3b) \)

5. \( \log_2 (8z) \)

6. \( \log_2 \left( \frac{16}{p} \right) \)

7. \( \log_2 \left( 32\sqrt{x} \right) \)

8. \( \ln \left( \frac{\sqrt[3]{x}}{(x + 2)^2} \right) \)
9. \( \log_6 \sqrt[3]{\frac{x - 2}{x + 1}} \)  
10. \( \log_4 \left[ \frac{x^3 (x - 3)}{\sqrt[3]{x + 1}} \right] \) 

9. \( \quad \) 
10. \( \quad \) 

In Problems 11 – 19, write each expression as a single logarithm.
11. \( \log_4 32 + \log_4 2 \) 
12. \( \log_2 48 - \log_2 3 \) 
13. \( 8 \log_2 z \) 
14. \( 4\log_2 a + 2\log_2 b \) 
15. \( \frac{1}{3} \log_4 z + 2 \log_4 (2z + 1) \) 
16. \( \log_7 x^4 - 2 \log_7 x \) 
17. \( \frac{1}{3} \left[ \ln(x - 1) + \ln(x + 1) \right] \) 
18. \( \log_5 \left[ x^2 + 3x + 2 \right] - \log_5 (x + 2) \) 
19. \( 10 \log_4 \sqrt{x} + 4 \log_4 \sqrt{x} - \log_4 16 \) 

11. \( \quad \) 
12. \( \quad \) 
13. \( \quad \) 
14. \( \quad \) 
15. \( \quad \) 
16. \( \quad \) 
17. \( \quad \) 
18. \( \quad \) 
19. \( \quad \) 

In Problem 20, use the Change-of-Base Formula and a calculator to evaluate each logarithm. Round your answer to three decimal places.
20a. \( \log_7 5 \) 
20b. \( \log_{\sqrt{3}} \sqrt{6} \) 

20a. \( \quad \) 
20b. \( \quad \)
In Problems 1 – 4, solve each equation.

1. \( \frac{2}{3}x - 7 = -1 \)

2. \( x^2 + 5x = 24 \)

3. \( 4n^2 = 2 - 7n \)

4. \( (x - 1)^2 - 5(x - 1) - 6 = 0 \)

5. Find the domain: \( f(x) = \log_2 (-2x + 6) \)
Guided Practice 8.5
Exponential and Logarithmic Equations

Objective 1: Solve Logarithmic Equations Using the Properties of Logarithms

1. We use the following property where \( M, N, \) and \( a \) are positive real numbers with \( a \neq 1 \) to solve logarithmic equations where the log function appears on both sides of the equation.

\[
\text{If } \log_a M = \log_a N, \text{ then } M = N.
\]

\( M \) and \( N \) are called arguments so we say, “if there is equality between two logarithmic expressions which have the same base, set the arguments equal.” Note that each log function has to be completely simplified to a single logarithm. Also, be careful to check for extraneous solutions.

Use this property to solve the following equations. (See textbook Examples 1 and 2)

(a) \( \log_2 x = \log_2 (3x - 5) \)

(b) \( \log_4 (x + 3) - \log_4 x = \log_4 10 \)

(c) \( \frac{1}{2} \ln x = 3 \ln 2 \)

1a. __________

1b. __________

1c. __________

Objective 2: Solve Exponential Equations

2. In Section 8.2, we were able to solve exponential equations of the form \( a^u = a^v \) by using the Property for Solving Exponential Equations. This states that if two exponential functions have the same base and the exponential functions are equal, it must be true that the exponents are equal.

Now we will encounter exponential equations which cannot be written on a common base. For this circumstance, we use the definition of a logarithm to convert from exponential form to logarithmic form. Use this approach to solve each of the following. Give both the exact and approximate solution. (See textbook Examples 3 and 4)

(a) \( 3^x = 12 \)

(b) \( \frac{1}{2} e^{3x} = 9 \)

2a. __________

2b. __________
Objective 3: Solve Equations Involving Exponential Models

3. Radioactive Decay The half-life of carbon-10 is 19.255 seconds. Suppose that a researcher possesses a 200-gram sample of carbon-10. The amount $A$ (in grams) of carbon-10 after $t$ seconds is given by

$$A(t) = 100 \cdot \left(\frac{1}{2}\right)^{t/19.255}$$

(See textbook Example 5)

(a) Write a model to find when there will be 90 grams of carbon-10 left in the sample. 3a. __________

(b) Use logarithms to solve the equation from part (a). 3b. __________

4. The population of a small town is growing at a rate of 3% per year. The population of the town can be calculated by the exponential function $P(t) = 2500e^{0.03t}$ where $t$ is the number of years after 1950. (See textbook Examples 5 and 6)

(a) What was the population in 1965? 4a. __________

(b) Write a model to find when the population reached 7500 people. 4b. __________

(e) Use logarithms to solve the equation from (b) and find when the population reached 7500 people. 4c. __________

(d) How long will it take for the population to double? (That is, for the town to have a population of 5000 people.) 4d. __________
### Do the Math Exercises 8.5

**Exponential and Logarithmic Equations**

In Problems 1 – 18, solve each equation. Express irrational solutions in exact form and as a decimal rounded to 3 decimal places.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Equation</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \log_5 x = \log_5 13 )</td>
<td>1. ( 13 ) ( \approx 13.000 )</td>
</tr>
<tr>
<td>2</td>
<td>( \frac{1}{2} \log_2 x = 2 \log_2 2 )</td>
<td>2. ( 4 ) ( \approx 4.000 )</td>
</tr>
<tr>
<td>3</td>
<td>( \log_2 (x - 7) + \log_2 x = 3 )</td>
<td>3. ( 7 ) ( \approx 7.000 )</td>
</tr>
<tr>
<td>4</td>
<td>( \log_3 (x + 5) - \log_3 x = 2 )</td>
<td>4. ( 8 ) ( \approx 8.000 )</td>
</tr>
<tr>
<td>5</td>
<td>( \log_5 (x + 3) + \log_5 (x - 4) = \log_5 8 )</td>
<td>5. ( 5 ) ( \approx 5.000 )</td>
</tr>
<tr>
<td>6</td>
<td>( 3^x = 8 )</td>
<td>6. ( \approx 1.893 )</td>
</tr>
<tr>
<td>7</td>
<td>( 4^x = 20 )</td>
<td>7. ( \approx 2.773 )</td>
</tr>
<tr>
<td>8</td>
<td>( e^x = 3 )</td>
<td>8. ( \approx 1.099 )</td>
</tr>
<tr>
<td>9</td>
<td>( 10^x = 0.2 )</td>
<td>9. ( \approx -0.7 )</td>
</tr>
<tr>
<td>10</td>
<td>( 2^{2x} = 5 )</td>
<td>10. ( \approx 0.7 )</td>
</tr>
<tr>
<td>11</td>
<td>( 3 \cdot 4^x = 15 )</td>
<td>11. ( \approx 1.5 )</td>
</tr>
<tr>
<td>12</td>
<td>( \log_6 x + \log_6 (x + 5) = 2 )</td>
<td>12. ( \approx 1.2 )</td>
</tr>
</tbody>
</table>
13. \(3 \log_2 x = \log_2 8\)  
14. \(5 \log_4 x = \log_4 32\)  
13. 
14. 

15. \(-4e^x = -16\)  
16. \(9^x = 27^{x-4}\)  
15. 
16. 

17. \(\log_7 x^2 = \log_7 8\)  
18. \(\log_3 (x - 5) + \log_3 (x + 1) = \log_3 7\)  
17. 
18. 

19. **A Population Model** According to the *United States Census Bureau*, the population of the world in 2012 was 7018 million people. In addition, the population of the world was growing at a rate of 1.26% per year. Assuming that this growth rate continues, the model 

\[
P(t) = 7018(1.0126)^{t-2012}
\]

represents the population \(P\) (in millions of people) in year \(t\). According to this model, when will the population of the world be 11.58 billion people?  

19. 

20. **Depreciation** Based on data obtained from the *Kelley Blue Book*, the value \(V\) of a Chevy Malibu that is \(t\) years old can be modeled by 

\[
V(t) = 25,258(0.84)^t
\]

According to the model, when will the car be worth $15,000?  

20. 

---

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In Problems 1 – 3, simplify each expression.

1. \( \sqrt{40} \)  
2. \( \sqrt{108} \)  
3. \( \sqrt{32p^4} \)

4. Simplify the expression: \( \sqrt{(2x - 5)^2} \)

In Problems 5 and 6 simplify each expression.

5. \(-2\sqrt{25}\)  
6. \(\sqrt{(-5 - 2)^2 + (16 - (-8))^2}\)

7. Find the area of a triangle whose base has length of \( \sqrt{18} \) cm and whose height has length of \( \sqrt{8} \) cm.

8. Find the length of the hypotenuse of a right triangle whose legs are 6 and \( 6\sqrt{3} \) cm.
Objective 1: Use the Distance Formula

1. We can find the distance between two points in the Cartesian plane using the Pythagorean Theorem. This can be accomplished by plotting the points, drawing a right triangle, and applying $a^2 + b^2 = c^2$ where $c$ is the length of the hypotenuse or, in this case, the distance between the points. If you get stuck or forget the distance formula, you can always use this approach.

We use the Distance Formula to find the length of a line segment quickly and easily. If the two points in the Cartesian plane are $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$, the distance between $P_1$ and $P_2$, denoted $d(P_1, P_2)$, is

2. Be sure to use the Rule for Order of Operations when finding the distance between two points. If we want to find the distance between $(a, b)$ and $(c, d)$, the steps are:

(a) subtract _______________; (b) subtract _______________; (c) square the value from part ________;

(d) square the value from part _______; (e) _______________; (f) _______________; (g) ____________.

3. Find the distance between $(9, 3)$ and $(1, -1)$. Find both the exact value and the approximate distance to two decimal places.  
(See textbook Example 1)
Guided Practice 9.1

**Objective 2: Use the Midpoint Formula**

4. A midpoint is a point (in this case an ordered pair) which divides a line segment into _________________.

5. To find the coordinates of the midpoint, we use the Midpoint Formula. This states that if a line segment has endpoints at \( P_1 = (x_1, y_1) \) and \( P_2 = (x_2, y_2) \), the midpoint, \( M \), is an ordered pair such that

\[
M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right).
\]

You can see that the Midpoint Formula averages the \( x \) values to find a coordinate in the middle and averages the \( y \) values to find a coordinate in the middle. You can verify that the midpoint divides the segment into two segments of equal length by finding the distance from \( P_1 \) to \( M \) and the finding the distance from \( M \) to \( P_2 \). These distances will be equal. This step is not necessary, but it is a good check.

6. Find the midpoint of the line segment joining \( P_1 = (-3, 2) \) and \( P_2 = (-5, -8) \). *(See textbook Example 4)*

6. ___________
Do the Math Exercises 9.1
Distance and Midpoint Formulas

In Problems 1 – 6, find the distance $d(P_1, P_2)$ between points $P_1$ and $P_2$.
1. $P_1 = (1, 3); P_2 = (4, 7)$
2. $P_1 = (-10, -3); P_2 = (14, 4)$
3. $P_1 = (-1, 2); P_2 = (-1, 0)$
4. $P_1 = (5, 0); P_2 = (-1, -4)$
5. $P_1 = (\sqrt{6}, -2\sqrt{2}); P_2 = (3\sqrt{6}, 10\sqrt{2})$
6. $P_1 = (-1.7, 1.3); P_2 = (0.3, 2.6)$

In Problems 7 – 12, find the midpoint of the line segment formed by joining points $P_1$ and $P_2$.
7. $P_1 = (1, 3); P_2 = (5, 7)$
8. $P_1 = (-10, -3); P_2 = (14, 7)$
9. $P_1 = (-1, 2); P_2 = (3, 9)$
10. $P_1 = (5, 0); P_2 = (-1, -4)$
11. $P_1 = (\sqrt{6}, -2\sqrt{2}); P_2 = (3\sqrt{6}, 10\sqrt{2})$
12. $P_1 = (-1.7, 1.3); P_2 = (0.3, 2.6)$
13. Consider the three points $A = (-2, 3)$, $B = (2, 0)$, and $C = (5, 6)$.

(a) Plot each point in the Cartesian plane and form the triangle $ABC$.

(b) Find the length of each side of the triangle.

14. Find all points having a $y$-coordinate of $-3$ whose distance from the point $(-4, 2)$ is $13$. 

13b. _________
In Problems 1 and 2, complete the square. Determine the number that must be added to the expression to make it a perfect square trinomial. Then factor the expression.

1. \( x^2 - 14x \)  
2. \( y^2 - 5y \)

1. __________  
2. __________

In Problems 3 and 4, factor completely.

3. \( y^2 + 16y + 64 \)  
4. \( 2x^2 - 12x + 18 \)

3. __________  
4. __________

5. Find (a) the area and (b) the circumference of a circle whose diameter is 15 inches. Give both the exact answer and then the answer rounded to 2 decimal places.

5a. __________  
   __________  
5b. __________  
   __________
Guided Practice 9.2
Circles

Objective 1: Write the Standard Form of the Equation of a Circle
1. A circle is the set of all points in the Cartesian plane that are a fixed distance $r$ from a fixed point $(h, k)$.
   (a) The point $(h, k)$ is called the ______________.
   (b) The fixed distance $r$ is called the ______________.
   (c) We also know that if $d$ is the length of the diameter of the circle, then ______________.

2. The standard form of an equation of a circle with radius $r$ and center $(h, k)$ is ______________.

3. Write the standard form of the equation of the circle with radius 5 and center $(13, -3)$.
   (See textbook Example 1)
   3. _________________________________________________________________________

Objective 2: Graph a Circle
4. Graph the equation: $(x - 2)^2 + (x + 3)^2 = 4$ (See textbook Example 2)
   (a) Identify the center: __________ (b) Length of radius: __________ (c) Graph:
Objective 3: Find the Center and Radius of a Circle Given an Equation in General Form

5. General form expands (multiplies) the binomials, regroups like terms, and has all the terms on one side of the equation and zero on the other side of the equation. What is the general form of the equation of a circle?

6. Graph the equation: \( x^2 + y^2 + 6x - 2y + 1 = 0 \) (See textbook Example 3)
   (a) Complete the square for the \( x \) terms and complete the square for the \( y \) terms:
   \[ x^2 + 6x + \underline{\quad} + y^2 - 2y + \underline{\quad} = -1 + \underline{\quad} + \underline{\quad} \]
   (b) Factor:
   \[ (x + \underline{\quad})^2 + (y - \underline{\quad})^2 = \underline{\quad} \]
   (c) Identify the center: \( \underline{\quad} \) and the length of radius: \( \underline{\quad} \)
   (d) Graph:

7. Are circles functions? Why or why not?

8. Is \( 3x^2 - 12x + 3y^2 - 15 = 0 \) the equation of a circle? Why or why not? If so, what is the center and radius? \( \underline{\quad} \)
In Problems 1 – 4, write the standard form of the equation of each circle whose radius is \( r \) and center is \((h, k)\). Graph each circle.

1. \( r = 5; \ (h, \ k) = (0, \ 0) \)
2. \( r = 2; \ (h, \ k) = (1, \ 0) \)
3. \( r = 4; \ (h, \ k) = (–4, \ 4) \)
4. \( r = \sqrt{7}; \ (h, \ k) = (5, \ 2) \)

In Problems 5 – 8, identify the center \((h, k)\) and radius \( r \) of each circle. Graph each circle.

5. \( x^2 + y^2 = 25 \)
6. \( (x - 5)^2 + (y + 2)^2 = 49 \)
7. \( (x - 6)^2 + y^2 = 36 \)
8. \( (x - 2)^2 + (y + 2)^2 = \frac{1}{4} \)
In Problems 9 – 11, find the center \((h, k)\) and radius \(r\) of each circle.

9. \[x^2 + y^2 + 2x - 8y + 8 = 0\]  
10. \[x^2 + y^2 + 4x - 12y + 36 = 0\]

11. \[2x^2 + 2y^2 - 28x + 20y + 20 = 0\]

In Problems 12 – 14, find the standard form of the equation of each circle.

12. Center at \((0, 3)\) and containing the point \((3, 7)\).

13. Center at \((2, -3)\) and tangent to the \(x\)-axis.

14. With endpoints of a diameter at \((-5, -3)\) and \((7, 2)\).

15. Find the area and circumference of the circle \((x - 1)^2 + (y - 4)^2 = 49\).
In Problems 1 – 3, use the function \( f(x) = 2(x - 2)^2 + 1 \).

1. Identify the vertex. 
2. Does the parabola open up or down? 
3. Name the axis of symmetry.

In Problems 4 and 5, complete the square. Determine the number that must be added to the expression to make it a perfect square trinomial. Then factor the expression.

4. \( x^2 + 10x \) 
5. \( -3x^2 + 12x \)

6. Solve: \( (x - 4)^2 = 16 \)
Guided Practice 9.3
Parabolas

1. A parabola is the set of all points $P$ in the plane that are the same distance from a fixed point $F$ as they are from a fixed line $D$. In other words, a parabola is the set points $P$ such that $d(F, P) = d(P, D)$.

   (a) The point $F$ is called the _____________ of the parabola.
   
   (b) The line $D$ is its _____________.
   
   (c) The turning point of the parabola is its _____________.
   
   (d) The line through the point $F$ and perpendicular to the line $D$ is called the _____________.

**Objective 1: Graph Parabolas Whose Vertex Is the Origin**

2. In this course, the axis of symmetry is parallel to either the $x$-axis or $y$-axis. This means that the parabola opens up, down, left, or right. In the equation of the parabola, the coefficient of the linear variable ($x^1$ or $y^1$) will determine the direction that the parabola opens. If $k$ is a real number (in the text $k = 4a$ as this coefficient can be used to determine the breadth of the parabola):

   (a) $y^2 = kx$ opens either

      (b) If $k > 0$, the parabola opens

      (c) If $k < 0$, the parabola opens

      2a. __________________________
      2b. __________________________
      2c. __________________________

      Notice that in (b), the positive $x$-axis has an arrow which points to the right and the parabola opens right.

   (d) $x^2 = ky$ opens either

      (e) If $k > 0$, the parabola opens

      (f) If $k < 0$, the parabola opens

      2d. __________________________
      2e. __________________________
      2f. __________________________

      Notice that in (e), the positive $y$-axis has an arrow which points up and the parabola opens up.

3. Determine which direction each parabola opens. (See textbook Examples 1 and 2)

   (a) $x^2 = -4y$  
   
   (b) $y^2 = 4x$  

   3a. __________
   
   3b. __________
Guided Practice 9.3

**Objective 2: Find the Equation of a Parabola**

4. Find the equation of a parabola with vertex at \((0, 0)\) if its axis of symmetry is the \(x\)-axis and its graph contains the point \((-2, -1)\). *(See textbook Examples 3 and 4)*

(a) Which direction does the parabola open?  \(4a. \underline{\text{_________}}\)

(b) Review the equations on page 702 of your text. Which matches the given information? \(4b. \underline{\text{_________}}\)

(c) Substitute the given information to find the equation for the parabola. \(4c. \underline{\text{_________}}\)

**Objective 3: Graph a Parabola Whose Vertex Is Not the Origin**

5. Graph the parabola \(x^2 - 8x + 4y + 20 = 0\). *(See textbook Example 5)*

(a) Isolate the terms involving the second-degree variable: \(5a. \underline{\text{_________}}\)

(b) Complete the square: \(5b. \underline{\text{_________}}\)

(c) Simplify: \(5c. \underline{\text{_________}}\)

(d) Factor: \(5d. \underline{\text{_________}}\)

(e) Which direction does the parabola open? \(5e. \underline{\text{_________}}\)

(f) Identify the vertex. \(5f. \underline{\text{_________}}\)

(g) Find two more points on the parabola and then graph.
In Problems 1 – 5, find the equation of the parabola described.

1. vertex at (0, 0); focus at (0, 5)  
2. vertex at (0, 0); focus at (–8, 0)  
3. vertex at (0, 0); contains the point (2, 2); axis of symmetry the x-axis  
4. vertex at (0, 0); directrix $x = –4$  
5. focus at (0, –2); directrix $y = 2$

In Problems 6 – 13, graph the parabola. Find (a) the vertex, (b) the focus, and (c) the directrix.

6. $x^2 = 28y$  
7. $y^2 = 10x$

6a.  
6b.  
6c.  
7a.  
7b.  
7c.
8. \( x^2 = -16y \)

9. \((x + 4)^2 = -4(y - 1)\)

8a. _________

8b. _________

8c. _________

9a. _________

9b. _________

9c. _________

10. \((y - 2)^2 = 12(x + 5)\)

11. \(x^2 + 2x - 8y + 25 = 0\)

10a. _________

10b. _________

10c. _________

11a. _________

11b. _________

11c. _________

12. \(y^2 - 8y + 16x - 16 = 0\)

13. \(x^2 - 4x + 10y + 4 = 0\)

12a. _________

12b. _________

12c. _________

13a. _________

13b. _________

13c. _________

14. **Suspension Bridge**  
The cables of a suspension bridge are in the shape of a parabola. The towers supporting the cable are 400 feet apart and 80 feet high. If the cables touch the road surface midway between the towers, what is the height of the cable at a point 100 feet from the center of the bridge?  

14. _________
In Problems 1 – 3, complete the square. Determine the number that must be added to the expression to make it a perfect square trinomial. Then factor the expression.

1. \(y^2 - 9y\)  
2. \(x^2 + 12x\)  
3. \(4y^2 - 2y\)

In Problems 4 – 7, use the function \(y = 9x^2 + 54x - 87\).

4. Write the equation in standard form by completing the square.  
5. Identify the vertex.  
6. Does the parabola open up or down?  
7. Name the axis of symmetry.
Guided Practice 9.4
Ellipses

**Objective 1: Graph an Ellipse Whose Center Is the Origin**

1. An ellipse is the set of all points in the plane such that the sum of the distances from two fixed points is a constant.
   
   (a) The fixed points are called the ________________.
   
   (b) The long axis contains the fixed points and is called the ________________.
   
   (c) The other axis is perpendicular to long axis and is called the ________________.
   
   (d) The point where the two axes intersect is the ________________ of the ellipse.
   
   (e) The major axis contains turning points of the ellipse called ________________.

2. The standard form of an ellipse is either \[ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \] or \[ \frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \]. (See textbook Examples 1 and 2)
   
   (a) In either of these equations, the larger of the two denominators is ________.
   
   (b) The distance from the center to the vertex is _______ units of length.
   
   (c) Therefore, the length of the major axis is _______.
   
   (d) The length of the minor axis is _______.
   
   (e) The term with larger denominator tell us which axis is the ____________ axis.
   
   (f) The foci are \( c \) units from the center, on the major axis, where \( c^2 = \) ________________.
   
   (g) For these two equations, the center of the ellipse is ________________.

3. Find the intercepts to graph each ellipse: (See textbook Examples 1 and 2)
   
   (a) \( \frac{x^2}{9} + \frac{y^2}{25} = 1 \)  
   
   (b) \( 4x^2 + 16y^2 = 64 \)
Objective 2: Find the Equation of an Ellipse Whose Center Is the Origin

4. Find the equation, in standard form, of the ellipse whose center is at (0, 0), one focus is at (0, 5) and one vertex is at (0, 13). (See textbook Example 3)

Objective 3: Graph an Ellipse Whose Center Is Not the Origin

In Problems 5 – 7, (a) write the equation in the form \( \frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \) or
\( \frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1 \), then identify (b) the center, (c) the vertices and (d) the foci. (See textbook Example 4)

5. \( \frac{(x+5)^2}{9} + \frac{(y-2)^2}{25} = 1 \)

6. \( 4(x-1)^2 + 9(y-3)^2 = 36 \)

7. \( 16x^2 + 9y^2 - 128x + 54y - 239 = 0 \)
In Problems 1 – 4, graph the ellipse. Find (a) the vertices and (b) the foci of each ellipse.

1. \( \frac{x^2}{25} + \frac{y^2}{4} = 1 \)
   1a. __________
   1b. __________

2. \( \frac{x^2}{16} + \frac{y^2}{36} = 1 \)
   2a. __________
   2b. __________

3. \( \frac{x^2}{64} + y^2 = 1 \)
   3a. __________
   3b. __________

4. \( 9x^2 + y^2 = 81 \)
   4a. __________
   4b. __________

In Problems 5 – 8, find an equation for each ellipse.

5. center at (0, 0); focus at (2, 0); vertex at (5, 0)
   5. __________

6. center at (0, 0); focus at (0, –1); vertex at (0, 5)
   6. __________

7. foci at (0, ±2); vertices at (0, ±7)
   7. __________

8. foci at (±6, 0); length of the major axis is 20
   8. __________
Do the Math Exercises 9.4

In Problems 9 and 10, graph each ellipse.

9. \( \frac{(x + 8)^2}{81} + (y - 3)^2 = 1 \)

10. \( 9(x - 3)^2 + (y - 4)^2 = 81 \)

11. Consider the graph of the ellipse: \( 25x^2 + 150x + 9y^2 - 72y + 144 = 0 \).
   (a) Write the equation of the ellipse in standard form.

   (b) Find the center.

   (c) Find the vertices.

   (d) Find the foci.

12. London Bridge  An arch in the shape of the upper half of an ellipse is used to support London Bridge. The main span is 45.6 meters wide. Suppose that the center of the arch is 15 meters above the center of the river.
   (a) Write the equation for the ellipse in which the x-axis coincides with the water and the y-axis passes though the center of the arch.

   (b) Can a rectangular barge that is 20 meters wide and sits 12 meters above the surface of the water fit through the opening of the bridge?
Five-Minute Warm-Up 9.5
Hyperbolas

In Problems 1 and 2, solve the equation.

1. \( y^2 = 16 \)
2. \( (y + 3)^2 = 4 \)

1. ___________
2. ___________

In Problems 3 – 4, complete the square. Determine the number that must be added to the expression to make it a perfect square trinomial. Then factor the expression.

3. \( x^2 + 8x \)
4. \( y^2 - \frac{4}{3}y \)

3. ___________
4. ___________

5. Graph \( y = \pm \frac{2}{3}x \). That is, graph \( y = \frac{2}{3}x \) and \( y = -\frac{2}{3}x \) on the same coordinate plane.

5. ___________
Objective 1: Graph a Hyperbola Whose Center Is the Origin

1. A hyperbola is the collection all points in the plane the difference of whose distances from two fixed points is a positive constant.
   (a) The two fixed points are called _______________.
   (b) The line containing these points, as well as the center and the vertices, is called the _______________.
   (c) The midpoints of the line segment joining the foci is the _______________.
   (d) The line through the center, perpendicular to the transverse axis, is called the _______________.
   (e) The branches of the hyperbola have turning points called _______________.

2. The standard form of a hyperbola is either \( \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \) or \( \frac{y^2}{a^2} - \frac{x^2}{b^2} = 1 \).
   (See textbook Examples 1 and 2)
   (a) In either of these equations, the first denominator is ________, whether it is larger or smaller.
   (b) The distance from the center to the vertex is _______ units of length.
   (c) Therefore, the length of the transverse axis is _______.
   (d) The length of the conjugate axis is ________.
   (e) If the first, or positive, term is \( \frac{x^2}{a^2} \), the hyperbola opens _____________.
   (f) If the first, or positive, term is \( \frac{y^2}{a^2} \), the hyperbola opens _____________.
   (g) The foci are \( c \) units from the center, on the transverse axis, where \( c^2 = \) _______________.
   (h) For these two equations, the center of the hyperbola is _______________.

3. Graph the hyperbola \( \frac{x^2}{16} - \frac{y^2}{9} = 1 \). (See textbook Example 1)
   (a) Find the center. \( 3a. \) __________________________
   (b) The equation is of the form \( \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \), where \( c^2 = a^2 + b^2 \). Find the values of \( a \) and \( b \). \( 3b. \) __________________________
Guided Practice 9.5

(e) Find the value of $c$.  
3c. __________________________

(d) Find the vertices and foci.  
3d. __________________________

(e) Let $x=\pm c$ to locate points above and below the foci.  
3e. __________________________

(f) Plot the vertices, foci, and the four points found in (e) to graph the hyperbola.

Objective 3: Find the Asymptotes of a Hyperbola Whose Center Is the Origin

4. In your own words, what is an asymptote?

5. The equations of the two asymptotes of the hyperbola can be found using the values of $a$ and $b$ as determined from the standard form. The equations for the asymptotes changes when the direction the hyperbola opening changes. Determine the equations of the asymptotes for each hyperbola:

<table>
<thead>
<tr>
<th>Hyperbola</th>
<th>Equation of Asymptotes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$</td>
<td>(a) $y = \pm$</td>
</tr>
<tr>
<td>$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$</td>
<td>(b) $y = \pm$</td>
</tr>
</tbody>
</table>
Do the Math Exercises 9.5
Hyperbolas

In Problems 1 – 4, graph each hyperbola. Find (a) the vertices and (b) the foci.

1. \( \frac{x^2}{9} - \frac{y^2}{16} = 1 \)  
   1a. __________  
   1b. __________  

2. \( \frac{y^2}{81} - \frac{x^2}{9} = 1 \)  
   2a. __________  
   2b. __________  

3. \( x^2 - 9y^2 = 36 \)  
   3a. __________  
   3b. __________  

4. \( 4y^2 - 9x^2 = 36 \)  
   4a. __________  
   4b. __________  

In Problems 5 – 8, find the equation for the hyperbola described.

5. center at (0, 0); focus at (–4, 0); and vertex (–1, 0)  
   5. __________  

6. vertices at (0, 6) and (0, –6); focus at (0, 8)  
   6. __________  

7. vertices at (0, –4) and (0, 4); asymptote the line \( y = 2x \)  
   7. __________  

8. foci at (–9, 0) and (9, 0); asymptote the line \( y = –3x \)  
   8. __________
In Problems 9 – 14, identify the graph of each equation as a circle, parabola, ellipse, or hyperbola.

9. \( x^2 + 4y = 4 \) \hspace{1cm} 10. \( 3y^2 - x^2 = 9 \)

11. \( y^2 - 4y - 4x^2 + 8x = 4 \) \hspace{1cm} 12. \( 4x^2 + 8x + 4y^2 - 4y - 12 = 0 \)

13. \( 4y^2 + 3x - 16y + 19 = 0 \) \hspace{1cm} 14. \( 9x^2 + 4y^2 - 18x + 8y - 23 = 0 \)

15. Consider the hyperbola \( \frac{y^2}{25} - \frac{x^2}{4} = 1 \).
   
   (a) Find the vertices.

16. Consider the hyperbola \( \frac{x^2}{16} - \frac{y^2}{36} = 1 \). Find the equation of the asymptotes.
Five-Minute Warm-Up 9.6
Systems of Nonlinear Equations

In Problems 1 and 2, solve the system using substitution.

1. \[ \begin{align*}
    y &= \frac{3}{4}x \\
    x - 4y &= -4
\end{align*} \]

2. \[ \begin{align*}
    -x + 3y &= 4 \\
    2x - 6y &= -8
\end{align*} \]

In Problems 3 and 4, solve the system using elimination.

3. \[ \begin{align*}
    6x - 5y &= 1 \\
    8x - 2y &= -22
\end{align*} \]

4. \[ \begin{align*}
    6x - 4y &= -5 \\
    -12x + 8y &= 2
\end{align*} \]
Objective 1: Solve a System of Nonlinear Equations Using Substitution

1. Solve the following system of equations using substitution: \[
\begin{align*}
  y &= x - 4 \quad (1) \\
  x^2 + y^2 &= 16 \quad (2)
\end{align*}
\]

(See textbook Example 1)

Step 1: Graph each equation in the system.

Graph of equation (1) is what? (a) ____________________

Graph of equation (2) is what? (b) ____________________

Graph each equation in the system: (c) ____________________

Based on (c), how many solutions will this system have? (d) ____________________

Step 2: Solve equation (1) for \( y \).

This is already done.

Step 3: Substitute the expression for \( y \) into equation (2).

Equation (2): (e) ____________________

Substitute the expression for \( y \) into equation (2): (f) ____________________

Simplify: (g) ____________________

Step 4: Solve for \( x \).

Factor: (h) ____________________

Zero-Product Property: \( i x = _____ \) or \( x = _____ \)

Step 5: Use your values from (i) and equation (1) to determine the ordered pairs that satisfy the system.

Equation (1): (j) ____________________

Substitute your first value of \( x \): (k) ____________________

Solve for \( y \): (l) ____________________

Equation (1): (m) ____________________

Substitute your second value of \( x \): (n) ____________________

Solve for \( y \): (o) ____________________

Step 6: Check

State the solution set: (p) ____________________
Objective 2: Solve a System of Nonlinear Equations Using Elimination

2. Solve the following system of equations by elimination: \[
\begin{align*}
\begin{cases}
x^2 + y^2 &= 4 \\ x^2 + 4y^2 &= 16
\end{cases}
\end{align*}
\] (See textbook Example 3)

Step 1: Graph each equation in the system.

Graph of equation (1) is what? (a) ____________________

Graph of equation (2) is what? (b) ____________________

Graph each equation in the system: (c) ____________________

Based on (c), how many solutions will this system have? (d) ____________________

Step 2: We want a pair of coefficients to be additive inverses so that when we add equation (1) and equation (2), one of the variables will be eliminated.

Let’s eliminate \( x^2 \). What will equation (1) need to be multiplied by in order to eliminate \( x^2 \)? (e) ____________________

Multiply equation (1) by the value determined in (e) and write the system: (f) \[
\begin{align*}
\begin{cases}
x^2 + y^2 &= 4 \\ 4x^2 + 4y^2 &= 16
\end{cases}
\end{align*}
\]

Step 3: Add equations (1) and (2) to eliminate \( x^2 \). Solve the resulting equation for \( y \).

Add: (g) ____________________

Divide by 3: (h) ____________________

Use the Square Root Property: (i) \( y = \) ____________________

Step 4: Solve for \( x \) using either equation (1) or equation (2). We will use your values from (i) and equation (1) to determine the ordered pairs that satisfy the system.

Equation (1): (j) ____________________

Substitute your first value of \( y \): (k) ____________________

Solve for \( x \): (l) ____________________

Equation (1): (m) ____________________

Substitute your second value of \( y \): (n) ____________________

Solve for \( x \): (o) ____________________

Step 5: Check

State the solution set: (p) ____________________
Do the Math Exercises 9.6
Systems of Nonlinear Equations

In Problems 1 – 4, solve the system of nonlinear equations by substitution.

1. \[
\begin{align*}
y &= x^3 + 2 \\
y &= x + 2
\end{align*}
\]

2. \[
\begin{align*}
y &= \sqrt{100 - x^2} \\
x + y &= 14
\end{align*}
\]

3. \[
\begin{align*}
x^2 + y^2 &= 16 \\
y &= x^2 - 4
\end{align*}
\]

4. \[
\begin{align*}
xy &= 1 \\
x^2 - y &= 0
\end{align*}
\]

In Problems 5 – 8, solve the system of nonlinear equations by elimination.

5. \[
\begin{align*}
x^2 + y^2 &= 8 \\
x^2 + y^2 + 4y &= 0
\end{align*}
\]

6. \[
\begin{align*}
4x^2 + 16y^2 &= 16 \\
2x^2 - 2y^2 &= 8
\end{align*}
\]

7. \[
\begin{align*}
2x^2 + y^2 &= 18 \\
x^2 - y^2 &= 9
\end{align*}
\]

8. \[
\begin{align*}
2x^2 - 5x + y &= 12 \\
14x - 2y &= -16
\end{align*}
\]
In Problems 9 – 12, solve the system of nonlinear equations by any method.

9. \[
\begin{align*}
y &= x^2 + 4x + 5 \\
x - y &= 9
\end{align*}
\]

10. \[
\begin{align*}
x^2 + y^2 &= 25 \\
x^2 - y^2 &= 25
\end{align*}
\]

11. \[
\begin{align*}
9x^2 + 4y^2 &= 36 \\
x^2 + (y - 7)^2 &= 4
\end{align*}
\]

12. \[
\begin{align*}
x^2 + y^2 &= 65 \\
y &= -x^2 + 9
\end{align*}
\]

13. **Fun with Numbers** The sum of two numbers is 8. The sum of their squares is 160. Find the numbers.

14. **Perimeter and Area of a Rectangle** The perimeter of a rectangle is 64 meters. The area of the rectangle is 240 square feet. Find the dimensions of the rectangle.


Five-Minute Warm-Up 10.1
Sequences

1. If \( f(x) = -2x^2 - 3x \), find the function value.
   (a) \( f(1) \)  
   (b) \( f(-3) \)  
   1a. __________  
   1b. __________

2. Evaluate the expression \((-1)^n (2n - 3)\) for each of the following.
   (a) \( n = 1 \)  
   (b) \( n = 2 \)  
   (c) \( n = 3 \)  
   (d) \( n = 4 \)  
   2a. __________  
   2b. __________  
   2c. __________  
   2d. __________

3. Evaluate the expression \( \left(-\frac{1}{3}\right)^{n+1} \) for each of the following.
   (a) \( n = 1 \)  
   (b) \( n = 2 \)  
   (c) \( n = 3 \)  
   3a. __________  
   3b. __________  
   3c. __________

4. If \( f(x) = \frac{1}{2x} \), find \( f(1) + f(2) + f(3) + f(4) \).  
   4. __________
Guided Practice 10.1
Sequences

1. A sequence is a function whose domain is the set of positive integers.

(a) The numbers in the ordered list are called _______________ of the sequence and we separate each entry on the list from the next entry by a comma.

(b) Sequences can be either infinite or finite. If the list does not end, it is called infinite and we use three dots, called _______________ , to indicate that the pattern continues indefinitely.

(c) By contrast, a _______________ sequence has a countable number of terms.

Objective 1: Write the First Few Terms of a Sequence

2. We use the notation $a_6$ to mean the sixth term of the sequence. The formula for the $n$th term, or general term, of the sequence is denoted

2. __________

3. Write the first five terms of the sequence $\{a_n\} = \left\{ \frac{2^n - 1}{n} \right\}$. (See textbook Example 1)

3. ________________
Guided Practice 10.1

**Objective 2: Find a Formula for the nth Term of a Sequence**

4. Sometimes a sequence is indicated by an observed pattern and it is our job to find the pattern. Try subtracting successive terms to find a constant difference or dividing successive terms to find a constant ratio. Sometimes the pattern is neither of these, but rather something you can recognize such as perfect squares.

Find the formula for the \(n\)th term of the sequence: *(See textbook Example 3)*

(a) 3, 6, 9, 12, … 
(b) \(\frac{1}{6}, \frac{1}{36}, \frac{1}{216}, \frac{1}{1296}, \ldots\)  

4a. __________  
4b. __________

**Objective 3: Use Summation Notation**

5. We use *summation notation* to indicate that the terms of the sequence should be added. The *index* of summation can be any variable, but typically we use \(i\). This tells you where to start the sum and where to end. When there are a finite number of terms to be added, the sum is called a *partial sum*.

Consider the partial sum: \(\sum_{i=1}^{4} \left(i^2 + 3\right)\). We substitute the values \(i = \_\), \(i = \_\), \(i = \_\), \(i = \_\) into the formula \(i^2 + 3\) to get the terms \_\_, \_\_, \_\_, \_\_. The sum is \_\_. *(See textbook Example 4)*

6. Write out the sum and determine its value: \(\sum_{k=0}^{5} (2k - 5)\)  

7. Express the sum using summation notation: 0 + 2 + 4 + 6 + 8 + 10 + 12 *(See textbook Example 5)*  

7. __________
In Problems 1 – 4, write down the first five terms of each sequence.

1. \(\{n - 4\}\)  
2. \(\left\{ \frac{n + 4}{n} \right\}\)  
3. \(\{3^n - 1\}\)  
4. \(\left\{ \frac{n^2}{2} \right\}\)

In Problems 5 – 8, find the \(n\)th term of each sequence suggested by the pattern.

5. 5, 10, 15, 20, …  
6. \(\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, \ldots\)  
7. 0, 7, 26, 63, …  
8. \(1, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \ldots\)

In Problems 9 – 14, determine the value of the sum.

9. \(\sum_{i=1}^{5}(3i + 2)\)  
10. \(\sum_{i=1}^{4} \frac{i^3}{2}\)  
11. \(\sum_{k=1}^{4} 3^k\)  
12. \(\sum_{k=1}^{8} (-1)^k \cdot k\)  
13. \(\sum_{j=1}^{8} 2\)  
14. \(\sum_{j=5}^{10} (k + 4)\)
In Problems 15 – 18, express each sum using summation notation.

15. $1 + 3 + 5 + \ldots + 17$

16. $\frac{1}{2} + \frac{1}{4} + \ldots + \frac{1}{2^{15}}$

17. $\frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \ldots + (-1)^{15+1} \left( \frac{2}{3} \right)^{15}$

18. $3 + 3 \cdot \frac{1}{2} + 3 \cdot \frac{1}{4} + \ldots + 3 \cdot \left( \frac{1}{2} \right)^{11}$

19. The Future Value of Money

Suppose that you place $5,000 into a company 401(k) plan that pays 8% interest compounded monthly. The balance in the account after $n$ months is given by $a_n = 5,000 \left( 1 + \frac{0.08}{12} \right)^n$.

(a) Find the value in the account after 1 month.

(b) Find the value in the account after 1 year.

(c) Find the value in the account after 10 years.
1. Determine the slope of $y = 4x + 5$.

2. If $g(x) = \frac{2}{3}x - 1$, find $g\left(-\frac{6}{5}\right)$.

3. Solve the following systems of linear equations:
   (a) $\begin{cases} 3x - 5y = 22 \\ x - y = 10 \end{cases}$
   (b) $\begin{cases} x - 2y = 9 \\ 2x + y = -2 \end{cases}$

4. Evaluate the expression: $\frac{9}{2} \left( -\frac{4}{3} + \left( -\frac{20}{3} \right) \right)$
Objective 1: Determine Whether a Sequence Is Arithmetic

1. If there is a constant difference between the successive terms, the sequence is called an arithmetic sequence.

   (a) In formulas for arithmetic sequences, we label the common difference _____.

   (b) We label the first term _____.

2. Determine if the sequence 5, 8, 11, 14, … is arithmetic. If it is, determine the first term and the common difference. (See textbook Example 1)

3. Show that the sequence \( \{a_n\} = \{2 + n^2\} \) is not arithmetic by listing the first six terms and calculating the difference between successive terms. (See textbook Example 3)

Objective 2: Find a Formula for the nth Term of an Arithmetic Sequence

4. The \( n \)th term of an arithmetic sequence whose first term is \( a_1 \) and whose common difference is \( d \), is determined by the formula:

   \[ a_n = a_1 + (n - 1)d. \]

   (a) Find a formula for the \( n \)th term of the arithmetic sequence whose first term is \(-3\) and whose common difference is 5. (See textbook Example 4)

   (b) Find the 8th term of this sequence.

5. The 5th term of an arithmetic sequence is 24, and the 11th term is 42. (See textbook Example 5)

   (a) Write and solve a system of linear equations to find the first term and the common difference.

   (b) Give a formula for the \( n \)th term.
Objective 3: Find the Sum of an Arithmetic Sequence

6. Let \( \{a_n\} \) be an arithmetic sequence with first term \( a_1 \) and common difference \( d \). The sum \( S_n \) of the first \( n \) terms of \( \{a_n\} \) is \( S_n = \frac{n}{2}(a_1 + a_n) \).

Find the sum of the first 10 terms of the arithmetic sequence 12, 16, 20, 24, \ldots (See textbook Example 6)

(a) Write a formula for the \( n \)th term.  

(b) Use the formula to find the 10th term of the sequence. 

(c) Find the sum of the first 10 terms of the arithmetic sequence. 

7. Find the sum of the first 10 terms of the arithmetic sequence \( \{10 - 13n\} \). (See textbook Example 7)

(a) Use the formula to find the 1st and 10th term of the sequence. 

(b) Find the sum of the first 10 terms of the arithmetic sequence.
### Do the Math Exercises 10.2

**Arithmetic Sequences**

In Problems 1 – 2, find the common difference and write out the first four terms.

1. \{10n + 1\}
2. \(\frac{1}{4}n + \frac{3}{4}\)

In Problems 3 – 5, find a formula for the nth term of the arithmetic sequence whose first term is \(a\) and common difference \(d\) is given. What is the fifth term?

3. \(a = 8, d = 3\)
4. \(a = 12, d = -3\)
5. \(a = -3; d = \frac{1}{2}\)

In Problems 6 – 8, write a formula for the nth term of each arithmetic sequence. Use the formula to find the 20th term of the sequence.

6. \(-5, -1, 3, 7, \ldots\)
7. \(20, 14, 8, 2, \ldots\)
8. \(10, \frac{19}{2}, 9, \frac{17}{2}, \ldots\)

In Problems 9 – 12, find the formula for the nth term of an arithmetic sequence using the given information.

9. 5th term is 7; 9th term is 19
10. 2nd term is -9; 8th term is 15
11. 6th term is -8; 12th term is -38
12. 5th term is 5; 13th term is 7
13. Find the sum of the first 40 terms of the sequence 1, 8, 15, 22, …

14. Find the sum of the first 75 terms of the sequence –9, –5, –1, 3 …

15. Find the sum of the first 50 terms of the sequence 12, 4, –4, –12, …

16. Find the sum of the first 80 terms of the arithmetic sequence \{2n – 13\}.

17. Find the sum of the first 35 terms of the arithmetic sequence \{–6n + 25\}.

18. Find the sum of the first 28 terms of the arithmetic sequence \(7 - \frac{3}{2}n\).

19. Find \(x\) so that 2\(x\), 3\(x\) + 2, and 5\(x\) + 3 are consecutive terms of an arithmetic sequence.

20. The Theater An auditorium has 40 seats in the first row and 25 rows in all. Each successive row contains 2 additional seats. How many seats are in the auditorium?
1. If \( f(x) = \left(\frac{2}{3}\right)^x \), find each of the following.

   (a) \( f(1) \)  
   (b) \( f(2) \)  
   (c) \( f(3) \)

   1a. ________  
   1b. ________  
   1c. ________

2. If \( g(n) = 3n^2 \), find each of the following.

   (a) \( g(1) \)  
   (b) \( g(2) \)  
   (c) \( g(3) \)

   2a. ________  
   2b. ________  
   2c. ________

3. Simplify each expression.

   (a) \( \frac{24x^5}{15x^2} \)

   3a. ________

   (b) \( (-3r^4)^2 \)

   3b. ________

4. Evaluate the expression:

\[
\frac{\frac{3}{2}}{1 - \frac{1}{4}}
\]

   4. ________
Guided Practice 10.3
Geometric Sequences and Series

Objective 1: Determine Whether a Sequence Is Geometric

1. If there is a constant ratio between the successive terms, the sequence is called a geometric sequence.

   (a) In formulas for geometric sequences, we label the common ratio _____.

   (b) We label the first term _____.

2. Determine if the sequence 36, 18, 9, \( \frac{9}{2} \),... is geometric. If it is, determine the first term and the common ratio.
   See Example 1

   2. ________________

3. Show that the sequence \( \{ b_n \} = \left\{ \frac{2}{5} \right\}^{n-1} \) is geometric by listing the first four terms and calculating the ratio between successive terms. (See textbook Example 3)

   3. ________________

Objective 2: Find a Formula for the nth Term of a Geometric Sequence

4. The \( n \)th term of a geometric sequence whose first term is \( a \) and whose common ratio is \( r \), is determined by the formula:
   \[ a_n = a r^{n-1}; \quad r \neq 0. \]

   (a) Find a formula for the \( n \)th term of the geometric sequence: 2, \( -\frac{2}{3}, \frac{2}{9}, -\frac{2}{27}, \ldots \)
   (See textbook Example 4)

   4a. __________

(b) Find the 8th term of this sequence.

   4b. __________
Objective 3: Find the Sum of a Geometric Sequence

5. Let \( \{a_n\} \) be a geometric sequence with first term \( a_1 \) and common ratio \( r \), where \( r \neq 0, r \neq 1 \).

The sum \( S_n \) of the first \( n \) terms of \( \{a_n\} \) is \( S_n = a_1 \cdot \frac{1 - r^n}{1 - r} \); \( r \neq 0, r \neq 1 \).

(a) Find the first term and the common ratio, \( r \), of the geometric sequence 6, 24, 96, 384, …
(See textbook Example 5)

\[ \text{5a. } \] __________

(b) Find the sum of the first 10 terms of this sequence.

\[ \text{5b. } \] __________

Objective 4: Find the Sum of a Geometric Series

6. An infinite sum of the terms of a geometric sequence is called a geometric series. We can find the sum of the series with the formula:

\[ \sum_{n=1}^{\infty} a_1r^{n-1} = \frac{a_1}{1-r} \quad \text{provided that } -1 < r < 1. \]

(a) Find the first term and the common ratio, \( r \), of the geometric series: \( 6 - \frac{2}{3} + \frac{2}{9} + \ldots \)

(See textbook Example 7)

\[ \text{6a. } \] __________

(b) Find the sum of this series.

\[ \text{6b. } \] __________

Objective 5: Solve Annuity Problems

7. If \( P \) represents the deposit in dollars made at each payment period for an annuity at \( i \) percent interest per payment period, the amount \( A \) of the annuity after \( n \) payment periods is:

\[ A = P \cdot \frac{(1 + i)^n - 1}{i}. \]

Retirement Raymond is planning on retiring in 15 years, so he contributes $1,500 into his IRA every 6 months (semiannually). What will be the value of the IRA when Raymond retires if earns 10% interest compounded semiannually? (See textbook Example 10)

\[ \text{7. } \] __________
Do the Math Exercises 10.3
Geometric Sequences and Series

In Problems 1 – 3, find the common ratio and write out the first four terms of each geometric sequence.

1. \(\{(-2)^n\}\) 
2. \(\left\{\frac{2^n}{3}\right\}\) 
3. \(\left\{\frac{3^n}{2^{n-1}}\right\}\)

In Problems 4 – 7, determine whether the given sequence is arithmetic, geometric, or neither.

4. \(\{8 - 3n\}\) 
5. \(\{n^2 - 2\}\) 
6. 100, 20, 4, \(\frac{4}{5}\), ...
7. 5, –2, 3, –1, 2, ...

In Problems 8 and 9, find a formula for the nth term of the geometric sequence whose first term and common ratio are given. Use the formula to find the 8th term.

8. \(a_1 = 30, r = \frac{1}{3}\) 
9. \(a_1 = 1, r = -4\)

In Problems 10 – 12, find the indicated term of each geometric sequence.

10. 12th term of 1, 3, 9, 27, ...
11. 8th term of 10, –20, 40, –80, ...
12. 10th term of 0.4, 0.04, 0.004, 0.0004, ...
In Problems 13 and 14, find the sum of each geometric series.

13. $3 + 9 + 27 + \ldots + 3^{10}$

14. $\sum_{n=1}^{12} 5 \cdot 2^n$

13. __________

14. __________

In Problems 15 – 17, find the sum of each infinite geometric series.

15. $1 + \frac{1}{3} + \frac{1}{9} + \ldots$

16. $12 - 3 + \frac{3}{4} - \frac{3}{16} + \ldots$

17. $\sum_{n=1}^{\infty} 10 \left( \frac{1}{3} \right)^n$

15. __________

16. __________

17. __________

In Problems 18 and 19, express each repeating decimal as a fraction in lowest terms.

18. $0.\overline{3}$

19. $0.4\overline{5}$

18. __________

19. __________

20. Depreciation of a Car

Suppose that you have just purchased a Chevy Impala for $16,000. Historically, the car depreciates by 10% each year, so that next year the car is worth $16,000(0.9). What will the value of the car be after you have owned it for four years?
In Problems 1 – 4, find the product.

1. \((x + 2)^2\)  
2. \((y - 3)^2\)  
3. \((4x - 5y)^2\)  
4. \(\left(3n + \frac{2}{3}\right)^2\)  

5. Simplify: \(\frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2}{3 \cdot 2 \cdot 4 \cdot 3 \cdot 2}\)
Guided Practice 10.4
The Binomial Theorem

**Objective 1: Compute Factorials**

1. If \( n \geq 0 \) is an integer, the factorion symbol \( n! \) (read “n factorial”) is defined as:
   \[ 0! = 1; \quad 1! = 1; \quad n! = n(n-1)(n-2) \ldots 3 \cdot 2 \cdot 1 \quad \text{for} \quad n \geq 2. \]

   Evaluate each of the following. (See textbook Example 1)

   \[
   \begin{align*}
   (a) \quad \frac{10!}{4!} & \quad \quad \quad (b) \quad \frac{6!}{2!(6-2)!} \\
   \end{align*}
   \]

   1a. __________  
   1b. __________

**Objective 2: Evaluate a Binomial Coefficient**

2. If \( j \) and \( n \) are integers with \( 0 \leq j \leq n \), the symbol \( \binom{n}{j} \) (read “n choose j”) is defined as
   \[ \binom{n}{j} = \frac{n!}{j!(n-j)!}. \]

   Evaluate each of the following. (See textbook Example 2)

   \[
   \begin{align*}
   (a) \quad \binom{5}{3} & \quad \quad \quad (b) \quad \binom{13}{7} \\
   \end{align*}
   \]

   2a. __________  
   2b. __________
Objective 3: Expand a Binomial

3. Multiplying binomials can become unwieldy when the exponents become large. We now introduce techniques for binomial expansion. We can find the binomial coefficients in a binomial expansion using Pascal’s Triangle or the Binomial Theorem, which states that for any positive integer \( n \),

\[
(x + a)^n = \binom{n}{0} x^n + \binom{n}{1} ax^{n-1} + \binom{n}{2} a^2 x^{n-2} + \ldots + \binom{n}{j} a^j x^{n-j} + \ldots + \binom{n}{n} a^n.
\]

Expand \((p + 2)^4\) using the Binomial Theorem. (See textbook Example 3)
Do the Math Exercises 10.4
The Binomial Theorem

In Problems 1 – 4, evaluate each expression.

1. \(5!\) 
2. \(\frac{6!}{2!}\)
3. \(\frac{10!}{2! \cdot 8!}\)
4. \(0!\)

In Problems 5 – 8, evaluate each expression.

5. \(\binom{5}{3}\) 
6. \(\binom{7}{5}\)
7. \(\binom{50}{49}\)
8. \(\binom{1000}{1000}\)

In Problems 9 – 16, expand each expression using the Binomial Theorem.

9. \((x - 1)^4\) 
10. \((x + 5)^5\)
Do the Math Exercises 10.4

11. \((2q + 3)^4\)  
12. \((3w - 4)^4\)  

13. \((y^2 - 3)^4\)  
14. \((3b^2 + 2)^5\)  

15. \((p - 3)^6\)  
16. \((3x^2 + y^3)^4\)  

17. Use the Binomial Theorem to find the numerical value of \((1.001)^4\) correct to five decimal places. [Hint: \((1.001)^5 = \left(1 + 10^{-3}\right)^5\).]  

17. ________
Chapter R Answers

Section R.2

Five-Minute Warm-Up  1. 2. 3. 4. 5. 6. 7. 8. 9. 10.
11. \( x < 0 \) 12 – 17. Answers may vary 12. \{1, 2, 3\…\} 13. \{0, 1, 2\…\} 14. \{-2, -1, 0, 1, 2\…\} 15. \(-\frac{1}{3}, 0.24, 0.\overline{7}\) 16. \(\pm \sqrt{2}, \pm \sqrt{13}, \pi\) 17. \(-4, 0, 2, \pi, \sqrt{6}, \frac{9}{4}\) 18. 0.111…; nonterminating 19. 0.375; terminating

Guided Practice 1. In order for \( A \) to be a proper subset of \( B \), every element of \( A \) is an element of \( B \), but there are elements of \( B \) that are not in \( A \). 2. yes 3. yes 4. There are elements of \( B \) that are not elements of \( A \). 5. \( \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \emptyset \) 6a. TRUE 6b. TRUE 6c. FALSE 6d. TRUE 6e. TRUE 6f. FALSE 6g. TRUE 6h. FALSE 6i. FALSE 6j. TRUE 6k. TRUE 6l. FALSE
7a. undefined, \(-1, 1.333\…, \pi, -2, 100,000, -\frac{5}{4}, 3, 0, \sqrt{5}, 0.25, \) not a real number\} 7b. \{100,000, \sqrt{9}\} 7c. \{-100,000, \sqrt{9}, 0, \frac{1}{2}\} 7d. \{100,000, \sqrt{9}, 0, -1, -\frac{6}{3}\} 7e. \{100,000, \sqrt{9}, 0, -1, -\frac{6}{3}, 1, \frac{1}{3}, -\frac{5}{4}, 0.25\} 7f. \{\pi, \sqrt{5}\} 7g. \{100,000, \sqrt{9}, 0, \frac{1}{2}, -1, -\frac{6}{3}, 1, \frac{1}{3}, -\frac{5}{4}, 0.25, \pi, \sqrt{5}\}

Do the Math 1. \{-3, -2, -1, 0, 1, 2, 3, 4, 5\} 2. \emptyset or \{ \} 3. TRUE 4. TRUE 5. TRUE 6. TRUE 7. \( \emptyset \) = 8. \( \emptyset \) \( \notin \) 9a. 4 9b. 5, 4 9c. 5, 4, \( \frac{4}{3}, -\frac{7}{5}\). \( 5, \pi \) 9d. \( \emptyset \) 9e. 5, 4, \( \frac{4}{3}, -\frac{7}{5}. \) 5, \( 5, \pi \) 10a. 13 10b. 0, 13 10c. -4.5656…, 0, 2.43, 13, \( \frac{8}{7} \) 10d. \( \sqrt{2} \) 10e. -4.5656…, 0, 2.43, 13, \( \frac{8}{7} \), \( \sqrt{2} \) 11a. -9.99 12b. -10.00 12. See Graphing Answer Section 13. \( x \geq 14. \) > 15. -\( \frac{37}{8} \) 16. 1.143 17. 1.142 18. 0.666…; repeating 19. -0.16; terminating 20a. no 20b. no 20c. The set of real numbers is the union of the set of rational numbers and the set of irrational numbers. Therefore, all real numbers are either rational or irrational. Since decimals cannot be simultaneously repeating and nonrepeating, there are no real numbers that are both rational and irrational.

Section R.3

Five-Minute Warm-Up 1. The distance a number is from zero. The absolute value of a number can never be negative. 2. Answer when two numbers are multiplied. 3. Answer when two numbers are divided. 4. Answer when two numbers are added. 5. Answer when two numbers are subtracted. 6. 0 7. 1 8. undefined 9a. 4 9b. 12 9c. 3 10a. 20 10b. 5 10c. 0.25 11. 0 12. 1 13. \( \frac{3}{10} \) 14. 1 \( \frac{5}{8} \)

Guided Practice 1a. positive 1b. negative 2. the same as the sign of the number with the larger absolute value 3. See Graphing Answer Section 4a. 12 = 2 \cdot 2 \cdot 3 4b. 18 = 2 \cdot 3 \cdot 3 4c. 2, 3 4d. 2, 3 4e. 2 \cdot 2 \cdot 3 \cdot 3 = 36 4f. 21 4g. \frac{2}{2} \cdot 10 5a-k. Answers will vary; 5a. a + 0 = a 5b. a \cdot 1 = a

5c. a + (-a) = 0 5d. a \cdot \frac{1}{a} = 1 5e. -(a) = a 5f. a + b = b + a 5g. a \cdot b = b \cdot a 5h. a \cdot 0 = 0 5i. (a + b) + c = a + (b + c) 5j. (a \cdot b) \cdot c = a \cdot (b \cdot c) 5k. a \cdot (b + c) = a \cdot b + a \cdot c 6a. 6x - 10 6b. -36y + 12 6c. 20n + 8 6d. 4x - 16

Do the Math 1a. \( \frac{1}{5} \) 1b. -5 2a. -10 2b. \( \frac{1}{10} \) 3. \(-2x - 10\) 4. \( \frac{5}{2} \) 5. 16 6. -105 7. \( \frac{4}{5} \) 8. -\( \frac{14}{3} \) 9. 12 10. -2 11. -\( \frac{481}{360} \) 12. \( \frac{1}{12} \) 13. 5.1 14. 16 15. -\( \frac{13}{2} \) 16. -\( \frac{7}{8} \) 17. Associative of Addition 18. Multiplicative Inverse 19. Multiplicative Identity 20. Commutative of Multiplication 21. 543 feet 22a. a \cdot 0 = 0 22b. a \cdot (b + (-b)) = 0 22c. ab + a(-b) = 0

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**Do the Math** 22d. $ab < 0$ since the product of a negative and a positive is negative; $a(-b)$ must be positive so that $ab + a(-b) = 0$.

**Section R.4**

<table>
<thead>
<tr>
<th>Five -Minute Warm-Up</th>
<th>1a. base</th>
<th>1b. exponent</th>
<th>1c. $4 \cdot 4 \cdot 4 = 64$</th>
<th>2a. squared</th>
<th>2b. $(3x)^2$</th>
<th>3a. cubed</th>
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</thead>
<tbody>
<tr>
<td>3b. $(y + 1)^3$</td>
<td>4a. 5</td>
<td>4b. $-2x$</td>
<td>5. $-\frac{27}{8}$</td>
<td>6. $-\frac{23}{18}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Guided Practice**

1a. 36 | 1b. 36 | 1c. $-36$ | 1d. $-36$ | 1e. $-\frac{64}{125}$ | 1f. $\frac{256}{625}$ |

2. positive | 3. negative |

4a. Parentheses | 4b. Exponents | 4c. Multiply or Divide | 4d. Add or Subtract |

5a. $\frac{3 \cdot 4}{-9 - 6 \cdot 3}$ | 5b. $\frac{12}{-9 - 18}$ |

5c. $\frac{12}{-27}$ | 5d. $-\frac{4}{9}$ |

**Do the Math**

1. $x + y = 3$ | 2. $3x + 2y = 7$ |

12. $180$ | 13. $-31$ | 14. $-46$ | 15. $24$ | 16. $8$ | 17. $\frac{1}{2}$ | 18. $\frac{2}{3}$ | 19. $603.19$ in$^2$ | 20a. $3 + 5 \cdot (6 - 3) = 18$ |

20b. $(3 + 5) \cdot (6 - 3) = 24$ | 21. We cannot use the Reduction Property across addition. |

**Section R.5**

<table>
<thead>
<tr>
<th>Five -Minute Warm-Up</th>
<th>1a. $-10x + 15$</th>
<th>1b. $36a + 27$</th>
<th>1c. $4n - 10$</th>
<th>1d. $-2z + \frac{6}{5}$</th>
<th>2a. $-8$</th>
<th>2b. $\frac{2}{3}$</th>
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<tbody>
<tr>
<td>2c. $28$</td>
<td>2d. $-\frac{11}{18}$</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

**Guided Practice**

1a. $-2x$ | 1b. $\frac{y}{2} - 6$ or $\frac{1}{2}y - 6$ | 1c. $\frac{m}{8}$ | 1d. $25 - x$ | 2. $8$ | 3a. $0$ | 3b. $6x^2$ | 4a. $3x - 1$ |

4b. $2x + 16$ | 5. domain | 6. $x = 0$ | 7. the denominator to equal zero | 8a. $0$, $-2$, $4$ | 8b. $3$, $5$, $0$ |

**Do the Math**

1. $2$ | 2. $-33$ | 3. $11$ | 4. undefined | 5. $7y$ | 6. $-x + 1$ | 7. $\frac{17}{30}y$ | 8. $-4x^2 + 3x$ |

9. $9y + 9$ | 10. $3$ | 11. $12x - 4$ | 12. $-6w + 11$ | 13. $\frac{82}{15}y + \frac{44}{15}$ | 14. $0.35x - 17.38$ | 15. $10 - y$ | 16. $\frac{x}{5}$ |

17. $z + 30$ | 18. $2x - \frac{y}{3}$ | 19. (a) and (d) only | 20. $60$ cm$^2$ | 21a. $3$ less than twice a number, $x$, or the difference of twice a number, $x$, and $3$ |

21b. Twice the difference of a number, $x$, and 3 |

**Chapter 1 Answers**

**Section 1.1**

<table>
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<tr>
<th>Five -Minute Warm-Up</th>
<th>1. $\frac{1}{2}$</th>
<th>2. $\frac{1}{6}$</th>
<th>3. $x$</th>
<th>4. $60$</th>
<th>5. $-6x + 15$</th>
<th>6. $-1$</th>
<th>7. $x + 7$</th>
<th>8. $7x$</th>
<th>9. $-19$</th>
<th>10. yes</th>
</tr>
</thead>
</table>

**Guided Practice**

1. Substitute the value into the original equation. If this results in a true statement, then the value for the variable is a solution to the equation. | 2a. Remove any parentheses using the Distributive Property. | 2b. Combine like terms on each side of the equation. | 2c. Use the Addition Property of Equality to get all variable terms on one side of the equation and all constants on the other side. | 2d. Use the Multiplication Property of Equality to get the coefficient of the variable to be 1. | 2e. Check your answer to be sure that is satisfies the original equation. |

3a. $12$ | 3b. $12$ | 12 | 3c. $\frac{2x + 3}{2} - 12\left(\frac{x - 1}{4}\right) = \left(\frac{x + 5}{12}\right) \cdot 12$ |

3d. $6(2x + 3) - 3(x - 1) = (x + 5) \cdot 1$ | 3e. the Distributive Property | 3f. $9x + 21 = x + 5$ |

3g. $9x + 21 - x = x + 5 - x$ | 3h. $8x + 21 = 5$ | 3i. $8x + 21 - 21 = 5 - 21$ | 3j. $8x = -16$ | 3k. $x = -2$ |

3l. $\{-2\}$ | 4a. false | 4b. $\emptyset$ | 4c. contradiction | 5a. true | 5b. $\{x | x \text{ is any real number}\}$ | 5c. identity |
**Do the Math**  1. yes  2. yes  3. \{-3\}  4. \{1\}  5. \{5\}  6. \{-1\}  7. \emptyset  8. \left\{ \frac{6}{5} \right\}  9. \left\{ -\frac{2}{3} \right\}  10. \{-3\}

11. \left\{ \frac{17}{2} \right\}  12. \{x | x \text{ is any real number}\}  13. \{-20\}  14. \emptyset  15. a = 2  16. a = 5  17. x = -\frac{8}{5}

18. x = \left\{ \frac{5}{2} \right\}

19. We “solve” an equation, which means to find the number or numbers that will replace the variable so that the left side of the equation will be equal to the right side of the equation. We “simplify” an expression (using the Distributive Property or by combining like terms) to form an equivalent algebraic expression.

**Section 1.2**

**Five-Minute Warm-Up**  1. Possible answers: sum, plus, more than, exceeds by, in excess of, added to, increased by  2. Possible answers: difference, minus, subtracted from, less, less than, decreased by  3. Possible answers: product, times, of, twice  4. Possible answers: quotient, divided by, per, ratio  5. 2x + 3

6. 5(y + 1)  7. 5z + 20  8. \frac{2n}{3}  9. \frac{x}{2} - 12  10. x - 15  11. 0.05  12. 7.5%

**Guided Practice**  1. Direct Translation: problems where we translate from English into Mathematics by using key words in the verbal description; Mixture: problems where two or more quantities are combined; Geometry: problems where the unknown quantities are related through geometric formulas; Uniform Motion: problems where an object travels at a constant speed; Work Problems: problems where two or more entities join forces to complete a job.  2. Identify what you are looking for; Give names to the unknowns; Translate the problem into the language of mathematics; Solve the equation(s); Check the reasonableness of your answer; Answer the question  3a. n + 1  3b. n + 2  4a. x + 2  4b. x + 4  4c. x + 6  5a. p + 2  5b. p + 4  5c. p + 6  6. I = Prt; I = Interest, P = Principal, r = interest rate expressed as a decimal, t = time

7a. 11 - p  7b. Chocolates: 4.50, p, 4.5p; Truffles: 7.50, 11 - p, 7.5(11 - p); Blend: no value, 11, 58.50

7c. 4.5p + 7.5(11 - p) = 58.5; p = 8  7d. Andy bought 8 pounds of chocolates and 3 pounds of truffles

8a. The distances add to 63 miles.  8b. Slower boat: r = 4.5 hours, 4.5r; Faster boat: r + 6, 4.55 hours, 4.5(r + 6) Total: no value, no value, 63 miles

8c. 4.5r + 4.5(r + 6) = 63; r = 4  8d. Slower boat is traveling at 4 mph; Faster boat is traveling at 10 mph

**Do the Math**  1. \frac{500}{9}  2. 75%  3. 10 - z = 6; z = 4  4. 2y + 3 = 16; y = \frac{13}{2}  5. 5x = 3x - 10; x = -5

6. 0.4x = x - 10; x = \frac{50}{3}  7. 24 and 32  8. 23, 25, 27, and 29  9. $3750 in savings; $6250 in stock

10. 6 nickels; 42 dimes  11. 25 l  12. 20 lb of almonds; 30 lb of peanuts  13. 3.4 hours

**Section 1.3**

**Five-Minute Warm-Up**  1a. \(A = s^2\); P = 4s  1b. \(A = lw\); P = 2l + 2w  1c. \(A = \frac{1}{2}bh\); P = a + b + c

1d. \(A = \frac{1}{2}(B + b)\); P = a + b + c + B  1e. \(A = ah\); P = 2a + 2b  1f. \(A = \pi r^2\); C = 2\pi r or \pi d

2a. 15.961  2b. 15.961  3a. -0.10  3b. -0.09  4a. 10  4b. 9  5a. 100.7  5b. 100.7  6a. $5.77  6b. $5.76

**Guided Practice**  1. Get the variable by itself, with a coefficient of 1, on one side of the equation and all of the constants and any other variables on the other side of the equation.  2a. \(h = \frac{3V}{B}\)  2b. \(P = \frac{A}{1 + rt}\)

3a. the sum of the measures of the angles of a triangle equals 180 degrees or \(x^\circ + y^\circ + z^\circ = 180\)

3b. \(x - 15; \frac{x}{2} + 45\)  3c. \(x - 15 + x + \frac{x}{2} + 45 = 180\)  3d. \(x = 60\)  3e. yes  3f. 45°; 60°; 75°

4a. \(w = P - \frac{2l}{2}\)  4b. width = 6.5 cm  4c. Answers will vary

**Do the Math**  1. \(k = \frac{y}{x}\)  2. \(m = \frac{y - b}{x}\)  3. \(C = \frac{5}{9}(F - 32)\)  4. \(h = \frac{2A}{b}\)  5. \(S = \frac{a(1 - r)^n}{1 - r}\)
Do the Math 6. \(b = \frac{2A - Bh}{h}\) or \(\frac{2A}{h} - B\) 7. \(y = -2x + 10\) 8. \(y = \frac{4}{15}x - 2\) 9. \(y = 4x - 5\) 10. 
y = \frac{-6}{5}x + 24 11a. \(A = \frac{217 - M}{0.85}\) 11b. 67 years old 12. 60°, 120° 13. 40°, 50° 14. 32°, 96°, 52°

15a. 27.53 ft² 15b. 23.71 ft 15c. using part (a) $227.12; using \pi key and not rounding $227.16

Section 1.4

Five-Minute Warm-Up 1. < 2. > 3. 4. = 5. > 6. < 7a. > or < 7b. ≥ or ≤ 7c. \(\frac{3}{8} < x < \frac{1}{2}\)

Guided Practice 1. none 2. TRUE 3. TRUE 4. FALSE 5a. -2; -2 5b. 5x > -15 5c. \(\frac{5x}{5} > \frac{-15}{5}\)

5d. \(x > -3\) 5e. \(\{x | x > -3\}\) 5f. \((-3, \infty)\) 5g. See Graphing Answer Section 6a. -7; -7 6b. 3x ≥ 7x - 8

6c. -7x; -7x 6d. -4x ≥ -8 6e. \(x \leq 2\) 6f. \(\{x | x \leq 2\}\) 6g. \((-\infty, 2]\) 6h. See Graphing Answer Section

7a. ≥ 7b. ≥ 7c. > 7d. > 7e. ≤ 7f. ≤ 7g. < 7h. < 8a. at least 8b. 25,000 + 0.20v ≥ 36,000

8c. v ≥ 55,000 8d. yes 8e. Nghiep must have annual sales of at least $55,000 in order to earn $36,000.

Do the Math 1 – 4. See Graphing Answer Section 1. \(\{x | x < 3\}\) 2. \(\{x | x \leq 4\}\) 3. \(\{x | x > -3\}\)

4. \(\{x | x \geq -4\}\) 5. \((-\infty, 6]\) 6. \((-\infty, -\frac{5}{2}]\) 7. \((3, 2, \infty)\) 8. \((\frac{7}{2}, \infty)\) 9. \((-\infty, -\frac{8}{5}]\) 10. \((-\infty, -\frac{2}{5}]\)

11. \((-\infty, \frac{1}{2}]\) 12. \([-3, \infty)\) 13. \((-\infty, 3)\) 14. \((5, \infty)\) 15. at least 91.5 16. 132 miles 17. As written, this notation says that \(x > 7\) and at the same time \(x > 4\), which simplifies to \(x > 7\).

Section 1.5

Five-Minute Warm-Up 1. See Graphing Answer Section 2. (b) only 3a. -14 3b. 6 3c. 4

4. \(y = \frac{4}{3}x + 4\) 5a. 12 5b. 0 5c. 125

Guided Practice 1. See Graphing Answer Section 2. origin 3. left 4. 2 5a. -2; -7; (-2, -7)

5b. -1; -4; (-1, -4) 5c. 0; -1; (0, -1) 5d. 1; 2; (1, 2) 5e. 2; 5; (2, 5) 6. \((a, 0)\) and \((b, 0)\) 7. \((0, c)\)

8. If one unit is manufactured and sold, there will be a gain (profit) of $50. 9. see after the ball leaves the hand of the thrower, it will be 8 ft above the ground.

Do the Math 1A. (4, -3); IV 1B. (-3, -1); III 1C. (-2, 1); II 2. See Graphing Answer Section

2D. y-axis 2E. 1 2F. x-axis 3. (b) only 4. (a) and (d) 5. (2, 0); (0, -4) 8. 6 – 9. See Graphing Answer Section 10. \(a = -\frac{7}{3}\) 11. \(b = -13\)

Section 1.6

Five-Minute Warm-Up 1. \(\{6\}\) 2. \(\left\{-\frac{2}{5}\right\}\) 3. \(y = \frac{5}{2}x + 10\) 4. \(y = \frac{4}{3}x - 2\) 5. -2 6. 2 7. -3x + 15

8. \(\frac{5}{4}x - 10\)

Guided Practice 1. standard 2. \(y = 0\) 3. \(x = 0\) 4. slope 5. \(m = \frac{y_2 - y_1}{x_2 - x_1}\) 6a. slants upward from left to right 6b. slants downward from left to right 6c. horizontal 6d. vertical 7. \(y - y_1 = m(x - x_1)\)

8a. -2; 4; -1 8b. \(y - y_1 = m(x - x_1)\)

8c. \(y - (-1) = -2(x - 4)\) 8d. \(y = -2x + 7\) 9a. \(m = \frac{y_2 - y_1}{x_2 - x_1}\) 9b. \(x_1 = 2, y_1 = -2, x_2 = -2, y_2 = 6\)

9c. \(m = \frac{6 - (-2)}{-2 - 2} = \frac{8}{-4} = -2\) 9d. \(y - y_1 = m(x - x_1)\)

9e. \(m = -2, x_1 = 2, y_1 = -2\) 9f. \(y - (-2) = -2(x - 2)\) 9g. \(y + 2 = -2x + 4\) 9h. 2

9i. \(y + 2 = -2x + 4\) 9j. yes 9k. yes 9l. (0, 2) 9m. See Graphing Answer Section

AN-4

Sullivan/Struve, Intermediate Algebra, 4e
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Do the Math  1 – 4. See Graphing Answer Section  5. \(-\frac{12}{5}\)  6. undefined  7. See Graphing Answer

Section 8.  \(y = -x\)  9.  \(y = 4x - 9\)  10.  \(y = -\frac{4}{3}x - \frac{5}{3}\)  11.  \(y = -\frac{1}{3}x - \frac{2}{3}\)  12.  \(m = 3; y\)-intercept: (0,2)

13.  \(m = -2; y\)-intercept is (0,3)  14.  \(m = \text{undefined}; \text{no } y\)-intercept  15.  \(m = 0; y\)-intercept is (0, -4)  16.  \(g(x) = 3x + 2; -7\)  17.  \(F(x) = -\frac{4}{5}x + \frac{33}{5}; \frac{39}{5}\)  18. Vertical line of the form \(x = a\), where \(a \neq 0\)  19.  Horizontal line of the form \(y = b\), where \(b \neq 0\)

Section 1.7

Five-Minute Warm-Up  1. \(2 - \frac{1}{5}\)  2. \(-\frac{2}{3}\)  3. \(\frac{3}{4}\)  4. \(\frac{2}{5}\)  5. \(y = \frac{7x + 9}{3}\) or \(y = \frac{7}{3}x + 3\)  6. \(-4\)  7. undefined

Guided Practice  1. Two lines are parallel if they never intersect. 2. slope; \(y\)-intercept  3. =; \#  4a. \(y = \frac{3}{4}x - \frac{1}{2}\)  4b. \(\frac{3}{4}\)  4c. \(\frac{3}{4}\)  4d. \(\frac{3}{4}\); 4; -1  4e. \(y - y_i = m(x - x_i)\)  4f. \(y - (-1) = \frac{3}{4}(x - 4)\)

4g. \(y = \frac{3}{4}x - 4\)  4h. See Graphing Answer Section  5. Two lines are perpendicular if they intersect at right angles (90°).

6. -1  7. -l; \(-\frac{1}{m_2}\)  8a. \(y = 3x - 6\)  8b. 3  8c. \(-\frac{1}{3}\)  8d. \(-\frac{1}{3}; -9; 1\)  8e. \(y - y_i = m(x - x_i)\)  8f. \(y - 1 = -\frac{1}{3}(x - (-9))\)  8g. \(y = -\frac{1}{3}x - 2\)  8h. See Graphing Answer Section

Do the Math  1a. \(-10; -\frac{1}{10}\)  1b. \(\frac{1}{2}\)  1c. \(\frac{3}{2}; \frac{3}{2}\)  1d. 0; undefined  1e. undefined; undefined

2. perpendicular  3. neither  4. perpendicular  5. \(y = -3x + 11\)  6. \(y = -\frac{1}{4}x + 2\)  7. \(x = 2\)  8. \(y = -\frac{5}{2}x + 2\)  9. \(y = -2x - 5\)  10. \(x = 2\)  11. \(y = \frac{1}{3}x - 2\)  12. \(y = -\frac{1}{4}x + \frac{1}{4}\)  13. \(B = -4\)

14. for \(\overline{AB}, \overline{CD}\) \(m = \frac{1}{3}\); for \(BC, \overline{DA}\) \(m = 4\); Yes, it is a parallelogram.

Section 1.8

Five-Minute Warm-Up  1. yes  2. no  3. \(\{x | x \geq 5\}\)  4. \(\{x | x \leq 0\}\)  5. \(\{n | n < 12\}\)  6. \(\{a | a > -12\}\)

Guided Practice  1. True  2a. dashed  2b. solid  3. half planes  4. shade  5a. \(3x - 4y = 12\)  5b. \((4,0)\)

5c. \((0,-3)\)  5d. dashed  5e. Answers may vary; \((0,0)\)  5f. false  5g. opposite half plane  5h. See Graphing Answer Section  6a. \(45x + 65y \geq 4000\)  6b. yes  6c. no

Do the Math  1. (a) and (b)  2. (a) only  3 – 8. See Graphing Answer Section  9. \(x \leq y + 12\)  10. \(x + y \geq -3\)  11a. \(0.10g + 0.25s \leq 3.00\)  11b. no  11c. yes  12a. \(0.50x + 2y \geq 1000\)  12b. no  12c. yes  13. \(y > \frac{4}{3}x - \frac{5}{3}\)

Chapter 2 Answers

Section 2.1

Five-Minute Warm-Up  1a. \((-1, 0]\)  1b. \([4, 7]\)  2a. \((-\infty, -4]\)  2b. \((-2, \infty)\)  3 – 5. See Graphing Answer Section

Guided Practice  1. A relation is a mapping that pairs elements of one set with elements of a second set. 2. inputs; \(x\)  3. outputs; \(y\)  4. mapping; a set of ordered pairs; graph; equation  5a. \((-\infty, \infty)\)  5b. \([-3, \infty)\)

6a. \([-3, 3]\)  6b. \([-4, 4]\)  7. \((1, -2), (-2, -1), (-3, 0), (-2, 1), (1, 2)\) See Graphing Answer Section  7a. \([-3, \infty)\)

7b. \((-\infty, \infty)\)
Do the Math 1. map: (30, 7.09), (35, 7.09), (40, 8.40), (45, 11.29); domain: \{30, 35, 40, 45\}; range: \{7.09, 8.40, 11.29\} 2. domain: \{-2, -1, 0, 1, 2\}; range: \{-3, 0, 3, 6\} 3. domain: \{-2, -1, 0, 1, 2\}; range: \{-8, -1, 0, 1, 8\} 4. domain: \{-3, 0, 3\}; range: \{-3, 0, 3\} 5. domain: \{-3, 3\}; range: \[-2, 2\] 6. domain: \{-2, 0, 1, 2\}; range: \{-2, -1, 0, 3\} 7. domain: \(-\infty, \infty\); range: \[-3, \infty\] 8. domain: \(-\infty, \infty\); range: \(-\infty, \infty\) 9. domain: \(-\infty, \infty\); range: \(-\infty, \infty\) 10. domain: \(-\infty, \infty\); range: \[-2, \infty\] 11. domain: \(-\infty, \infty\); range: \(-\infty, 8\) 12. domain: \(-\infty, \infty\); range: \[-2, \infty\] 13. domain: \[2, \infty\); range: \(-\infty, \infty\) 14. domain: \(-\infty, \infty\); range: \(-\infty, \infty\)

Section 2.2

Five-Minute Warm-Up 1. 17 2. 26 3. \(-16, \infty\) 4. \(\{x | x \leq 100\}\) 5a. domain: \{-2, -4, -6, -8\}

5b. range: \[1, 3, 5, 7\] 6. \(\frac{1}{5}\) 7. -5 8. 3, 9. 10. 83, 0 in.3

Guided Practice 1. A function is a relation in which each element of the domain (inputs) corresponds to exactly one element in the range (outputs). 2a. yes; domain: \{3, 4, 5\}; range: \{-6, -1\} 2b. no 3a. 2x; -2x 3b. no 4a. yes 4b. no 5. A graph represents a function if and only if every possible vertical line intersects the graph in at most one point. 6. (c) and (d) only 7. independent; domain 8. dependent; range 9. 52 10. -6a + 2 11. 4 12. -3k + 1 13a. N, the number of trucks produced 13b. t, time worked in hours 13c. 300; After 5 hours, the factory produced 300 trucks.


11. Yes 12a. 10 12b. -5 12c. 6x + 1 12d. 3x + 7 13a. -9 13b. 1 13c. 4x - 3 13d. -2x - 7


Section 2.3

Five-Minute Warm-Up 1. \[\left\{\frac{1}{2}\right\}\] 2. \{6\} 4 - 4. See Graphing Answer Section 5a. domain: \([-4, 4]\)

5b. range: \([-1, 2]\) 6a. domain: \([-3, 2]\) 6b. range: \([-\frac{7}{6}, \frac{10}{3}]\)

Guided Practice 1. zero 2. \[\left\{x | x \neq \frac{3}{2}\right\}\] 3. \((-\infty, \infty)\) \((-2, 8)(-1, 6)(0, 4)(1, 2)(2, 0)(3, 2)(4, 4)\) 4. See Graphing Answer Section 5a. \((-\infty, \infty)\) 5b. \([-2, \infty)\) 5c. x-intercepts: \((-2, 0), (2, 0)\); y-intercept: \((-0, -2)\) 6a. yes 6b. -7; \((6, -7)\) 6c. 2; \((2, 3)\) 7. x; zero 8. \((2, 0), (-2, 0)\)

Do the Math 1. \((-\infty, \infty)\) 2. \((-\infty, -\frac{1}{2}) \cup \left(-\frac{1}{2}, \infty\right)\) 3. \((-\infty, \infty)\) 4. \((-\infty, -\frac{5}{6}) \cup \left(-\frac{5}{6}, \infty\right)\)

5. \((-\infty, \infty)\) 6 - 7. See Graphing Answer Section 8a. \((-\infty, \infty)\) 8b. \((-\infty, \infty)\) 8c. x-intercept: \((3, 0)\); y-intercept: \((0, -1)\)

8d. x = 3 9a. \((-\infty, \infty)\) 9b. \((-\infty, \infty)\) 9c. x-intercepts: \((-2, 0), (1, 0)\); y-intercept: \((0, 2)\)

9d. x = -2, x = 1, x = 4 10a. 8 10b. 0, 7 10c. \((-4, 10)\) 10d. \(0, 5\) 10e. yes; \(x = -4\)

11a. no 11b. 17; \((4, 17)\) 11c. -3; \((-3, -4)\) 11d. no 12a. \([0, \infty)\) 12b. 381.7 cm³

Section 2.4

Five-Minute Warm-Up 1–2. See Graphing Answer Section 3. \(y = -3x - 1\) 4. \([-0.9]\) 5. \(\{y | y \geq -\frac{1}{24}\}\)

Guided Practice 1. linear; line 2. slope; y-intercept 3a. \((0, -2)\) 3b. \(\frac{3}{2}\) 3c. See Graphing Answer

Section 4. \(mx + b = 0\) 5a. \((0, \infty)\) 5b. 66 ft 5c. 15 ft 5d. when the width exceeds 3 ft

6. fixed; \(a\) 7a. $4500 7b. negative 7c. \(V(x) = 22,500 - 4000x\) 7d. \([0, 5]\) 7e. $4500 7f. after 2.75 years 7g. time, \(x\) 7h. the value, \(V\) 8. 2 9a. slope 9b. \(y - y_1 = m(x - x_1)\)
Do the Math 1 – 4. See Graphing Answer Section 5. \( x = 8 \) 6. \( x = 4 \) 7a. \( \{-4\} \) 7b. \(-\frac{1}{3}\)

7c. \( -\frac{1}{3} \) 7d. \( \{x \mid x > -4\} \) 8. See Graphing Answer Section; \( \left( -4, -\frac{1}{3} \right) \) 9a. \( g(x) = 3x + 2 \) 9b. \(-7\)

10a. age, \( a \) 10b. Birth rate, \( B \) 10c. \([15, 44]\) 10d. 23.5 per 1000 women 10e. 37 years old

Section 2.5

Five-Minute Warm-Up 1a. \( \{x \mid -4 \leq x < 1\} \) 1b. \([-4, -1)\) 2. See Graphing Answer Section

3. \((-3, 5]\) 4. \([-3\} \) 5. \(\{x \mid x \leq -6\} \) 6. \(\{x \mid x > -2\} \)

Guided Practice 1. \( A \cap B \); and 2. \( A \cup B \); or 3a – 3b. See Graphing Answer Section 4a. \( 5x < 10 \)

4b. \( x < 2 \) 4c. \( x \geq -4 \) 4d – 4f. See Graphing Answer Section 5. \( a < x < b \)

6a. \(-8 + 3 \leq 5x - 3 + 3 \leq 7 + 3 \) 6b. \(-5 \leq 5x \leq 10 \) 6c. \(-1 \leq x \leq 2 \) 6d. See Graphing Answer Section

7a. \(-\frac{3}{2} x > 3 \) 7b. \( x < -2 \) 7c. \( 7x > 14 \) 7d. \( x > 2 \) 7e – 7g. See Graphing Answer Section

7h. \((-\infty, -2) \cup (2, \infty) \)

Do the Math 1. \([2, 3, 4, 5, 6, 7, 8, 9]\) 2. \([4, 6]\) 3. \(\varnothing \) 4. \([-2, 2]\) 5. \((-\infty, \infty) \)

6. \((0, 5]\) 7. \((-\infty, 0) \cup [6, \infty) \) 8. \( x = 1 \) 9. \((-\infty, 2) \cup [5, \infty) \) 10. \((-2, \frac{4}{7}] \) 11. \((-\infty, -2) \cup (3, \infty) \)

12. \((-\infty, -10) \cup (1, \infty) \) 13. \((2, 4]\) 14. \((-\frac{5}{3}, \frac{11}{4}] \) 15. \((-4, 1]\) 16. \((-\frac{9}{4}, 3]\) 17. \([-\frac{7}{3}, 3]\)

18. \( a = -15; b = -1 \) 19. \( 60 < x < 90 \) 20. approximately 756.7 kwh up to approximately 1953.1 kwh

Section 2.6

Five-Minute Warm-Up 1a. \(\{\} \) 1b. \(0 \) 1c. \(\frac{3}{4} \) 1d. \(5.2 \) 2. \(45 \) 3. \(-12 \) 4a. \(\{4\} \) 4b. \(\{7\} \)

5a. \(\{x \mid x > -6\} \) 5b. \(\{x \mid x \leq 3\} \)

Guided Practice 1. \( u = a; u = -a \) 2. isolate the absolute value expression 3a. \( |3x - 1| = 2 \)

3b. \(3x - 1 = 2 \) or \(3x - 1 = -2 \) 3c. \( l; -\frac{1}{3} \) 3d. \(\left\{1, -\frac{1}{3}\right\}\) 4. \( u = v; u = -v \) 5. \(-a < u < a; -a \leq u \leq a \)

6a. \(-9 \leq 4x - 3 \leq 9 \) 6b. \(-6 \leq 4x \leq 12 \) 6c. \(-\frac{3}{2} \leq x \leq 3 \) 6d. See Graphing Answer Section 6e. \(\left[-\frac{3}{2}, 3\right] \)

7. \( u > a \) or \( u < -a \); \( u \geq a \) or \( u \leq -a \) 8a. \(8x + 3 > 3 \) 8b. \(8x + 3 \geq 3 \) or \(8x + 3 < -3 \)

8c. \( x > 0 \) or \( x < -\frac{3}{4} \) 8d. See Graphing Answer Section 8e. \(\left(-\infty, -\frac{3}{4}\right) \cup (0, \infty) \)

Do the Math 1. \(\left(\left\{-\frac{7}{2}, \frac{13}{2}\right\}\right) \) 2. \(\{0, 8\} \) 3. \(\{y \mid -10 < y < 2\} \) 4. \(\{x \mid -1 \leq x \leq \frac{7}{3}\}\)

5. \(\{x \mid x \leq -11 \) or \( x \geq 3\}\) 6. \(\{z \mid z \) is any real number\} \) 7. \(\varnothing \) 8. \(\left(-\frac{4}{5}, 2\right) \) 9. \(\varnothing \) 10. \((-\infty, \infty) \)

11. \(\left\{-1, -\frac{1}{2}\right\}\) 12. \(\left(-\infty, -\frac{1}{2}\right) \cup (1, \infty) \) 13. \(\varnothing \) 14. \(\{y \mid y \) is any real number\} \) 15. \(\left|x - (4)\right| < 2 \)

16. \(2x - 7 > 3 \) 17. \(<234.64 \) days or \(>297.36 \) days 18. The absolute value of every real number is positive or 0 and therefore greater than any negative number.
Chapter 3 Answers
Section 3.1

Five-Minute Warm-Up  1. 17  2. Yes  3. See Graphing Answer Section  4. \( y = x + 1 \)
5. slope: \( \frac{1}{3} \); y-intercept: \((0, 3)\)  6. \(-15\)  7. \{1\}

Guided Practice  1a. inconsistent; the lines are parallel  1b. consistent and dependent; the lines coincide
1c. consistent and independent; the lines intersect  2a. \(-y = -3x + 14\)  2b. \(y = 3x + 14\)  2c. \(5x + 2y = -5\)
2d. \(5x + 2(3x + 14) = -5\)  2e. \(5x + 6x + 28 = -5\)  2f. \(11x + 28 = -5\)  2g. \(11x = -33\)  2h. \(x = -3\)  2i. 5
2j. \((-3, 5)\)
3a. \((1) -10x - 5y = 20\); \((2) 3x + 5y = 29\)  3b. \(-7x = 49\)  3c. \(x = -7\)  3d. \(2(-7) + y = -4\)
3e. \(y = 10\) 3f. \((-7, 10)\)  4. The variables will both be eliminated and a true statement will remain  5. The variables will both be eliminated and a false statement will remain
6. \(\{ (x, y) \mid -x + 3y = 1 \}\) or
   \(\{ (x, y) \mid 2x - 6y = -2 \}\)

Do the Math  1a. no  1b. yes  2a. no  2b. no  3. \((2, 0)\)  4. \((1, -4)\)  5. \((4, 2)\)  6. \(\left(\frac{5}{3}, -\frac{5}{2}\right)\)
7. \(\emptyset\)  8. \(\left(\frac{2}{9}, -\frac{13}{9}\right)\)  9. \((5, -11)\)  10. \(\{ (x, y) \mid y = \frac{1}{2}x + 2 \}\) or \(\{ (x, y) \mid x - 2y = -4 \}\)  11. \(\emptyset\)
12. no solution  13. exactly one solution  14a. \(y = -\frac{1}{2}x + \frac{5}{2}\)  14b. \(y = 2x\)  14c. \((1, 2)\)

Section 3.2

Five-Minute Warm-Up  1. \(x + 12 = 5\)  2. \(y - 6 = 2y\)  3. \(2(l - 6) = w\)  4. \(2h - 10 = \frac{2}{w}\)  5. \(12000 - s\)
6. \(37.50\)  7. \(C(x) = 42x + 4000\)

Guided Practice  1a. The number of child’s tickets.  1b. (1) \(a + c = 13\); (2) \(26a + 18.50c = 278\)
1c. \((5, 8)\)  1d. Yes; Yes  1e. 5 adult and 8 child’s tickets were purchased  2a. \(P = 2l + 2w\)  2b. \(l = w + 10\)
2c. \((1) 2l + 2w = 125\); \(l - w = 10\)  2d. \((36.25, 26.25)\)  2e. The dimensions are \(36.25\) ft by \(26.25\) ft
3a. Peanuts: \(p; 2; 2p\); Trail Mix: \(b, 5, 5b\); Blend \(10, 3.20, 32.00\)  3b. \((1) p + t = 10\); \(2p + 5t = 32\)
3c. \((6, 4)\)  The backpacker used 6 pounds of peanuts and 4 pounds of trail mix  4a. \(r - c\)  4b. \(r + c\)
5a. With the wind: \(36, r + c, 3\); Against the wind: \(32, r - c, 4\)  5b. \((1) 3r + 3c = 36; 4r - 4c = 32\)
5c. \((10, 2)\); The speed of the cyclist is 10 mph and the speed of the wind is 2 mph.
6. The point(s) of intersection of the two graphs.  7. revenue = costs

Do the Math  1. \(14\) and \(11\)  2. \(15\) and \(8\)  3. \(25\) dimes \(4.72.5\) cm by \(57.5\) cm  5. \$32,000 in bonds
6. 10 lb. of chocolate covered peanuts and 40 lb. of chocolate covered almonds  7. 25 units of powder A
   and 10 units of powder B  8. 150 mph; 20 mph  9. 0.3 hour or 18 minutes

Section 3.3

Five-Minute Warm-Up  1. \(-10\)  2. \((-5, -15)\)  3. \(\emptyset\)  4. \(\{(x, y) | 4x + y = 3\}\) or \(\{(x, y) | 8x + 2y = 6\}\)

Guided Practice  1a. a plane  2a. consistent; independent  2b. inconsistent  2c. consistent; dependent
3a. \(-3z - 9y - 9z = -27\)  3b. \(3x + 5y + 4z = 8\)  3c. \(-4y - 5z = -19\)  3d. \(-5x - 15y - 15z = -45\)
3e. \(5x + 3y + 7z = 9\)  3f. \(-12y - 8z = -36\)  3g. \(12y + 15z = 57\)  3h. \(-12y - 8z = -36\)  3i. \(7z = 21\)
3j. \(z = 3\)  3k. \(y = 1\)  3l. \(x = -3\)  3m. \((-3, 1, 3)\)  4. inconsistent; \(\emptyset\)  5. consistent and dependent
6a. \(x = 5z - 3\)  6b. \(y = 10z - 8\)  6c. \(x = 5z - 3, \ y = 10z - 8\)  7a. \(a + b + c = 600\)
7b. \(80a + 60b + 25c = 33,500\)  7c. \(80a + \frac{3}{5}b(60) + \frac{4}{5}c(25) = 24,640\)
Do the Math 1a. Yes 1b. Yes 2. \((-2, 6, 6)\) 3. \(\left(\frac{13}{2}, 0, -\frac{3}{2}\right)\) 4. \((0, -3, 1)\) 5. \(\emptyset\)

6. \(\left(\frac{2}{3}, \frac{1}{2}, \frac{1}{2}\right)\) 7a. \(a + b + c = 2; 4a + 2b + c = 9\) 7b. \(a = 3; b = -2; c = 1\) 7c. \(f(x) = 3x^2 - 2x + 1\)

8. Potato: 1.5; Chicken: 1.5; Coke: 1

Section 3.4

Five-Minute Warm-Up 1. \(-2, 1, -3\) 2. 12 3. \(y = \frac{7}{5}x - 2\) 4. \(x = -\frac{4}{9}y - \frac{4}{3}\) 5. \(-6x + 27y - 3z\)

Guided Practice 1. 2 \times 4 2. standard; 0 3. See Graphing Answer Section 4. \(\begin{cases} x + y = 2 \\ -3x + y = 10 \end{cases}\)

5a. Interchange any 2 rows. 5b. Replace a row by a non-zero multiple of that row. 5c. Replace a row by the sum of that row and a non-zero multiple of another row. 6a – 6b. See Graphing Answer Section 7. When there are ones on the main diagonal (when the row and column number are the same) and zeros below the ones. This is also called triangular form. 8a – 8d. See Graphing Answer Section

8e. \((1, 2, -2)\)

9. \(\{(x, y)|x + 4y = 2\}\) 10. \(\emptyset\)

Do the Math 1 – 2b. See Graphing Answer Section 3. \(\{(x, y)|5x - 2y = 3\}\) 4. \(\left(\frac{4}{3}, -\frac{5}{3}\right)\)

5. \(\emptyset\) 6. \((0, -5, 4)\) 7. \$7000 in savings; \$3000 in Treasury bonds; \$2000 in the mutual fund

Section 3.5

Five-Minute Warm-Up 1. Yes 2. No 3. \(\{x|x < -3\}\) 4. See Graphing Answer Section

Guided Practice 1a. 14 1b. -25 2a. coefficient 2b. the constants 2c. second; \(y\) 3. \(\begin{cases} 3x - 5y = 9 \\ x + 2y = 2 \end{cases}\)

4a. \(\begin{vmatrix} 3 & -5 \\ 1 & 2 \end{vmatrix}\) 4b. \(\begin{vmatrix} 9 & -5 \\ 2 & 2 \end{vmatrix}\) 4c. \(\begin{vmatrix} 3 & 9 \\ 1 & 2 \end{vmatrix}\) 5a. \(x = \frac{D_x}{D}\) 5b. \(y = \frac{D_y}{D}\) 6. See Graphing Answer Section.

7. See the determinants in the Graphing Answer Section 7b. 22 7c. -66 7d. 11 7e. 22 7f. \(-\frac{66}{22} = -3\)

7g. \(\frac{D_x}{D} = \frac{11}{22} = \frac{1}{2}\) 7h. \(\frac{D_y}{D} = \frac{22}{22} = 1\) 7i. \((-3, \frac{1}{2}, 1)\) 8. inconsistent; \(\emptyset\) 9. consistent; dependent; infinitely many

Do the Math 1. 14 2. -4 3. -5 4. \((5, -4)\) 5. \(\left(\frac{7}{3}, \frac{5}{6}\right)\) 6. \((3, 2, -1)\) 7. Cramer’s Rule does not apply 8. \(x = -3\) 9. area is 12.5 sq units

Section 3.6

Five-Minute Warm-Up 1. Yes 2. No 3. \(\{x|x < -3\}\) 4. See Graphing Answer Section

Guided Practice 1a. graphing 2. dashed 3. solid 4. the opposite half-plane 5. intersection 6. See Graphing Answer Section 7a. \(5x + 8y \leq 40\) 7b. \(y \geq 2x\) 7c. \(x \geq 0; y \geq 0\)

7d. See Graphing Answer Section 7e. \((0, 5)\) 8a. See Graphing Answer Section 8b. \(\left(0, \frac{800}{3}\right)\) \((0, 0)\) \(\left(\frac{400}{3}, 0\right)\)

Do the Math 1. (b) 2. (b) 3-7. See Graphing Answer Section 7. The graph is unbounded. Corner points: \((0, 8); (6, 2); (12, 0)\) 8a. See Graphing Answer Section 8b. \(\left(0, \frac{800}{3}\right)\) \((0, 0)\) \(\left(\frac{400}{3}, 0\right)\)
Chapter 4 Answers

Section 4.1

Five-Minute Warm-Up  1. $-1$  2. $5$  3. $4x^2 + 2x$  4. $-48x - 16$  5. $5p + 12$  6. $xy^2$  7. $27$  8. $-29$

Guided Practice  1a. A single term which is the product of a constant and a variable with whole number exponents.  1b. The constant multiplied with the variable.  1c. The degree is the same as the exponent on the variable.  1d. The degree is the sum of the exponents on the variables.  2a. A polynomial is the sum of two or more monomials.  2b. The degrees of the terms are in descending order.  2c. The degree of the polynomial is the same as the highest degree of any of the terms in the polynomial.  3a. monomial  3b. binomial  3c. trinomial  3d. polynomial  4. Like terms have the same variables, raised to the same powers.

Do the Math  1. $-6x^3 - 3x^2 - 3t + 7$  10. $-10xy^2 - 4xy - y^2$  11. $\frac{5}{4}y^3 + \frac{7}{24}y - \frac{1}{6}$  12. $-w^3 + 4w^2 + 6w + 4$  13. $-114$  14. $-11$

Section 4.2

Five-Minute Warm-Up  1. $x^9$  2. $21y^7$  3. $64z^3$  4. $16a^6$  5. $\frac{25}{4}x^2$  6. $\frac{2x^6}{9y}$  7. $21a - 14b$  8. $\frac{4x}{5} + \frac{3}{4}$

Guided Practice  1. The Distributive Property  2. $-2a^2b^7 - 14a^2b^3 + 6ab^4$  3a. The Distributive Property  3b. FOIL  4. $27x^3 + 39x - 10$  5. $12a^2 - 17ab - 7b^2$  6a. $(3x - 1) \cdot 2x^2 + (3x - 1) \cdot x + (3x - 1) \cdot (-5)$  6b. $6x^3 - 2x^2 + 3x^3 - x - 15x + 5$  6c. $6x^3 + x^2 - 16x + 5$  7. $A^2 - B^2$  8a. $A^2 + 2AB + B^2$  8b. $A^2 - 2AB + B^2$

Do the Math  1. $-27a^5b^7$  2. $9x^6y^4$  3. $-12m^3n^3 + 3m^2n^4 - 15mn^5$  4. $z^2 - 5z - 24$  5. $-14y^2 - 31y + 10$  6. $2y^2 + \frac{23}{6}y - 4$  7. $3m^2 + mn - 10n^2$  8. $21p^3 - 41p^2 + 31p - 6$

Section 4.3

Five-Minute Warm-Up  1. $r^5$  2. $\frac{7x^4}{3}$  3. $25$  4. $\frac{a^3}{2b^3}$  5. $\frac{81s^2}{4t^{10}}$  6. $\frac{2x^3y^2}{z}$  7. $\frac{4}{3}$

Guided Practice  1a. $x^2$  1b. $x^3 + x^3$  1c. See Graphing Answer Section

1d. $x^3 - 3x + 4 + \frac{2}{x + 1}$  2. standard; $0$  3. $x - c; x + c$  4a. $x^4 + 3x^3 - 9x^2 + 0 + 18; 1, 3, -9, 0, 18$

4b. $\frac{3}{3} \frac{1}{2} \frac{3}{3} \frac{9}{10} \frac{9}{18}$  4c. $\frac{3}{3} \frac{1}{2} \frac{3}{3} \frac{9}{10} \frac{9}{18}$  4d. $\frac{3}{3} \frac{1}{2} \frac{3}{3} \frac{9}{10} \frac{9}{18}$  4e. $\frac{3}{3} \frac{1}{2} \frac{3}{3} \frac{9}{10} \frac{9}{18}$

4f. $\frac{3}{3} \frac{1}{2} \frac{3}{3} \frac{9}{10} \frac{9}{18}$  4g. quotient: $x^3 + 6x^2 + 9x + 27$; remainder: $99$  4h. $x^3 + 6x^2 + 9x + 27 + \frac{99}{x - 3}$

5. $f(c)$  6. $f(c) = 0$

AN-10

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**Do the Math**

1. $2z + 3$
2. $\frac{1}{2}z + \frac{2}{z}$
3. $y + \frac{9y}{2x} + \frac{8}{y}$
4. $x - 7$
5. $x - 6 + \frac{9}{4x + 7}$
6. $x^2 - x - 20$
7. $a^2 - 8a + 15$
8. $x - 5$
9. $k + 5 + \frac{-3k + 7}{2k^2 - 3}$
10. $x + 6 + \frac{7}{x - 4}$
11. $x^2 - 3x - 4 - \frac{5}{x + 3}$
12. $a^3 + 8a^2 - a - 8 - \frac{9}{a - 8}$
13. $\frac{3}{2}x - \frac{15}{4} + \frac{35}{4(x + 1)}$
14. $1$
15. $-68$
16. not a factor
17. $(5x + 2) ft$
18. $(x + 7) ft$

**Section 4.4**

**Five-Minute Warm-Up**
1. $2 \cdot 2 \cdot 3 = 2^2 \cdot 3$
2. $2 \cdot 3 \cdot 3 = 2 \cdot 3^2$
3. $2^3 \cdot 3^2$
4. $5^3$
5. $-24x + 16$
6. $7u^3 + 28u^4 - 7u^3$
7. $2, 3, 5, 7, 11, 13, 19, 23, 29$
8a. $12, 8$
8b. $96$
9. $3x^2 y$

**Guided Practice**

1a. $2m^2 n^2$
1b. $2m^2 n^2 \cdot 3m + 2m^2 n^2 \cdot 9mn^2 - 2m^2 n^2 \cdot 11n^3$
1c. $2m^2 n^2 (3m^3 + 9mn^2 - 11n^3)$
2. $(7a - 1)(a - 3)$
3. four
4. true
5a. $2x$
5b. $3y$
5c. $2x(x - 2) + 3y(x - 2)$
5d. $(x - 2)(2x + 3y)$

**Do the Math**

1. $(z + 6) - 4(b - 8)$
2. $3a(4a + 15)$
3. $-6q(q^2 - 6q + 8)$
4. $-2b(9b^2 - 5b - 3)$
5. $(5z + 3)(6z + 5)$
6. $4ab^2(2a^3 + 3a^2 b - 9b^2)$
7. $(x - y)(8 + b)$
8. $(y + 3)(3y^2 - 5)$
9. $3(a - 5)(a - 3)$
10. $2y(y + 7)(y - 2)$
11. $6(x - 3)$
12. $(c - 1)(c^2 + 5)$
13. $2\pi r(r + 4)$ sq in
14. $0.08x$
15b. $0.80x - 0.15(0.80x)$
15c. $0.68x$
15d. $0.442$
16. (a) Receiving a 30% discount is the better deal. Explanations may vary.

**Section 4.5**

**Five-Minute Warm-Up**
1. $3, -4, -7$
2a. $-13$
2b. $36$
3a. $-9$
3b. $-36$
4. $7, -4$
5. $-9, -6$
6. $12, 4$
7. $21, -3$
8. $4a$
9. $-12, 3$

**Guided Practice**

1a. $19; 11; 9; -19; -11; -9$
1b. $3$ and $6$
1c. $(y + 3)(y + 6)$
2. List all possible combinations of factors of $c$ and check to see if any sum to $b$
3a. Grouping
3b. Trial and Error
4a. $3; 12$
4b. $36$
4c. $-37; -20; -15; -13; -12$
4d. $-4$ and $-9$
4e. $3x^2 - 4x - 9x + 12$
4f. $x(3x - 4) - 3(3x - 4) = (x - 3)(3x - 4)$
5. $2(4x + 3y)(6x - 5y)$
6a. $p^2$
6b. $12u^2 - u - 1$
6c. $(3u - 1)(4u + 1)$
6d. $(3p^2 - 1)(4p^2 + 1)$
7a. $-2(8x^2 + 2x - 3)$
7b. $8; 2; -3; -24$
8. $-3n(2n - 3)(3n + 1)$

**Section 4.6**

**Do the Math**

1. $(z + 7)(z - 4)$
2. $(q + 10)(q - 8)$
3. $-(p + 6)(p - 9)$
4. $(m + 2n)(m + 5n)$
5. $-4(s + 6)(s + 2)$
6. $(6x - 1)(x - 6)$
7. $(3r + 5)(4r - 3)$
8. $(4r + 3)(5r + 2)$
9. $(m + 3n)(3m - 2n)$
10. $4x(x - 9)(x - 4)$
11. $3xy(2x + 3)(9x - 8)$
12. $(y^2 + 3)(y^2 + 2)$
13. $(rs + 12)(rs - 4)$
14. $(3z + 11)(z + 7)$
15. $(4m + 9n)(6m + n)$
16. $(r^2 - 4)(r^2 - 2)$
17. $(p - 5q)(p - 9q)$
18. $(9a + 17)(a + 1)$
19. $(a + 3)(a - 2)$
20. $3(y + 2)(8y - 3)$
21. $-3mn(4m - 3n)(2m + 3n)$
22. $3xy(2x + 3)(9x - 8)$

**Guided Practice**

1a. $A^2 + 2AB + B^2$
1b. $A^2 - 2AB + B^2$
2a. $p$
2b. $9$
2c. $p^2 + 2(p \cdot 9) + 9^2$
2d. $(p + 9)^2$
3. $81$
4. $A^2 - B^2$
5a. $(x - 8)(x + 8)$
5b. $(5m^3 - 6n^2)(5m^3 + 6n^2)$

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Guided Practice  6a. \((x^2 + 4x + 4) - 4y^2\)  6b. \((x + 2)^2 - (2y)^2\)  6c. \((x + 2 + 2y)(x + 2 - 2y)\)
7a. \(A^3 + B^3\)  7b. \(A^3 - B^3\)  7c. \(A^3 + 3AB^2 + B^3\)  8a. \(p^3\)  8b. \(p + 4\)  8c. \(p^3 + 4^3\)
8d. \((p + 4)(p^2 - 4p + 16)\)  9a. \(3x(8x^3 + 125)\)  9b. \(3x\left((2x)^3 + 5^3\right)\)  9c. \(3x(2x + 5)(4x^2 - 10x + 25)\)

Do the Math  1. \((3z - 1)^2\)  2. \((6b + 7)^2\)  3. \((2a + 5b)^2\)  4. \((b^2 + 4)^2\)  5. \((9 - a)(9 + a)\)
6. \((x^2 - 3y^2)(x^2 + 3y^2)\)  7. \(4z(3x - 4y)(3x + 4y)\)  8. \((z + 4)(z^2 - 4z + 16)\)  9. \((6 - n)(36 + 6n + n^2)\)
10. \(2(2m + 3n)(4m^2 - 6mn + 9n^2)\)  11. \((5z + 3)(7z^2 + 3z + 9)\)  12. \((m^3 + n^3)(m^6 - m^3n^4 + n^8)\)
13. \((3a - b)(3a + b)\)  14. \((4x - 5)(16x^2 + 20x + 25)\)  15. \(3m(m - 3n)(m^2 + 3mn + 9n^2)\)
16. \((p - 3)^2(p + 3)^2\)  17. \((3mn - 5)^2\)  18. \((p + 4 - q)(p + 4 + q)\)  19. \(-5(a + 2)(a^2 - 2a + 4)\)
20. \((6m + n - 9)(6m + n + 9)\)  21. prime  22. \((p - 2 + q)(p - 2 - q)\)  23. \((x + 0.3)^2\)  24. \(\left(\frac{a}{6} + \frac{b}{7}\right)\left(\frac{a}{6} - \frac{b}{7}\right)\)

Section 4.7

Five-Minute Warm-Up  1. \(-8x^3 y^7\)  2. \(6a^3b^2 - a^4b + \frac{5}{2}ab^2\)  3. \(20x^2 + 7xy - 6y^2\)  4. \(a^2 + 8ab + 16b^2\)
5. \(-9a\left(3a^2 - a + 2\right)\)  6. \((4p + 3)(p - 2q)\)

Guided Practice  1. factor out the GCF  2a. \(-4x\)  2b. \(-4x(3x^2 + 5xy - 2y^2)\)  2c. \(3\)
2d. \(-4x(3x - y)(x + 2y)\)  3a. 2b. \(8a - 5b^2\)(\(8a + 5b^2\)) 4a. There is no GCF. 4b. \((-x^3 - 2x^2) + (5x + 10)\) 4c. \(-x^2(x + 2) + 5(x + 2)\) 4d. \((x + 2)(-x^2 + 5)\) 5. All factors are monomials or are prime.

Do the Math  1. \(3(x + 7)(x - 5)\)  2. \(-5(a + 4)(a - 4)\)  3. \((4m - 7)(2m - 7)\)
4. \(2(3p^2 - q)(9p^4 + 3p^2q + q^2)\)  5. \(-4c(c^2 - 4c + 7)\)  6. \((6t + 5)(3t - 4)\)  7. \(2(6p^2 + 25q^2)\)
8. \(4w^2 + 1)(2w - 1)(2w - 1)\)  9.prime  10. \(2pq(5p + 2)(2p - 1)\)  11. \(2p^3(3p + 2q)(9p^2 - 6pq + 4q^2)\)
12. \((2z^2 + 5)(2z^2 - 5)\)  13. \((2b^2 + 5)(2b^2 - 3)\)  14. \(3(x + 4)(3x + 2)\)  15. \((a - 2b + 6)(a + 2b + 6)\)
16. \((w^3 + 5)(w - 1)(w^2 + w + 1)\)  17. \((q^4 + 1)(q^4 - q^2 + 1)\)  18. \(-2(y + 2)(y + 4)(y - 4)\)
19. \(-5z(1 + 4z^2)\)  20. \(2h^2 + 9)(3h - 2)(3h + 2)\)  21. \(f(x) = (x + 8)(x - 5)\)  22. \(H(p) = (5p - 2)(p + 6)\)
23. \(g(x) = -4(5x - 3)(5x + 3)\)  24. \(G(x) = (2x - 1)(x - 3)(x + 3)\)

Section 4.8

Five-Minute Warm-Up  1. \(\left\{\frac{3}{2}\right\}\)  2. \(\left\{-\frac{1}{5}\right\}\)  3a. 204  3b. \(\frac{37}{4}\)  3c. \(3, -3\)  5. \(-7; (-5, -7)\)  6. \(x = -\frac{3}{2}\)

Guided Practice  1. If the product of two numbers is zero, then one of the factors must be zero. That is, if \(a \cdot b = 0\), then \(a = 0\) or \(b = 0\) or both \(a\) and \(b\) are zero.  2. quadratic equation  3. List the terms from the highest power to the lowest power.  4a. \(2x^2 - 3x - 2 = 0\)  4b. \((2x + 1)(x - 2) = 0\)  4c. \(2x + 1 = 0\)
4d. \(x - 2 = 0\)  4e. \(x = -\frac{1}{2}\)  4f. \(x = 2\)  4g. \(\left\{-\frac{1}{2}, 2\right\}\)  5a. Simplify the left side of the equation.

5b. Standard form  5c. \(\{10, -5\}\)  6a. \(p^3 + 2p^2 - 9p - 18 = 0\)  6b. \((p - 3)(p + 3)(p + 2) = 0\)
6c. \(p = -3\)  6d. \(p + 3 = 0\)  6e. \(p + 2 = 0\)  6f. \(p = 3\)  6g. \(p = -3\)  6h. \(p = -2\)  6i. \(\{3, -3, -2\}\)
7a. \(3x^2 - 13x - 10 = 0\)  7b. \(-\frac{2}{3}\)  7c. \(-\frac{2}{3}, 0\)  7d. \((5, 0)\)  8a. \(n + 2\)  8b. \(A = l \cdot w\)  8c. \(n; n + 2\)
8d. \(n(n + 2) = 255\)  8e. \(\{-17, 15\}\)  8f. no; distance cannot be negative  8g. The dimensions of the rectangle
are: width of 15 cm by length of 17 cm.

**Do the Math** 1. \[ \left\{0, -\frac{4}{3}\right\} \] 2. \{-11, 0, 9\} 3. \{-3, 0\} 4. \{-8, 5\} 5. \{-6\} 6. \left\{\frac{1}{4}, 6\right\} 7. \left\{-\frac{1}{3}, -\frac{5}{2}\right\}

8. \left\{-4, \frac{5}{2}\right\} 9. \left\{-5, \frac{3}{2}\right\} 10. \{-9, 5\} 11. \{-5, -4, -4\} 12. \left\{-\frac{3}{2}, 0, \frac{3}{4}\right\} 13. \{-8, 4\} 14. \{0, 6\}

15. \(x = 0\) or \(x = -5\) 16. \(x = 2\) or \(x = -7\) 17. \(-\frac{5}{4}, 0\) and \(\left\{\frac{7}{2}, 0\right\}\) 18. height: 12 m; base: 8 m 19. 3 ft 20. width: 13 in.; length: 21 in.

**Chapter 5 Answers**

**Section 5.1**

**Five-Minute Warm-Up**

1. \((x - 5)(x + 2)\) 2. \((2z - 1)(3z + 2)\) 3. \{-4, -5\} 4. \(\frac{2}{9}\) 5. (a); (b) 6. \(\frac{2}{5}\) 7. \(\frac{9}{20}\)

**Guided Practice**

1. the denominator to equal zero. 2a. \(\{x|x \neq -2\}\) 2b. \(\{a|a \neq 3, a \neq 6\}\)

3a. \(\frac{(x - 10)(x + 2)}{x + 2(2x - 1)}\) 3b. \(\frac{x - 10}{2x - 1}\) 4a. \(\frac{x(x - 4)}{(x - 2)(x + 2)}\) 4b. \(\frac{(x - 3)(x + 2)}{(x - 2)(x + 2)(x - 4)}\)

4c. \(\frac{x(x - 3)}{(x - 2)(x + 2)(x - 4)}\) 4d. \(\frac{x + p}{q} = \frac{x}{p}\) 6a. \(x^2 - 12y^2 + 24y^5\)

6b. \(\frac{27 \cdot 4 \cdot x - 7 \cdot y^3}{10x^2}\) 6c. \(\frac{3y^3}{10x^2}\) 7. \(q(x) = 0\) 8. \(\{x|x \neq -8, x \neq 5\}\)

**Do the Math**

1. \(\{x|x \neq 7\}\) 2. \(\{x|x\) is any real number\} 3. \(\{x|x \neq -4\}\)

6. \(\frac{x - 3y}{x - 2y}\) 7. \(\frac{v^2 - 5}{v + 3}\) 8. \(\frac{x(x + 4)}{4}\) 9. \(\frac{x + 5}{x + 6}\) 10. \(-4y - 3\) 11. \(-\frac{1}{3}\) 12. \(\frac{(x - 3)^2}{x(2x - 5)}\) 13. \(\frac{5(m - 4)}{m(m + 4)}\)

14. \(\frac{1}{15}\) 15. \(12m^2n^2\) 16. \(\frac{y - 2}{3y + 1}\) 17. \(\frac{1}{x - y}\) 18. \(\{x|x \neq -5, x \neq \frac{1}{4}\}\) 19. \(\{x|x\) is any real number\}

20a. \(\frac{(x + 1)(x + 4)}{x - 2}\) 20b. \(\frac{(x + 1)(x + 9)}{2x - 5}(x + 5)\)

**Section 5.2**

**Five-Minute Warm-Up**

1. 21 2. 600 3. \(\frac{7}{24} = \frac{175}{600}; \frac{14}{75} = \frac{112}{600}\) 4. 6 5. \(\frac{89}{90}\) 6. \(-\frac{3}{14}\) 7. \(\frac{25}{48}\)

**Guided Practice**

1a. \(\frac{x^2 - 3 + (x^2 + x)}{2x + 3}\) 1b. \(\frac{2x^2 + x - 3}{2x + 3}\) 1c. \(\frac{(2x + 3)(x - 1)}{(2x + 3)}\) 1d. \(x - 1\)

2a. \(\frac{x^2 - 11 + (-1)(-3x - 1)}{x^2 - 25}\) 2b. \(\frac{x^2 - 11 + 3x + 1}{x^2 - 25}\) 2c. \(\frac{x^2 + 3x - 10}{x^2 - 25}\) 2d. \(\frac{(x + 5)(x - 2)}{(x + 5)(x - 5)}\) 2e. \(\frac{x - 2}{x + 5}\)

3. **Step 1:** Factor each denominator completely. When factoring, write the factored form using powers.

**Step 2:** The LCD is the product of each prime factor the greatest number of times it appears in any factorization.

**Step 1:** Find the LCD. **Step 2:** Rewrite each rational expression as an equivalent rational expression with the common denominator. You will need to multiply out the numerator, but leave the denominator in factored form. **Step 3:** Add or subtract the rational expressions found in Step 2.

**Step 4:** Simplify the result. 5a. \((x - 2)(x + 2)\) 5b. \(\frac{3}{x + 2} = \frac{3x - 6}{(x + 2)(x - 2)}; \frac{8 - 2x}{(x + 2)(x - 2)}\) 5c. \(\frac{1}{x - 2}\)

6a. \((a + 10)(a + 2)(a - 2)\) **continued next page**
Guided Practice

6b. \( \frac{a}{a+10}(a+2) = \frac{a^2-2a}{(a+10)(a+2)(a-2)} \);

6c. \( \frac{a^2-3a-2}{(a+10)(a+2)(a-2)} \);

7a. \( x^2(x-1)^2 \);

7b. \( \frac{x}{x^2(x-1)^2} \);

7c. \( \frac{x^3-3x^2+2x-1}{x^2(x-1)^2} \);

Do the Math

1. \( \frac{5x+2}{x-3} \)
2. \( \frac{2}{x-5} \)
3. \( \frac{3x-2}{x-6} \)
4. \( 2x+5 \)
5. \( 24a^3b^2 \)
6. \( (m+6)(m-3)(m-4) \)
7. \( x^2(x-3)(x+3) \)
8. \( \frac{2x+15}{9x^2} \)
9. \( \frac{4(x+2)}{(x-3)(x+1)} \)
10. \( \frac{z-7}{z-4} \)
11. \( \frac{2x^2-5x-1}{(x-1)(x+1)(x+3)} \)
12. \( \frac{2m^2-4mn+4n^2}{(m+2n)^2(m-3n)} \)
13. \( \frac{1}{(x+1)(x+2)} \)
14. \( \frac{15}{m(m-3)} \)
15. \( \frac{2(x-2)}{x(x+2)^2} \)
16. \( \frac{4(x-2)}{x-4} \)

18a. \( \frac{8x+1}{(x+2)(x-1)} \)
18b. \( \{x| x \neq -2, x \neq 1\} \)
19a. \( S(r) = \frac{2 \pi r^3 + 400}{r} \)
19b. \( S(4) \approx 200.53 \text{ sq cm} \)

Section 5.3

Five-Minute Warm-Up

1. \( (4x+3)(2x-1) \)
2. \( \frac{4p^3}{5} \)
3. \( \frac{9y}{20x^3} \)
4. \( 16r^{12} \)
5. \( \frac{1}{y^2 z^5} \)
6. \( \frac{4}{9} \)
7. \( -\frac{7}{8} \)

Guided Practice

1. When sums and/or differences of rational expressions occur in the numerator or denominator of a quotient, the quotient is called a complex rational expression. Rational expressions with more than one fraction bar are complex and need to be simplified.

2a. \( \frac{x-1}{x-1} \)
2b. \( \frac{x-1+1}{x-1} = \frac{x}{x-1} \)

2c. \( \frac{x+1}{x} \)
2d. \( \frac{x-1}{x} \)
2e. \( \frac{x-1}{x+1} \)
3a. Multiplicative Inverse (a number multiplied by its reciprocal is one) or alternately, any quotient with the same numerator and denominator is a representation of one whole.
3b. Multiplicative Identity
3c. Distributive Property

4a. \( (x-4)(x+4) \)
4b. \( \left[ \frac{x^3}{(x-4)(x+4)} - \frac{x}{(x+4)} \right] = \left[ \frac{(x-4)(x+4)}{(x-4)(x+4)} \right] \)
4c. \( \left[ \frac{x^2(x-4)(x+4)}{x} - \frac{1}{x} \right] \)
4d. \( \frac{x^2-x(x-4)}{x-(x+4)} \)

4e. \( \frac{x^2-x^2+4x}{x-x-4} = \frac{4x}{-4} = -x \)
5a. \( \frac{1}{x} + \frac{1}{3} = \frac{1}{3} \)
5b. \( \frac{3x}{3-x} \) or \( -\frac{3x}{x-3} \)

Do the Math

1. \( \frac{x^2+1}{(x+1)(x-1)} \)
2. \( \frac{7x+9w}{9x-7w} \)
3. \( \frac{a}{(a-3)(a+1)} \)
4. \( \frac{1}{(x-2)(x-1)} \)
5. \( \frac{-2(x-2)}{(x-1)(x+1)} \)
6. \( \frac{1}{z+4} \)
7. \( n^2 + nm + m^2 \)
8. \( \frac{x+4}{x} \)
9. \( -x \)
10. \( \frac{x-3}{x+3} \)
11. \( \frac{6}{x+5} \)
12. \( \frac{2}{x-y} \)
13. \( -\frac{xy}{(x-y)^2} \)
14. \( \frac{b^2-2ab+4a^2}{ab(b-2a)} \)
15a. \( \frac{R_1R_2R_3}{R_2R_3 + R_1R_3 + R_1R_2} \)
15b. \( \frac{60}{31} \approx 1.94 \text{ ohms} \)
Section 5.4

Five-Minute Warm-Up
1. \( \{4\} \) 2. \( \{-4, 6\} \) 3. \( (3z - 2)(z - 4) \) 4. (b) only 5. \( \{15, -3\} \) 6. \((-2, 12)\)

Guided Practice
1. the denominator equal to zero
2a. \( x = 7, x = -7 \) 2b. \( \{x \mid x \neq 7, x \neq -7\} \)
2c. \( (x - 7)(x + 7) \) 2d. \( (x - 7)(x + 7) \frac{x + 5}{x - 7} = \frac{x - 3}{x + 7}(x - 7) \) 2e. \( (x + 7)(x + 5) = (x - 3)(x - 7) \)
2f. \( x^2 + 12x + 35 = x^2 - 10x + 21 \) 2g. \( x = -\frac{7}{11} \) 2h. \( \left\{-\frac{7}{11}\right\} \) 3a. \( \{x \mid x \neq 0\} \) 3b. \( 12x^2 \)
3c. \( x^3 + 6x^2 = 4x + 24 \) 3d. \( \{2, -2, -6\} \) 4a. \( \{x \mid x \neq 1, x \neq -1\} \) 4b. \( (x - 1)(x + 1) \) 4c. \( x = -1 \) 4d. \( \emptyset \)
5a. \( \{x \mid x \neq 4, x \neq 2\} \) 5b. \( (x - 4)(x - 2) \) 5c. \( x = 5 \) or \( x = 4 \) 5d. \( \{5\} \) 5e. \( x + \frac{7}{x} = 8 \) 5f. \( \{x \mid x \neq 0\} \)
6a. \( x = 7 \) or \( x = 1 \) 6d. \( \{7, 1\} \) 6f. \( \{7, 8\} \) and \( (1, 8) \)

Do the Math
1. \( \{6\} \) 2. \( \emptyset \) 3. \( \{2, 4\} \) 4. \( \left\{-\frac{1}{2}, \frac{3}{4}\right\} \) 5. \( \left\{-\frac{3}{2}, 4\right\} \) 6. \( \{0, 8\} \) 7. \( \{2\} \) 8. \( \emptyset \) 9. \( \{5\} \)

Section 5.5

Five-Minute Warm-Up
1. \( \{-3, 2\} \) 2. \( \{x \mid x < -1\} \) See Graphing Answer Section 3. \( \left[-\frac{5}{3}, \infty\right) \) 4. yes

Guided Practice
1a. \( 0; 0 \) 1b. \( 0; 0 \) 2. positive; negative 3. positive; negative; negative; positive 4. denominator equal to zero 5a. \( x = -2 \) 5b. \( x = 4 \) 5c. \( -3 \) (answers may vary) 5d. negative 5e. negative 5f. positive 5g. 0 (answers may vary) 5h. positive 5i. negative 5j. negative 5k. 5 (answers may vary) 5l. positive 5m. positive 5n. positive 5o. negative 5p. \( x = 4 \)
5q. See Graphing Answer Section 6. \( \frac{-2x + 12}{(x - 1)(x + 1)} > 0 \) 7. \( \frac{x - 11}{x + 2} < 0 \) 8. See Graphing Answer Section 9. See Graphing Answer Section

Do the Math
1. \( (-\infty, -8) \cup (-2, \infty) \) 2. \( (-\infty, -12) \cup (2, \infty) \) 3. \( (-5, 10) \) 4. \( (-\infty, -4) \cup \left[\frac{2}{5}, 5\right) \)
5. \( \left[-1, \frac{2}{3}\right) \cup [6, \infty) \) 6. \( (4, \infty) \) 7. \( (-2, 11] \) 8. \( (-\infty, -6) \cup (-5, \infty) \) 9. \( (-\infty, -3) \cup \left[-\frac{3}{2}, 0\right) \)
10. \( (-\infty, -\frac{1}{2}) \cup (4, 13] \) 11. \( (-\infty, -3] \cup (8, \infty) \) 12. \( \left[-\frac{2}{3}, 4\right) \) 13. \( (-4, 0) \) and \( \left[\frac{5}{3}, 0\right) \) 14. \( (-3, 0) \)
15. 125 or more bicycles

Section 5.6

Five-Minute Warm-Up
1. \( x = -\frac{3}{4}y - 6 \) 2. \( y = -\frac{9}{2}x + 18 \) 3. 65 mph

Guided Practice
1. \( y = \frac{xz}{2z - 6x} \) 2. \( \frac{3}{x} \) Answers may vary 3. \( \frac{3}{x} = \frac{x + 2}{9} \) Answers may vary 4. \( \left\{\frac{13}{3}\right\} \)
5. The figures are the same shape, but they are a different size. The corresponding angles are equal and the corresponding sides are proportional. 6a. \( \frac{6}{3.2} = \frac{x}{8} \) 6b. 15 ft 7a. \( \frac{1}{12} \) 7b. \( \frac{1}{x} \) 7c. \( \frac{1}{t + 2} \) 8a. \( \frac{1}{3} \) 8b. \( \frac{1}{5} \)
8c. \( \frac{1}{t} \) 8d. \( \frac{1}{3} + \frac{1}{5} = \frac{1}{t} \) 8e. It takes 1.875 hours to clean the building when Josh and Ken work together.
Guided Practice 9a. $180 + w$; where $w$ is the speed of the wind 9b. $180 - w$ 9c. $\frac{1000}{180 + w}$

9d. $\frac{600}{180 - w}$ 9e. $\frac{1000}{180 + w} = \frac{600}{180 - w}$ 9f. The speed of the wind is 45 mph.

Do the Math 1. $V_2 = \frac{V_1 P_1}{P_2}$ 2. $r = \frac{A}{P} - 1$ 3. $x_i = \frac{y_i - y + mx}{m}$ 4. $m_i = \frac{m v_2}{2 v_1 - v_2}$

5. about 21.02 million flight hr 6. approximately 5.83 hr 7. 15 hours 8. 3 minutes; $\frac{1}{2}$ mile 9. 6 hours

10. 135 miles per hour 11. 10 miles per hour

Section 5.7

Five-Minute Warm-Up 1a. $\{ -5 \}$ 1b. $\left\{ -\frac{4}{9} \right\}$ 2. $\{-48\}$ 3 – 4. See Graphing Answer Section

Guided Practice 1. $y = kx$ 2. constant of proportionality 3. linear; zero 4. $y = -\frac{25}{3}$ 5. $y = \frac{k}{x}$

6. $y = \frac{8}{3}$ 7. $r = 42$ 8a. $V = \frac{k - T}{P}$ 8b. $T = 300; P = 15; V = 100$ 8c. $k = 5$ 8d. $V = \frac{5 - T}{P}$ 8e. 22.5 Atm

Do the Math 1a. $k = 5$ 1b. $y = 5x$ 1c. $y = 25$ 2a. $k = \frac{1}{5}$ 2b. $y = \frac{1}{5}x$ 2c. $y = 7$ 3. $B = 80$

4. $r = 64$ 5. $1225.62$ 6. $80.50$ 7. 96 feet per second 8a. $k = 80$ 8b. $y = \frac{80}{x}$ 8c. $y = \frac{16}{7}$ 9a. $k = \frac{1}{3}$

9b. $y = \frac{1}{3}x$ 9c. $y = 40$ 10a. $k = \frac{21}{10}$ 10b. $Q = \frac{21x}{10y}$ 10c. $Q = \frac{28}{5}$ 11. 24 amps 12. $\approx 0.033$ foot-candles

Chapter 6 Answers

Section 6.1

Five-Minute Warm-Up 1. 0.5 2. $\frac{4}{9}$ 3. $\left| 4x + 3 \right|$ 4. $\left| x - y \right|$ 5. $\frac{1}{49}$ 6. $x^2$ 7. $\frac{9x^2}{y^4}$ 8. $8b^{12}$

Guided Practice 1. index; radicand; cube root 2. $\geq 0$; any real number 3. principal root 4a. $-3$

4b. not a real number 4c. $\frac{2}{3}$ 5. $\approx 2.83$ 6. $\left| 2x - 1 \right|$ 7a. $\sqrt{144} = 12$ 7b. $\sqrt{-64}$ not a real number

7c. $2\sqrt{x}$ 8. $\sqrt[3]{a^m} \cdot \left( \sqrt[3]{a} \right)^w$ 9a. $216$ 9b. $-1024$ 9c. $16$ 10. $\approx 8.55$ 11a. $(2x)^3$ 11b. $(2x^2y)^{\frac{3}{2}}$

12a. $\frac{1}{64^2} = \frac{1}{8}$ 12b. $2 \cdot 16^{\frac{2}{3}} = 2\sqrt[3]{16} = 8$ 12c. $\frac{1}{(4x)^2} = \frac{1}{32x^2}$

Do the Math 1. 6 2. $-4$ 3. $-4$ 4. $\frac{2}{5}$ 5. 6 6. $n$ 7. $\left| 2x - 3 \right|$ 8. 4 9. $-5$ 10. $-3$

11. not a real number 12. $-100,000$ 13. 8 14. $\frac{1}{11}$ 15. 343 16. $\frac{1}{81}$ 17. $x^\frac{3}{4}$ 18. $(3x)^\frac{2}{3}$

19. $(3pq)^\frac{7}{4}$ 20. 4.40 21. 3.16 22. 1000 23. not a real number 24. $\frac{1}{5}$ 25. 26. $-\frac{1}{5}$ 27. $(-9)^{\frac{1}{2}} = -\sqrt{-9}$

but there is no real number whose square is $-9$. However, $-9^{\frac{1}{2}} = -1 \cdot 9^{\frac{1}{2}} = -\sqrt{9} = -3$.

Section 6.2

Five-Minute Warm-Up 1. $54z^7$ 2. $\frac{3u^2}{2}$ or $\frac{3}{2}u^2$ 3. $\frac{1}{625}$ 4. $\frac{27}{64}$ 5. $\frac{3x^6}{y}$ 6. $\frac{1}{25p^{10}}$ 7. $\frac{64b^{10}}{a^7}$

8. $\frac{1}{27xy^7}$ 9. $\frac{9}{7}$
Guided Practice  1. All exponents are positive; each base only occurs once; there are no parentheses in the expression; there are no powers written to powers.  
2a. 36  
2b. \(-2a^{\frac{1}{7}}\)  
3. \(\frac{x^{\frac{11}{3}}}{y^{\frac{15}{8}}}\)  
4a. 9  
4b. \(2x^2\left|y\right|\sqrt[3]{8}\)
4c. \(\sqrt[4]{x}\)  
4d. \(\sqrt[4]{p}\)  
5. \(2x^3\left(3x^3 + 5x - 2\right)\)

Do the Math  1. 9  
2. 10  
3. \(\frac{1}{7}y^{10}\)  
4. \(\frac{2}{9}\)  
5. \(a^\frac{1}{2}b^\frac{3}{5}\)  
6. \(\frac{b^2}{a^\frac{3}{2}}\)  
7. \(\frac{5p^5}{q^\frac{5}{2}}\)  
8. \(\frac{8m^4}{n^6}\)  
9. \(\frac{x^3}{y^9}\)
10. \(x^2 + 4x^\frac{1}{2}\)  
11. \(\frac{2}{a^\frac{3}{2}}\)  
12. \(8p^2 - 32\)  
13. \(x^2\)  
14. \(25\)  
15. \(5x^2y^3\)  
16. \(\sqrt[3]{p^{13}}\)  
17. \(\sqrt[5]{7}\)
18. \(3(x - 5)^\frac{1}{2}(5x - 9)\)  
19. \(\frac{2(6x + 1)}{x^3}\)  
20. \(\frac{6x + 13}{(x + 3)^2}\)

Section 6.3

Five-Minute Warm-Up  1. 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144  
2. 1, 8, 27, 64, 125  
3. 1, 16, 81
4. not a real number  
5. 10  
6. \(\left| x \right|\)  
7. \(\left| 4x + 1 \right|\)  
8. \(\left| 2p - 1 \right|\)  
9. 13

Guided Practice  1a. \(\sqrt[3]{35}\)  
1b. \(\sqrt[2]{x^2 - 25}\)  
1c. \(\sqrt[14]{x^2}\)  
2. The radicand does not contain factors that are perfect powers of the index. For square roots, there can be no factor with exponent two or higher in the radicand. For cube roots, there can be no factor with exponent three or higher in the radicand. 3a. 25
3b. \(\sqrt[5]{25} \cdot \sqrt[3]{3}\)  
3c. \(5\sqrt[3]{3}\)  
4a. 16 \cdot 3  
4b. \(\sqrt[6]{16} \cdot \sqrt[3]{3} = \frac{\sqrt[6]{16} \cdot \sqrt[3]{3}}{2}\)  
4c. \(-6 + 4\sqrt[3]{3}\)  
4d. \(\frac{2(-3 + 2\sqrt[3]{3})}{2}\)
5. Rewrite the variable expression as the product of two expressions, where one of the factors has an exponent that is a multiple of the index. 6. \(\frac{2x\sqrt[3]{3}}{11}\)  
7. \(\frac{6}{a^2}\)  
8a. Rewrite \(\sqrt[3]{6} \cdot \sqrt[4]{48}\) as \(6\sqrt[3]{3} \cdot 4\sqrt[4]{3}\)  
8b. 4
8c. \((6^3 \cdot 48)^\frac{1}{3}\)  
8d. \((6^3 \cdot 48)^\frac{1}{3}\)  
8e. 1728\(^{\frac{1}{3}}\)  
8f. \(\sqrt[2]{6} \cdot 3^3 = 2\sqrt[10]{108}\)

Do the Math  1. \(\frac{3}{4}a^2b^2\)  
2. \(\sqrt[2]{p^2 - 25}\)  
3. \(\sqrt[3]{-3x}\)  
4. \(\sqrt[4]{2}\)  
5. \(2a(\sqrt[5]{x})\)  
6. \(-4p\)  
7. \(12\sqrt[3]{b}\)
8. \(s^4\sqrt[5]{s}\)  
9. \(x^2\sqrt[3]{x^5}\)  
10. \(-3q^4\sqrt[2]{2}\)  
11. \(\sqrt[5]{x^3} \sqrt[3]{y}\)  
12. \(-1\)  
13. \(-\frac{3}{4} + 2\sqrt[3]{3}\)  
14. 6  
15. \(6x\sqrt[5]{x}\)  
16. \(3a\sqrt[2]{2}\)
17. \(6m^3\sqrt[3]{2mn^2}\)  
18. \(\frac{\sqrt[5]{5}}{6}\)  
19. \(\frac{\sqrt[5]{5}}{2}\)  
20. \(-\frac{3x^3}{4y^4}\)  
21. 2  
22. \(3y^2\sqrt[3]{2}\)  
23. \(-4x^3\)  
24. \(\sqrt[4]{392}\)  
25. \(\sqrt[5]{45}\)

Section 6.4

Five-Minute Warm-Up  1. \(3z^3 - 5z^2 - 4z\)  
2. \(-3a^2b^2 - a^2 + 6ab - b^2\)  
3. \(-8x^4 - 12x^3y + 20x^2y^2\)
4. \(\frac{2}{9}x^5 - \frac{4}{7}x^4 + x^3\)  
5. \(18x^2 - 23c - 6\)  
6. \(a^2b^2 - 4\)  
7. \(49n^2 - 42n + 9\)

Guided Practice  1. Like radicals have the same index and the same radicand. 2a. \(9\sqrt{x}\)  
2b. \(8\sqrt[3]{3}p\)
3a. \(-10\sqrt{2}\)  
3b. \(-n^4\sqrt[6]{6} + 4n^4\sqrt[6]{6n}\)  
4. \(18 + 4\sqrt[6]{6} - 27\sqrt[2]{2} - 12\sqrt[3]{3}\)  
5a. \((A + B)(A - B) = A^2 + B^2\)
5b. \(-49 + 3 = 46\)  
6. 72 square units

Do the Math  1. \(14\sqrt{3}\)  
2. \(7\sqrt[2]{2}\)  
3. \(11\sqrt[5]{5} - 11\sqrt[5]{5}\)  
4. \(8\sqrt[3]{3}\)  
5. \(2\sqrt[3]{3}\)  
6. \(3\sqrt[3]{2}\)  
7. \(13\sqrt[5]{7}\)  
8. \(2\sqrt[3]{3}\)
9. \(3(2x - 1)^\frac{3}{2}\)  
10. \(5\sqrt[3]{5} + 3\sqrt[3]{15}\)  
11. \(\sqrt[3]{12} + 2\sqrt[3]{2}\)  
12. \(15 + 3\sqrt[5]{5} + 5\sqrt[6]{+\sqrt[3]{30}}\)  
13. \(-141 - 22\sqrt[10]{10}\)
14. \(-2\sqrt{3}\)  
15. \(7 - 4\sqrt[3]{3}\)  
16. 2  
17. 18  
18. \(\frac{1}{2}y^2 - 3\sqrt[3]{y} - 18\)  
19. \(-2\sqrt{3}\)  
20. \(z + 2\sqrt[5]{5z} + 5\)  
21. \(8 - 4\sqrt[15]{15}\)
**Do the Math**  22. \(19 + 2x + 8\sqrt{2x + 3}\)  23. 3a  24. 9 – \(4\sqrt{14}\)  25. \(\frac{2\sqrt{5}}{25}\)

### Section 6.5

**Five-Minute Warm-Up**  1. 3  2. 11a  3. \(\frac{3\sqrt{2}}{2}\)  4. 1  5. \(24 - \sqrt{15}\)

**Guided Practice** 1. The process of rationalizing the denominator of a quotient requires rewriting the quotient, using properties of rational expressions, so that the denominator of the equivalent expression does not contain any radicals. 2. a perfect square 3a. \(\frac{\sqrt{3}}{3}\)  3b. \(\frac{\sqrt{5}}{\sqrt{5}}\)  3c. \(\frac{\sqrt{2a}}{2a}\)  4a. \(\frac{\sqrt[4]{25}}{25}\)  4b. \(\frac{\sqrt[4]{18}}{\sqrt[4]{18}}\)  4c. \(\frac{\sqrt[3]{3x^2y^3}}{\sqrt[3]{3x^2y^3}}\)

5. conjugate 6a. \(3 - \sqrt{5}\)  6b. \(2\sqrt{3} + 5\sqrt{2}; -38\)  7. \(\frac{\sqrt{2} + 3}{\sqrt{2} + 3}\)  8. \(-4(\sqrt{2} + \sqrt{5})\)

**Do the Math**  1. \(\frac{2\sqrt{3}}{3}\)  2. \(-\frac{\sqrt{3}}{2}\)  3. \(\frac{\sqrt{5}}{\sqrt{5}}\)  4. \(\frac{\sqrt{33}}{11}\)  5. \(\frac{\sqrt{5z}}{z}\)  6. \(\frac{4\sqrt{2a}}{a^3}\)  7. \(-\frac{\sqrt[3]{4p^2}}{p}\)  8. \(-\frac{\sqrt[3]{15}}{6}\)

9. \(\frac{\sqrt[3]{6z}}{3z}\)  10. \(\frac{2\sqrt[4]{9b^2}}{b} = \frac{2\sqrt[3]{3b}}{b}\)  11. \(2(\sqrt[7]{7} + 2)\)  12. \(10(\sqrt[3]{10} - 3)\)  13. \(\frac{\sqrt[5]{5} + \frac{3}{2}}{3}\)  14. \(2 + \sqrt{3}\)  15. \(\frac{4\sqrt{3}}{5}\)

16. \(\frac{11\sqrt{5}}{23}\)  17. \(\frac{\sqrt{10} - 5\sqrt{5}}{5}\)  18. \(\sqrt{3}\)  19. \(\frac{1}{3}\)  20. \(\frac{3\sqrt{5}}{5}\)  21. \(\frac{10\sqrt{2} - 27}{23}\)  22. \(\frac{4 - \sqrt{6}}{2}\)  23. 5

24. \(\frac{a - b}{2(2 - \sqrt{3})}\)  25. \(\frac{a - b}{\sqrt{2a} + \sqrt{2b}}\)

### Section 6.6

**Five-Minute Warm-Up**  1. 12  2. \(n\)  3. \(\{x | x \leq -3\}\)  4. -40  5. See Graphing Answer Section

**Guided Practice** 1a. \(4\sqrt{5}\)  1b. \(\sqrt[4]{9}\)  1c. \(\sqrt{2}\)  2. \(\geq 0\)  3. any real number 4a. \(\{x | x \geq \frac{3}{2}\}\) or \(\left[ \frac{3}{2}, \infty \right)\)

4b. \(\{x | x \text{ is any real number}\}\) or \((-\infty, \infty)\)  4c. \(\{t | t \leq 2\}\) or \((-\infty, 2]\)  5a. \([-3, \infty)\)  5b. See Graphing Answer Section  5c. \([0, \infty)\)  6a. \((-\infty, \infty)\)  6b. See Graphing Answer Section  6c. \((-\infty, \infty)\)

**Do the Math**  1a. 4  1b. \(2\sqrt{3}\)  1c. 2  2a. 1  2b. -3  2c. \(2\sqrt{2}\)  3a. \(\frac{1}{3}\)  3b. \(\frac{\sqrt{3}}{3}\)  3c. \(\frac{\sqrt{5}}{2}\)  4. \([-4, \infty)\)

5. \(\left[ -\infty, \frac{5}{2} \right]\)  6. \((-\infty, \infty)\)  7. \(\left[ \frac{2}{3}, \infty \right)\)  8. \((3, \infty)\)  9a. \([1, \infty)\)  9b. \([0, \infty)\)  10a. \((-\infty, 4]\)  10b. \([0, \infty)\)

11a. \([0, \infty)\)  11b. \([1, \infty)\)  12a. \((-\infty, \infty)\)  12b. \((-\infty, \infty)\)  9 – 12(c). See Graphing Answer Section

### Section 6.7

**Five-Minute Warm-Up**  1. \{-3\}  2. \{1, 2\}  3. \(2x\)  4. \(2x + 10\sqrt{2x + 25}\)  5. \(3x + 8\)  6. \(4\sqrt{3}\)

**Guided Practice** 1a. \(\sqrt{4x + 1} = 5\)  1b. \((\sqrt{4x + 1})^2 = 5^2\)  1c. \(4x + 1 = 25\)  1d. \(x = 6\)  1e. yes  1f. \(\{6\}\)

2. Extraneous solutions are apparent solutions; they are algebraically correct but do not satisfy the original equation. 3. Isolate the radical on one side of the equation. If there are two radicals, one radical should be on the left side and the other radical on the right side of the equation. 4a. \(x = 1\)  4b. no  4c. \(\emptyset\)  4d. The square root function is equal to a negative number. 5a. \(x = 4\) or \(x = -1\)  5b. \(x = 4, \text{ yes} ; x = -1, \text{ no}\)

5c. \(\{4\}\)  6a. \((3x + 1)^3 = 8\)  6b. \(\frac{2}{3}\)  6c. \(x = 1\)  6d. \(\{1\}\)  7a. \(\sqrt{2x^2 - 5x - 20} = \sqrt{x^2 - 3x + 15}\)

7b. \(2x^2 - 5x - 20 = x^2 - 3x + 15\)  7c. \(x = 7, x = -5\)  7d. yes  7e. \(\{7, -5\}\)

**Do the Math**  1. \{14\}  2. \(\emptyset\)  3. \{3\}  4. \{49\}  5. \{13\}  6. \{9\}  7. \{4\}  8. \{-2\}  9. \{6\}  10. \{-6, 3\}

11. \{1, 5\}  12. \{12\}  13. \{-6\}  14. \{3\}  15. \{5\}  16. \{9\}  17. \{12\}  18. \{9\}  19. \(a = \frac{\sqrt{r}}{r}\)  20. \(S = 4\pi r^2\)

Section 6.8

Do the Math 21. \( U = \frac{CV^2}{2} \)  

22. $1102.50

Section 7.1

Chapter 7 Answers
Section 7.2

Five-Minute Warm-Up  1. $5 \sqrt{5}$  2. $\frac{5}{2} + \sqrt{2}$  3. $2 \sqrt{7} i$  4. $1 - \frac{2}{3}i$  5. $3x^2 - 9x + 1$  6. $2\sqrt{10}$

Guided Practice  1. $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$  2. standard form  3a. $2x^2 - x - 4 = 0$; $a = 2, b = -1, c = -4$
  3b. $3x^2 + 6 = 0$; $a = 3, b = 0, c = 6$  3c. $3x^2 - 6x = 0$; $a = 3, b = -6, c = 0$  4a. $8; -2; -3$
  4b. $n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$  4c. $n = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(8)(-3)}}{2(8)}$
  4d. $100$  4e. $2 \pm \sqrt{10} = \frac{1 \pm 5}{8}$  4f. $\frac{1 + 5}{8}, \frac{1 - 5}{8}$
  4g. $\left\{\frac{3}{4}, -\frac{1}{2}\right\}$  5. $b^2 - 4ac$  6a. $169$; two rational  6b. $-20$; two complex, not real  6c. $0$; one repeated real

7a. $200$  7b. $200 = -16r^2 + 150t + 2$  7c. $\{1.6, 7.8\}$  7d. The rocket will be at a height of 200 feet at two different times. While it is going up, 1.6 seconds after launch, and again when it comes down, at 7.8 seconds.
  7e. no  7f. At 9.4 seconds the rocket will hit the ground ($h = 0$).

Do the Math  1. $\{-4, 8\}$  2. $\left\{-\frac{1}{2}, \frac{2}{3}\right\}$  3. $\left\{1 \pm \frac{\sqrt{2}}{2}\right\}$  4. $\left\{\frac{3}{2} \pm \frac{\sqrt{5}}{2}\right\}$  5. $\left\{1 \pm \frac{\sqrt{10}}{2}i\right\}$  6. $\left\{-\frac{3}{5} \pm \frac{\sqrt{14}}{5}\right\}$

7. $24$, irrational  8. $0$, one repeated real  9. $-95$, two complex, not real  10. $\left\{\frac{7}{2}, \frac{\sqrt{21}}{2}\right\}$
  11. $\left\{-2, \frac{1}{3}\right\}$

12. $\left\{\frac{2}{5} \pm \frac{\sqrt{29}}{5}\right\}$  13. $\left\{\frac{5}{2}\right\}$  14. $\left\{1 + \frac{\sqrt{6}}{2}\right\}$  15. $\left\{\frac{1}{2} \pm \frac{\sqrt{19}}{2}i\right\}$
  16a. $x = -4$ or $x = 2$  16b. $(-2, -8); (0, -8)$

17. width $\approx 5.307$ in.; length $\approx 11.307$ in.  18. base $\approx 9.426$ in.; height $\approx 7.426$ in.  19. $\approx 49.0$ mph

20. Let $x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$; $x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$. $x_1 + x_2 = -\frac{b}{a}$; $x_1 \cdot x_2 = \frac{b^2 - (b^2 - 4ac)}{2a \cdot 2a} = \frac{4ac}{4a^2} = c$

Section 7.3

Five-Minute Warm-Up  1. $(a - 3)(a + 3)(a^2 + 2)$  2. $(2x - 5)(3x - 5)$  3. $\frac{25}{x^2}$  4. $\frac{4}{9}x^6$  5. $\left\{\frac{3}{2}, -2\right\}$

Guided Practice  1a. $\sqrt{y}$; $2u^2 - 11u + 15 = 0$  1b. $1v^2 + u^2 + 10u + 1 = 0$  1c. $(x - 1); u^2 - 5u + 6 = 0$

1d. $x^2 ; u^2 - u - 3 = 0$  1e. $x^2$ or $\frac{1}{x}$; $4u^2 + u - 3 = 0$  2a. $x^2$  2b. $u^2 - 6u - 16 = 0$  2c. $(u - 8)(u + 2) = 0$

2d. $u = 8$ or $u = -2$  2e. $x^2 = 8$ or $x^2 = -2$  2f. $\sqrt{x^2} = \pm \sqrt{8}$ or $\sqrt{x^2} = \pm \sqrt{-2}$

2g. $x = \pm \sqrt{2}$ or $x = \pm \sqrt{2}i$  2h. $\left\{2\sqrt{2}, -2\sqrt{2}, \sqrt{2}i, -\sqrt{2}i\right\}$

3a. $(x^2 + 3)$  3b. $\{1, -1, i, -i\}$  4a. $\sqrt{x}$

4b. $\{4\}$  5a. $n^2$  5b. $\{-1, 27\}$  6a. For $\sqrt{x}, x \geq 0$.  6b. For $\frac{1}{x}, x \neq 0$.

Do the Math  1. $\{-3, -1, 1, 3\}$  2. $\{-1, -\frac{1}{2}, \frac{1}{2}, 1\}$  3. $\{3, -4\}$  4. $\{36\}$  5. $\emptyset$  6. $\left\{-\frac{1}{5}, \frac{1}{3}\right\}$

7. $\left\{-\frac{2}{5}, 5\right\}$  8. $\{-1, 27\}$  9. $\left\{\frac{1}{3}, \frac{1}{4}\right\}$

10. $\{-1, 2, -1 \pm \sqrt{3}i, \frac{1}{2} \pm \frac{\sqrt{3}}{2}i\}$  11. $\{2, -3\}$  12. $\{-2i, -1, 1, 2i\}$

13. $\{81\}$  14. $\left\{\frac{7}{8}, 2\right\}$  15. $0, \sqrt{5}i, -\sqrt{5}i$  16. $-\sqrt{7}i, \sqrt{7}i, -\sqrt{2}, \sqrt{2}$

17. $0, -\sqrt{3}, \sqrt{3}$  18. $-\sqrt{2}i, \sqrt{2}i, -\sqrt{5}, \sqrt{5}$  19. $-\sqrt{6}, \sqrt{6}, -\sqrt{7}, \sqrt{7}$

20. $\frac{49}{4}$

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Section 7.4

Five-Minute Warm-Up 1 – 2. See Graphing Answer Section 3. –29 4. 33 5. \( \{ x | x \text{ is any real number} \} \)

Guided Practice 1. (b) 2. (a) 3a. vertically 3b. 2 3c. down 4a. horizontally 4b. 1 4c. right 5a. up
5b. down 6a. \( a > 1 \) 6b. \( 0 < |a| < 1 \) 7a. \( f(x) = (3x^2 - 12x) + 7 \) 7b. \( f(x) = 3(x^2 - 4x + ___) + 7 + ___ \)
7c. 4 7d. Add the additive inverse of \((3)(4)\) or \(-12\) 7e. \( f(x) = 3(x^2 - 4x + 4) + 7 + (-12) \)
7f. \( f(x) = 3(x - 2)^2 - 5 \) 7g. (2, -5) 7h. up 7i. \( x = 2 \)

Do the Math 1 – 6. See Graphing Answer Section 7. \( h(x) = \left( x - \frac{7}{2} \right)^2 - \frac{9}{4} ; V \left( \frac{7}{2}, -\frac{9}{4} \right) ; x = \frac{7}{2} \)
8. \( f(x) = 3(x + 3)^2 - 2 ; V(-3, -2) ; x = -3 \) 9. \( g(x) = -(x + 4)^2 + 2 ; V(-4, 2) ; x = -4 \)

Section 7.5

Five-Minute Warm-Up 1. \( x \)-intercept: \((-4, 0)\); \(y\)-intercept: \((0, 3)\) 2. \( \left\{ 8, -\frac{3}{2} \right\} \) 3. \((6, 0)\) and \((-1, 0)\) 4. \(-12\)

Guided Practice 1. \( \frac{b}{2a} \) 2. setting \( f(x) = 0 \) and solving for \( x \) 3a. 1; 2; \(-8\) 3b. up 3c. \( x = -1 \)
3d. \( y = -9 \) 3e. \((-1, -9)\) 3f. \( x = -1 \) 3g. \(-8\) 3h. \( 36 \) 3i. 2 3j. \((-4, 0)\); \((2, 0)\) 3k. See Graphing
Answer Section 4. \( y \)-coordinate 5. maximum 6. minimum 7a. \( x \cdot y \) 7b. \( 2x + y = 50 \)
7c. \( y = 50 - 2x \) 7d. \( A = x(50 - 2x) \) or \( A = -2x^2 + 50x \) 7e. \( x = 12.5 \) 7f. \( y = 25 \)
7g. The pen has two sides of 12.5 ft and one side 25 ft. 7h. 312.5 ft²

Do the Math 1. 2; \( \left[ \left(-\frac{1}{2}, 0 \right) \right] \) and \((4, 0)\) 2. \((1, 3)\) 3. \((2, -0.28, 0)\) and \((1.78, 0)\) 4 – 9. See Graphing
Answer Section 10. maximum; 11 11. minimum; \(-6\) 12. \( 25 \) and \( 25 \) 13. \(-5\) and \( 5 \) 14a. \( \approx 4.84 \) seconds
14b. \( \approx 383.39 \) feet 14c. \( \approx 9.74 \) seconds

Section 7.6

Five-Minute Warm-Up 1. \([-4, -2)\) 2. \((-3, 1]\) 3. \((-2, \infty)\) 4. \((-\infty, 5]\) 5. \( \{ x | x \geq 2 \} \)
6. \( \{ 1 + \sqrt{6}, 1 - \sqrt{6} \} \)

Guided Practice 1a. above 1b. below 2a. graphical 2b. algebraic 3a. \( f(x) = x^2 + 2x - 3 \)
3b. \((-3, 0)\); \((1, 0)\) 3c. \((-1, -4)\) 3d. See Graphing Answer Section 3e. negative 3f. \([-3, 1] \)
4a. positive 4b. negative 5a. \((x + 7)(x - 5) = 0 \) 5b. \( x = -7 ; x = 5 \) 5c. \((-\infty, -7) ; (-7, 5) ; (5, \infty) \)
5d. See Graphing Answer Section 5e. positive 5f. no 5g. \((-\infty, -7) \cup (5, \infty) \)

Do the Math 1. \([-1, 8)\) 2. \((-\infty, 4) \cup (10, \infty)\) 3. \((-4, -1)\) 4. \( \left(-\frac{7}{2}, 1 \right) \) 5. \((-\infty, -2) \cup (3, \infty) \)
6. \( \left(-\infty, 3 - \frac{\sqrt{29}}{2} \right] \cup \left[ 3 + \frac{\sqrt{29}}{2}, \infty \right) \) 7. \( (-\infty, -3 - \frac{\sqrt{69}}{6}) \cup \left(3 + \frac{\sqrt{69}}{6}, \infty \right) \) 8. \((-\infty, \infty)\) 9. \( \emptyset \) 10. \( \{ 4 \} \)
11. \((-\infty, -4) \cup (0, \infty)\) 12. \([-8, 6] \) 13. \((-\infty, 0] \cup [5, \infty) \) 14. \((-\infty, -9] \cup [7, \infty) \) 15. between \$50 and \$70
16. \( \left[ \frac{4}{3}, 2 \right) \cup [6, \infty) \) 17. \((-\infty, -2) \cup \left(-\frac{5}{3}, 2 \right) \)
Chapter 8 Answers

Section 8.1

Five-Minute Warm-Up 1. \( \{x \mid x \neq 2, x \neq 1\} \) 2a. -47 2b. \(-3a^2 + 12a - 11\)
2c. \(-3x^2 - 6xh - 3h^2 + 1\) 3a. not a function 3b. is a function

Guided Practice 1. \((f \circ g)(x)\) 2a. -3 2b. 25 2c. 25 3a. 39 3b. 38 3c. 38
4. A function is one-to-one if each unique element in the domain corresponds to a unique element in the range and each unique element in the range corresponds to a unique element in the domain. 5a. yes 5b. no 6. the horizontal line test
7. one-to-one 8. \(f^{-1}(x)\) \(; \ (b, a)\) 9a. \((15, -3), (-5, -1), (0, 0), (10, 2)\) 9b. \(-3, -1, 0, 2\)
9c. \(\{15, -5, 0, 10\}\) 9d. \(\{15, -5, 0, 10\}\) 9e. \(-3, -1, 0, 2\) 10. \(y = x\) 11a. \(y = 4x - 8\)
11b. \(x = 4y - 8\) 11c. \(x + 8 = 4y\) 11d. \(y = \frac{x + 8}{4}\) 11e. \(f^{-1}(x) = \frac{x + 8}{4}\)

Do the Math 1. 253 2. 6 3. 1 4. -94 5. \((f \circ g)(x) = 4x - 3\) 6. \((g \circ f)(x) = \sqrt{x + 2} - 2\)
7. \((f \circ f)(x) = \frac{2(x - 1)}{3 - x}; x \neq 1, 3\) 8. one-to-one 9. not one-to-one 10. one-to-one
11. \((1, -10), (4, -5), (3, 0), (2, -5)\) 12. \((f \circ g)(x) = (g \circ f)(x) = x\) 13. \((f \circ g)(x) = (g \circ f)(x) = x\)
14. \(g^{-1}(x) = x - 6\) 15. \(H^{-1}(x) = \frac{x - 8}{3}\) 16. \(f^{-1}(x) = \frac{3x - 2}{x}\)
17. \(G^{-1}(x) = 3 - \frac{2}{x}\) 18. \(R^{-1}(x) = \frac{4x}{2 - x}\) 19. \(g^{-1}(x) = (x + 3)^3 - 2\)
20. \(V(t) = 36\pi t; 1080\pi \approx 3392.92\ m^3\)

Section 8.2

Five-Minute Warm-Up 1. 16 2. \(\frac{1}{4}\) 3. 1 4. \(\frac{1}{10}\) 5a. 6.0235 5b. 6.0234 6. \(\frac{10}{x^2}\) 7. \(\frac{2a^6}{5}\)
8. \(64p^{15}\) 9. \(\left\{-\frac{1}{2}, \frac{5}{3}\right\}\)

Guided Practice 1. a positive real number; 1 2. \(\approx 2.63902\) 3. \(\approx 2.66514\) 4. (b) 5. (a) 6. \(\approx 2.718\)
7a. \(\approx 20.09\) 7b. \(\approx 0.14\) 8. the exponents are equal 9a. \(3^{x^2} = 3^3\) 9b. \(2x + 1 = 3\) 9c. \(2x = 2\)

9d. \(x = 1\) 9e. \(\{1\}\) 10. \(\approx 64.7\) grams 11. \(A = P\left(1 + \frac{r}{n}\right)^{nt}\); A = final amount; t = years that the interest accumulates; \(r =\) annual interest rate written in decimals; \(n =\) number of pay periods per year
12a. \(P=100, r = .07, n=360, t=5\) 12b. \$141.48 12c. \(P=100, r = .07, n=360, t=5\) 12d. \$141.90

Do the Math 1a. 9.518 1b. 9.673 1c. 9.735 1d. 9.738 1e. 9.739 2a. 20.086 2b. 4.482
3 – 5. See Graphing Answer Section 6. \(-2\) 7. \(\{4\}\) 8. \(\{1\}\) 9. \(-1, 0, \frac{1}{3}\) 10. \(-2\) 11. \(\left\{\frac{2}{3}\right\}\)

12. \(f(2) = 9; (2, 9)\) 13. \(x = -4; \left(-4, \frac{1}{81}\right)\) 14a. \(\approx 7287\) million people 14b. \(\approx 8259\) million people

Section 8.3

Five-Minute Warm-Up 1. \(\{x \mid x > -\frac{3}{2}\}\) 2. \(\{3\}\) 3. \(-4, -3\) 4. \(\frac{1}{16}\) 5. 8 6. 4 7. \(\frac{1}{4}\)

Guided Practice 1. \(x = a^y\) 2. \(\log_a \left(\frac{1}{8}\right) = -3\) 3. \(\log_a 3 = 4\) 4. \(4^p = 30\) 5. \(n^{-5} = 3\) 6a. \((0, \infty)\)
6b. \((-\infty, \infty)\) 6c. \(x \mid x < 5\) 7. \(y = 5^x\); See Graphing Answer Section 8. \(x = e^y\) 9. \(x = 10^y\) 10. \(-2\)

11. \(e^{-2} \approx 0.1353\) 12a. decibels (loudness) 12b. Richter Scale (earthquakes) 12c. pH (chemistry)

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Do the Math 1. \( \log_2 64 = 3 \) 2. \( \log_b 23 = 4 \) 3. \( \log z = -3 \) 4. \( 3^4 = 81 \) 5. \( 6^{-4} = x \) 6. \( a^2 = 16 \) 7. 2 8. 2 9. \((2, \infty)\) 10. \((-\infty, \frac{3}{5})\) 11. \(-0.108\) 12. \(-0.693\) 13. \(\{6\}\) 14. \(\left\{ \frac{27}{4} \right\}\) 15. \(\{9\}\) 16. \(\left\{ \frac{7}{2} \right\}\) 17. \(\{4\}\) 18. \([-2\sqrt{2}, 2\sqrt{2}\] 19. \(\approx 9.2\) on the Richter scale

Section 8.4

Five-Minute Warm-Up 1a. 1.140 1b. 1.139 2. \(x^2\) 3. \(a^\frac{3}{2}\) 4. 2\(^{-4}\) 5. \(\left\{ \frac{3}{2} \right\}\) 6. 1 7. 6 8. \(y = 3^x\)
Guided Practice 1a. \(\sqrt{2}\) 1b. 0 1c. 1 1d. 7 2a. \(\log_a M + \log_a N\) 2b. \(\log_a M - \log_a N\) 2c. \(\log_a M\) 3. \(\log 9 + \log x\) 4. \(\ln (2 - \ln x)\) 5. \(\log_4 \left(\frac{1}{x - 1}\right)\) 6. \(\frac{\log_a M}{\log_a a}\) 7. \(\approx 3.170\) 8. Answers may vary.

Do the Math 1a. \(-3\) 1b. \(\sqrt{2}\) 1c. 10 2a. \(a\) 2b. \(a + 2b\) 3. \(\log_4 a - \log_4 b\) 4. \(3\log_3 a + \log_3 b\) 5. \(3 + \log_2 z\) 6. \(4 - \log_2 p\) 7. \(5 + \frac{1}{4} \log_2 z\) 8. \(\frac{1}{5} \ln x - 2 \ln (x + 2)\) 9. \(\frac{1}{3} \left[ \log_6 (x - 2) - \log_6 (x + 1) \right]\) 10. \(3 \log_4 x + \log_4 (x - 3) - \frac{1}{3} \log_4 (x + 1)\) 11. 3 12. 4 13. \(\log_2 z^8\) 14. \(\log_2 (a^4 b^2)\) 15. \(\log_4 \left[ \sqrt{2} (2z + 1)^2 \right]\) 16. \(\log_7 (x^2)\) 17. \(\ln \sqrt{x^2 - 1}\) 18. \(\log_5 (x + 1)\) 19. \(\log_4 \left( \frac{x^4}{16} \right)\) 20a. 0.827 20b. 1.631

Section 8.5

Five-Minute Warm-Up 1. \(\{9\}\) 2. \((-8, 3)\) 3. \(\left\{ \frac{1}{4}, -2 \right\}\) 4. \(\{7, 0\}\) 5. \(\{x | x < 3\}\)
Guided Practice 1a. \(\left\{ \frac{5}{2} \right\}\) 1b. \(\left\{ \frac{1}{3} \right\}\) 1c. \(\{64\}\) 2a. \(x = \frac{\log 12}{\log 3} \approx 2.262\) 2b. \(x = \frac{\ln 18}{3} \approx 0.963\)
3a. \(90 = 100 \left( \frac{1}{2} \right)^{\frac{\log 25}{\log 3}}\) 3b. \(\approx 2.927\) sec. 4a. 3921 people 4b. \(7500 = 2500e^{0.03t}\) 4c. 36.6 years or 1986 4d. 23.1 years or 1973

Do the Math 1. \(\{13\}\) 2. \(\{16\}\) 3. \(\{8\}\) 4. \(\left\{ \frac{5}{8} \right\}\) 5. \(\{5\}\) 6. \(\frac{\log 8}{\log 3} \approx 1.893\) 7. \(\frac{\log 20}{\log 4} \approx 2.161\)
8. \(\ln 3 \approx 1.099\) 9. \(\log 0.2 \approx -0.699\) 10. \(\frac{\log 5}{2 \log 2} \approx 1.161\) 11. \(\frac{\log 5}{\log 4} \approx 1.161\) 12. \(\{4\}\) 13. \(\{2\}\) 14. \(\{2\}\)
15. \(\ln 4 \approx 1.386\) 16. \(\{12\}\) 17. \(\pm 2\sqrt{2} \approx -2.828, 2.828\) 18. \(\{6\}\) 19. 2052 20. \(\approx 2.989\) years

Chapter 9 Answers

Section 9.1

Five-Minute Warm-Up 1. \(2\sqrt{10}\) 2. \(6\sqrt{3}\) 3. \(4\sqrt{2}\) 4. \(2x - 5\) 5. \(-10\) 6. 25 7. 6 cm\(^2\) 8. 12
Guided Practice 1. \(d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\) 2a. \(a\) and \(c\) 2b. \(b\) and \(d\) 2c. \((a)\) 2d. \((b)\) 2e. add (c) and (d) 2f. take the square root of (e) 3. \(4\sqrt{5} \approx 8.94\) 4. two equal parts 5. \(x_1 + x_2; y_1 + y_2\) 6. \((-4, -3)\)

Do the Math 1. 5 2. 25 3. 2 4. \(2\sqrt{13} \approx 7.21\) 5. \(2\sqrt{78} \approx 17.66\) 6. \(\sqrt{5.69} \approx 2.39\) 7. \(3, 5\) 8. \(2, 2\) 9. \(\left\{ \frac{1}{2}, \frac{11}{2} \right\}\) 10. \((2, -2)\) 11. \((2\sqrt{6}, 4\sqrt{2})\) 12. \((-0.7, 1.95)\) 13a. See Graphing Answer Section
13b. \(d(A, B) = 5; d(B, C) = 3\sqrt{5}; d(A, C) = \sqrt{58} \approx 7.62\) 14. \((-16, -3)\) and \((8, -3)\)
Section 9.2

Five-Minute Warm-Up 1. 49; (x - 7)²  2. \( \frac{25}{4}; \left( y - \frac{5}{2} \right)² \)  3. (y + 8)²  4. 2(x - 3)²

5a. \( \frac{225\pi}{4} \approx 176.71 \text{ in.}^² \)  5b. 15\( \pi \approx 47.12 \text{ in.} \)

Guided Practice 1a. center  1b. radius  1c. \( d = 2r \)  2. \((x - h)^2 + (y - k)^2 = r^2 \)

3. \((x + 1)^2 + (y - 3)^2 = 25 \)  4a. \((2, -3) \)  4b. 2  4c. See Graphing Answer Section

5. \( x^2 + y^2 + ax + by + c = 0 \)  6a. 9; 1; 9  6b. 3; 1; 9  6c. \((-3, 1) \); 3  6d. See Graphing Answer Section

7. No; all circles fail the vertical line test.  8. Yes; Center: \((2, 0)\); radius = 3

Do the Math 1 – 8. See Graphing Answer Section  1. \( x^2 + y^2 = 25 \)  2. \((x - 1)^2 + y^2 = 4 \)

3. \((x + 4)^2 + (y - 4)^2 = 16 \)  4. \((x - 5)^2 + (y - 2)^2 = 7 \)  5. \(C(0, 0), r = 5 \)  6. \(C(5, -2), r = 7 \)

7. \(C(6, 0), r = 6 \)  8. \(C(2, -2), r = 0.5 \)  9. \(C(-1, 4), r = 3 \)  10. \(C(-2, 6), r = 2 \)  11. \(C(7, -5), r = 8 \)

12. \((x - 2)^2 + (y + 3)^2 = 9 \)  13. \((x - 1)^2 + \left( y + \frac{1}{2} \right)^2 = \frac{169}{4} \)

15. \(A = 49\pi \text{ square units}; C = 14\pi \text{ units} \)

Section 9.3

Five-Minute Warm-Up 1. \((2, 1) \)  2. up  3. \(x = 2 \)  4. \(25; (x + 5)^2 \)  5. \(-12; -3(x - 2)^2 \)  6. \(\{8, 0\} \)

Guided Practice 1a. focus  1b. directrix  1c. vertex  1d. axis of symmetry  2a. left or right  2b. right

2c. left  2d. up or down  2e. up  2f. down  3a. down  3b. right  4a. left  4b. \(y^2 = -4ax \)  4c. \(y^2 = -\frac{1}{2}x \)

5a. \(x^2 - 8x = -4y - 20 \)  5b. \(x^2 - 8x + 16 = -4y - 20 + 16 \)  5c. \(x^2 - 8x + 16 = -4y - 4 \)

5d. \((x - 4)^2 = -4(y + 1) \)  5e. down  5f. \((4, -1) \)  5g. See Graphing Answer Section

Do the Math 1. \(x^2 = 20y \)  2. \(y^2 = -32x \)  3. \(y^2 = 2x \)  4. \(y^2 = 16x \)  5. \(x^2 = -8y \)

6 – 13. See Graphing Answer Section  6a. \(V(0, 0) \)  6b. \(F(0, 7) \)  6c. \(y = -7 \)  7a. \(V(0, 0) \)  7b. \(F\left(\frac{5}{2}, 0\right) \)

7c. \(x = -\frac{5}{2} \)  8a. \(V(0, 0) \)  8b. \(F(0, -4) \)  8c. \(y = 4 \)  9a. \(V(-4, 1) \)  9b. \(F(-4, 0) \)  9c. \(y = 2 \)  10a. \(V(-5, 2) \)

10b. \(F(-2, 2) \)  10c. \(x = -8 \)  11a. \(V(-1, 3) \)  11b. \(F(-1, 5) \)  11c. \(y = 1 \)  12a. \(V(2, 4) \)  12b. \(F(-2, 4) \)

12c. \(x = 6 \)  13a. \(V(2, 0) \)  13b. \(F\left(2, -\frac{5}{2}\right) \)  13c. \(y = \frac{5}{2} \)  14. 20 feet

Section 9.4

Five-Minute Warm-Up 1. \(\frac{81}{4}; \left( y - \frac{9}{2} \right)² \)  2. \(36; (x + 6)² \)  3. \(\frac{3}{4}; 4\left( x - \frac{1}{4} \right)² \)  4. \(y = 9(x + 3)² - 6 \)

5. \((-3, -6) \)  6. up  7. \(x = -3 \)

Guided Practice 1a. foci  1b. major axis  1c. minor axis  1d. center  1e. vertices  2a. \(a² \)  2b. \(a \)

2c. \(2a \)  2d. \(2b \)  2e. major  2f. \(a² - b² \)  2g. the origin  3a – 3b. See Graphing Answer Section

4. \(\frac{x^2}{144} + \frac{y^2}{169} = 1 \)  5a. \(\frac{(x + 5)^2}{9} + \frac{(y - 2)^2}{25} = 1 \)  5b. \(C(-5, 2) \)  5c. \(V₁(-5, 7); V₂(-5, -3) \)

5d. \(F₁(-5, 6); F₂(-5, -2) \)  6a. \(\frac{(x - 1)^2}{9} + \frac{(y - 3)^2}{4} = 1 \)  6b. \(C(1, 3) \)  6c. \(V₁(4, 3); V₂(-2, 3) \)  6d. \(F\left(1 \pm \sqrt{5}, 3\right) \)

7a. \(\frac{(x - 4)^2}{36} + \frac{(y + 3)^2}{64} = 1 \)  7b. \(C(4, -3) \)  7c. \(V₁(4, -11); V₂(4, 5) \)  7d. \(F\left(4, -3 \pm 2\sqrt{7} \right) \)
Do the Math 1–4. See Graphing Answer Section  1a. \( V(\pm 5, 0) \)  1b. \( F(\pm 2\sqrt{5}, 0) \)  2a. \( V(0, \pm 6) \)  2b. \( F(0, \pm 2\sqrt{5}) \)  3a. \( V(\pm 8, 0) \)  3b. \( F(\pm 3\sqrt{7}, 0) \)  4a. \( V(0, \pm 9) \)  4b. \( F(0, \pm 6\sqrt{2}) \)  5. \( \frac{x^2}{25} + \frac{y^2}{21} = 1 \)  6. \( \frac{x^2}{24} + \frac{y^2}{25} = 1 \)  7. \( \frac{x^2}{45} + \frac{y^2}{49} = 1 \)  8. \( \frac{x^2}{100} + \frac{y^2}{64} = 1 \)  9–10. See Graphing Answer Section  11a. \( \frac{(x + 3)^2}{9} + \frac{(y - 4)^2}{25} = 1 \)  11b. \( C(-3, 4) \)  11c. \( V(-3, -1) \) and \( (3, 9) \)  11d. \( F(-3, 0) \) and \( (3, 0) \)  12a. \( \frac{x^2}{519.84} + \frac{y^2}{225} = 1 \)  12b. yes

Section 9.5

Five-Minute Warm-Up 1. \([-4, 4]\)  2. \([-1, -5]\)  3. 16; \( (x + 4)^2 \)  4. \( \frac{4}{9}; \left( y - \frac{2}{3} \right)^2 \)  5. See Graphing Answer Section  
Guided Practice 1a. foci  1b. transverse axis  1c. center  1d. conjugate axis  1e. vertices  2a. \( a^2 \)  2b. \( a \)  2c. \( 2a \)  2d. \( 2b \)  2e. left and right  2f. up and down  2g. \( a^2 + b^2 \)  2h. the origin  3a. center: \((0, 0)\)  3b. \( a = 4, b = 3 \)  3c. \( c = 5 \)  3d. vertices: \((\pm 0, 4)\)  3e. \((\pm 5, 9), (\pm 5, -9)\)  3f. See Graphing Answer Section

4. A boundary line that the graph approaches but does not cross as \( x \) (or \( y \)) \( \rightarrow \pm \infty \).  
5a. \( \frac{b}{a} x \)  5b. \( \frac{a}{b} y \)  

Do the Math 1–4. See Graphing Answer Section  1a. \( V(\pm 3, 0) \)  1b. \( F(\pm 5, 0) \)  2a. \( V(0, \pm 9) \)  2b. \( F(0, \pm 3\sqrt{10}) \)  3a. \( V(\pm 6, 0) \)  3b. \( F(\pm 2\sqrt{10}, 0) \)  4a. \( V(0, \pm 3) \)  4b. \( F(0, \pm \sqrt{13}) \)  5. \( x^2 - \frac{y^2}{25} = 1 \)  6. \( \frac{y^2}{36} - \frac{x^2}{28} = 1 \)  7. \( \frac{y^2}{16} - \frac{x^2}{4} = 1 \)  8. \( \frac{x^2}{8.1} - \frac{y^2}{72.9} = 1 \)  9. parabola  10. hyperbola  11. hyperbola  12. circle  13. parabola  14. ellipse  15a. \((0, 5)\) and \((0, -5)\)  15b. \((0 \pm \sqrt{29})\)  16. \( y = \pm \frac{3}{2} x \)

Section 9.6

Five-Minute Warm-Up 1. \( \left( \frac{2}{3}, \frac{3}{2} \right) \)  2. \( \{ (x, y) \} | -x + 3y = 4 \} \)  3. \((-4, -5)\)  4. \( \emptyset \)  
Guided Practice 1a. a line  1b. a circle  1c. See Graphing Answer Section  1d. \( 2 \)  1e. \( x^2 + y^2 = 16 \)  1f. \( x^2 + (x - 4)^2 = 16 \)  1g. \( 2x^2 - 8x = 0 \)  1h. \( 2x(x - 4) = 0 \)  1i. \( x = 0 \) or \( x = 4 \)  1j. \( y = x - 4 \)  1k. \( y = 0 - 4 \)  1l. \( y = -4 \)  1m. \( y = x - 4 \)  1n. \( y = 4 - 4 \)  1o. \( y = 0 \)  1p. \( \{(0, -4), (4, 4)\} \)  2a. a circle  2b. an ellipse  2c. See Graphing Answer Section  2d. \( 2 \)  2e. \( -1 \)  2f. \( \begin{cases} -x^2 - y^2 = -4 \quad (1) \\ x^2 + 4y^2 = 16 \quad (2) \end{cases} \)  2g. \( 3y^2 = 12 \)  2h. \( y^2 = 4 \)  2i. \( y = \pm 2 \)  2j. \( x^2 + y^2 = 4 \)  2k. \( x^2 + (2)^2 = 4 \)  2l. \( x = 0 \)  2m. \( x^2 + y^2 = 4 \)  2n. \( x^2 + (-2)^2 = 4 \)  2o. \( x = 0 \)  2p. \( \{(0, 2), (0, -2)\} \)  
Do the Math 1. \((-1, 1), (0, 2), and (1, 3)\)  2. \((6, 8)\) and \((8, 6)\)  3. \((0, -4), (\pm \sqrt{7}, 3)\)  4. \((1, 1)\)  5. \((-2, -2)\) and \((2, 2)\)  6. \((-5, 0)\) and \((2, 0)\)  7. \((-3, 0)\) and \((3, 0)\)  8. \((-2, -6)\) and \((1, 15)\)  9. \( \emptyset \)  10. \((-5, 0)\) and \((5, 0)\)  11. \( \emptyset \)  12. \((-4, -7), (4, -7), (-1, 8), (1, 8)\)  13. \(-4\) and \(12\)  14. 20 meters by 12 meters

Chapter 10 Answers

Section 10.1

Five-Minute Warm-Up 1a. \(-5\)  1b. \(-9\)  2a. \(1\)  2b. \(1\)  2c. \(-3\)  2d. \(5\)  3a. \(\frac{1}{9}\)  3b. \(-\frac{1}{27}\)  3c. \(\frac{1}{81}\)  4. \(\frac{25}{24}\)
Guided Practice  1a. terms  1b. ellipse  1c. finite  2. \(a_n\)  3. \(1, \frac{3}{2}, \frac{7}{3}, \frac{15}{4}, \frac{31}{5}\)  4a. \(a_n = 3n\)

4b. \(an = (-1)^{n+1} \left(\frac{1}{6}\right)^n\)

5. 1; 2; 3; 4; 7; 12; 19; 42  6. \(-5 + (-3) + (-1) + 1 + 3 + 5 = 0\)  7. \(\sum_{i=0}^{6} (2i)\)

Do the Math  1. \(-3, -2, -1, 0, 1\)  2. 5, 3, \(\frac{7}{3}\), 2, \(\frac{9}{5}\)  3. 2, 8, 26, 80, 242  4. \(\frac{1}{2}, \frac{9}{2}, \frac{8}{3}, \frac{25}{2}\)  5. \(a_n = 5n\)

6. \(a_n = \frac{n}{2}\)  7. \(a_n = n^3 - 1\)  8. \(a_n = \left(-\frac{1}{2}\right)^{n-1}\)

9. 55  10. 50  11. 120  12. 4  13. 16  14. 69

15. \(\sum_{k=1}^{9} (2k - 1)\)  16. \(\sum_{k=1}^{16} \frac{1}{2k - 1}\)  17. \(\sum_{i=1}^{15} (-1)^{i+1} \left(\frac{2}{3}\right)^i\)  18. \(\sum_{k=1}^{12} \left[3 \cdot \left(\frac{1}{2}\right)^{k-1}\right]\)  19a. $503.33

19b. $5415.00  19c. $11,098.20

Section 10.2

Five-Minute Warm-Up  1. 4  2. \(-\frac{9}{5}\)  3a. (14, 4)  3b. (1, -4)  4. -36

Guided Practice  1a. \(d\)  1b. \(a_i\)  2. yes; \(a_1 = 5\); \(d = 3\)  3. 3, 6, 11, 18, 27, 38; 3, 5, 7, 9, 11

4a. \(a_n = 5n - 8\)  4b. \(a_n = 32\)  5a. \(a_i = 12\); \(d = 3\)  5b. \(a_n = 3d + 12\)  6a. \(a_n = 4n + 8\)  6b. \(a_{10} = 48\)

6c. \(S_{10} = 300\)  7a. \(a_1 = 3\), \(a_{10} = -87\)  7b. \(S_{10} = -420\)

Do the Math  1. \(d = 10\); 11, 21, 31, 41  2. \(d = \frac{1}{4}\); 1, \(\frac{5}{4}\), \(\frac{3}{2}\), \(\frac{7}{4}\)  3. \(a_n = 3n + 5\); \(a_5 = 20\)

4. \(a_n = -3n + 15\); \(a_5 = 0\)  5. \(a_n = \frac{1}{2}n - \frac{7}{2}\); \(a_5 = -1\)  6. \(a_n = 4n - 9\); \(a_{20} = 71\)  7. \(a_n = -6n + 26\); \(a_{20} = -94\)

8. \(a_n = \frac{1}{2}n + \frac{21}{2}\); \(a_{20} = \frac{1}{2}\)  9. \(a_n = 3n - 8\)  10. \(a_n = 4n - 17\)  11. \(a_n = -5n + 22\)  12. \(a_n = \frac{1}{4}n + \frac{15}{4}\)


Section 10.3

Five-Minute Warm-Up  1a. \(\frac{2}{3}\)  1b. \(\frac{4}{9}\)  1c. \(\frac{8}{27}\)  2a. 3  2b. 12  2c. 27  3a. \(\frac{8x^3}{5}\)  3b. \(9r^8\)  4. 2

Guided Practice  1a. \(r\)  1b. \(a_i\)  2. yes; \(a_1 = 36\); \(r = \frac{1}{2}\)  3. \(\frac{1}{5}\), \(\frac{4}{25}\), \(\frac{8}{125}\); \(r = \frac{2}{5}\)  4a. \(\{a_n\} = \left\{2 \cdot \left(-\frac{1}{3}\right)^{n-1}\right\}\)

4b. \(-\frac{2}{2187}\)  5a. \(a_i = 6\), \(r = 4\)  5b. 1,048,575  6a. \(a_i = 6\); \(r = -\frac{1}{3}\)  6b. \(\frac{9}{2}\)  7. \$99,658.27

Do the Math  1. \(r = -2\); -2, 4, -8, 16  2. \(r = 2\); \(\frac{2}{3}, \frac{4}{3}, \frac{8}{3}, \frac{16}{3}\)  3. \(r = \frac{1}{6}, \frac{1}{3}, \frac{1}{18}, \frac{1}{108}\)  4. arithmetic

5. neither  6. geometric  7. neither  8. \(a_n = 30 \cdot \left(\frac{1}{3}\right)^{n-1}\); \(a_8 = \frac{10}{729}\)  9. \(a_n = (-4)^{n-1}\); \(a_8 = -16,384\)

10. 177,147  11. -1280  12. 0.00000000004  13. 88,572  14. 40,950  15. \(\frac{3}{2}\)  16. \(\frac{48}{5}\)  17. 5  18. \(\frac{1}{3}\)

19. \(\frac{5}{11}\)  20. \$10,497.60

Section 10.4

Five-Minute Warm-Up  1. \(x^2 + 4x + 4\)  2. \(y^2 - 6y + 9\)  3. \(16x^2 - 40xy + 25y^2\)

4. \(9n^2 + 4n + \frac{4}{9}\)  5. 35

Guided Practice  1a. 151,200  1b. 15  2a. 10  2b. 1716  3. \(p^4 + 8p^3 + 24p^2 + 32p + 16\)

Do the Math  1. 120  2. 360  3. 45  4. 1  5. 10  6. 21  7. 50  8. 1  9. \(x^4 - 4x^3 + 6x^2 - 4x + 1\)
10. \( x^5 + 25x^4 + 250x^3 + 1250x^2 + 3125x + 3125 \)  
11. \( 16q^4 + 96q^3 + 216q^2 + 216q + 81 \)  
12. \( 81w^4 - 432w^3 + 864w^2 - 768w + 256 \)  
13. \( y^8 - 12y^6 + 54y^4 - 108y^2 + 81 \)  
14. \( 243b^{10} + 810b^8 + 1080b^6 + 720b^4 + 240b^2 + 32 \)  
15. \( p^6 - 18p^5 + 135p^4 - 540p^3 + 1215p^2 - 1458p + 729 \)  
16. \( 81x^8 + 108x^6y^3 + 54x^4y^6 + 12x^2y^9 + y^{12} \)  
17. 1.00501
Graphing Answer Section

Section R.2 Do the Math

12. 

\[ \begin{array}{cccc}
-\frac{5}{3} & -0.5 & 2.5 & 4 \\
-4 & -2 & 0 & 2 & 4 \\
\end{array} \]

Section R.3 Guided Practice

3. 

\[ \begin{array}{ccc}
+ & + & - \\
+ & + & - \\
- & - & + \\
\end{array} \]

Section 1.4 Guided Practice

5g. 

\[ \begin{array}{cccc}
-4 & -2 & 0 & 2 & 4 \\
\end{array} \]

6h. 

\[ \begin{array}{cccc}
-7 & -5 & -3 & -1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\
\end{array} \]

Section 1.4 Do the Math

1. 

\[ \begin{array}{cccc}
-4 & -2 & 0 & 2 & 4 \\
\end{array} \]

2. 

\[ \begin{array}{cccc}
-4 & -2 & 0 & 2 & 4 \\
\end{array} \]

3. 

\[ \begin{array}{cccc}
-4 & -2 & 0 & 2 & 4 \\
\end{array} \]

Section 1.5 Warm-Up

1. 

\[ \begin{array}{cccc}
-2 & 0 & \frac{3}{2} & 3 \\
\end{array} \]

Section 1.5 Guided Practice

1. 

\[ \begin{array}{cccc}
\text{II} & \text{I} & \text{III} & \text{IV} \\
\text{x-axis} & \text{y-axis} \\
\end{array} \]

5. 

\[ \begin{array}{cccc}
(2,5) & (1,2) & (-1,4) & (-2,7) \\
\end{array} \]

Section 1.5 Do the Math

2. 

\[ \begin{array}{cccc}
\end{array} \]

6. 

\[ \begin{array}{cccc}
\end{array} \]

7. 

\[ \begin{array}{cccc}
\end{array} \]

8. 

\[ \begin{array}{cccc}
\end{array} \]

Section 1.5 Do the Math

9. 

\[ \begin{array}{cccc}
\end{array} \]

9m. 

\[ \begin{array}{cccc}
\end{array} \]
### Section 1.6 Do the Math

1.  
2.  
3.  
4.  

### Section 1.7 Guided Practice

5.  
6.  
7.  
8.  

### Section 1.8 Guided Practice

9.  
10.  
11.  
12.  

### Section 1.8 Do the Math

13.  
14.  

### Section 2.1 Warm-Up

15.  
16.  
17.  

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Section 2.1 Guided Practice

6.

Section 2.3 Warm-Up

3. 4.

Section 2.3 Guided Practice

4.

Section 2.3 Do the Math

6. 7.

Section 2.4 Warm-Up

1. 2.

Section 2.4 Guided Practice

3c.

Section 2.4 Do the Math

1.

Section 2.4 Do the Math

2. 3. 4. 8.
Section 2.5 Warm-Up

2.

Section 2.5 Guided Practice

3a.

3b.

4d-f.

Section 2.5 Guided Practice

6d.

7e-g.

Section 2.6 Guided Practice

6d.

8d.

Section 3.1 Warm-Up

5.

Section 3.4 Guided Practice

3.

6a.

6b.

Section 3.4 Guided Practice

8a.

8b.

8c.

8d.

Section 3.4 Do the Math

1.

2a.

2b.

Section 3.5 Guided Practice

6.

\[
(1) \cdot \begin{bmatrix} -3 \\ 4 \\ -2 \end{bmatrix} - (-1) \cdot \begin{bmatrix} 5 \\ 3 \\ -1 \end{bmatrix} + (2) \cdot \begin{bmatrix} 5 \\ -3 \\ 1 \end{bmatrix}
\]
Section 3.5 Guided Practice

7a. \[
\begin{array}{ccc}
1 & 2 & -1 \\
2 & -4 & 1 \\
-2 & 2 & -3
\end{array}
\]

7c. \[
\begin{array}{ccc}
-3 & 2 & -1 \\
-7 & -4 & 1 \\
4 & 2 & -3
\end{array}
\]

7d. \[
\begin{array}{ccc}
1 & -3 & -1 \\
2 & -7 & 1 \\
-2 & 4 & -3
\end{array}
\]

7e. \[
\begin{array}{ccc}
1 & 2 & -3 \\
2 & -4 & -7 \\
-2 & 2 & 4
\end{array}
\]

Section 3.6 Warm-Up

Section 3.6 Guided Practice

4.

6.

7d

Do the Math 3.6

3.

4.

5.

6.

Do the Math 3.6

Guided Practice 4.3

Warm-Up 5.5

7.

8a.

1c. \[
\frac{x^2 - 3x + 4}{x + 1} = \frac{(x^3 - 2x^2 + x + 6)}{- (3x^2 - 3x)} - \frac{4x + 6}{2}
\]

2. \[
\begin{array}{c}
\text{Factors of } -24: 1, -24, 2, -12, 3, -8, 4, -6, 6, -4, 8, -3, 12, -2, 24, -1 \\
\text{Sum of 2: } -23, -10, -5, -2, 2, 5, 10, 23
\end{array}
\]

Guided Practice 4.5

7b.

Guided Practice 5.5

5q.
### Guided Practice 5.5

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<th>Interval</th>
<th>$(-\infty, -4)$</th>
<th>$-4$</th>
<th>$(-4, 2)$</th>
<th>$2$</th>
<th>$2.5$</th>
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<td>$4$</td>
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<tr>
<td>Sign of $x + 4$</td>
<td>$-$</td>
<td>$0$</td>
<td>$+$</td>
<td>$+$</td>
<td>$+$</td>
<td>$+$</td>
<td>$+$</td>
</tr>
<tr>
<td>Sign of $x - 3$</td>
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<td>$-$</td>
<td>$-$</td>
<td>$-$</td>
<td>$-$</td>
<td>$0$</td>
<td>$+$</td>
</tr>
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<td>Sign of quotient</td>
<td>$-$</td>
<td>$0$</td>
<td>$+$</td>
<td>Undef.</td>
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<td>$0$</td>
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<td>Conclusion</td>
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<td>Included</td>
<td>Included</td>
<td>Not Included</td>
<td>Not Included</td>
<td>Included</td>
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### Section 5.7 Warm-Up

5. [Graph of a linear function]

6. [Graph of an increasing function]

### Warm Up 6.6

5b. [Graph of a quadratic function]

### Guided Practice 6.6

5b. [Graph of a quadratic function]

6b. [Graph of a quadratic function]

### Do the Math 6.6

9. [Graph of a quadratic function]

10. [Graph of a quadratic function]

11. [Graph of a quadratic function]

12. [Graph of a quadratic function]
Warm Up 7.4

1. 

2. 

Do the Math 7.4

1. 

2. 

3. 

4. 

5. 

6. 

Guided Practice 7.5

3k. 

Do the Math 7.5

4. 

5. 

6. 

7.
Do the Math 7.5

8.

9.

Guided Practice 7.6

3d. 5d.

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<td>$-$</td>
<td>$-$</td>
<td>$-$</td>
<td>$0$</td>
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<tr>
<td>Sign of product</td>
<td>$+$</td>
<td>$0$</td>
<td>$-$</td>
<td>$0$</td>
<td>$+$</td>
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<tr>
<td>Conclusion</td>
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Do the Math 8.2

3.

4.

5.

Guided Practice 8.3

7.

Do the Math 9.1

13a.
Guided Practice 9.5

3.

Do the Math 9.5

1. 
2. 
3. 
4. 

Guided Practice 9.6

1c. 
2c. 