1) Which of the following is the sum of the two solutions of $x^2 + x - 12 = 0$?

a) -4 b) 4 c) 1 d) -1 e) none of these [Problem submitted by Vin Lee, LACC Professor of Mathematics, Source: Vin Lee]

Solution: $x^2 + x - 12 = 0$

$(x - 3)(x + 4) = 0$

$x = -4, 3$

So the answer is d) -1.
2) If \( f(x) = \frac{1}{2}x + 2 \) and \( g(x) = -2x + 7 \), which of the following is the point of intersection of the graphs of \( f \) and \( g \)?

a) (2, 2)  b) (3, 2)  c) (1, 5)  d) (2, 3)  e) none of these [Problem submitted by Vin Lee, LACC Professor of Mathematics, Source: Vin Lee]

Solution: \( f(x) = g(x) \)

\[
\frac{1}{2}x + 2 = -2x + 7
\]

\[
\frac{5}{2}x = 5
\]

\[
x = 2
\]

\[
f(2) = g(2) = 3
\]

So the answer is d) (2, 3).
3) Suppose a store owner wants to make a 100 pound mixture of peanuts and cashews to sell for $4.30 per pound. If peanuts sell for $2.50 per pound and cashews sell for $7.00 per pound, how many pounds of each should be used?

a) 60 pounds of peanuts and 40 pounds of cashews  
b) 50 pounds of peanuts and 50 pounds of cashews  
c) 40 pounds of peanuts and 60 pounds of cashews  
d) 30 pounds of peanuts and 70 pounds of cashews  
e) none of these  

[Problem submitted by Vin Lee, LACC Professor of Mathematics, Source: Vin Lee]

Solution: Let P be the number of pounds of peanuts needed and C the number of pounds or cashews.

\[ P + C = 100 \quad \rightarrow \quad C = 100 - P \]

\[ 2.5P + 7C = 430 \quad \rightarrow \quad 2.5P + 7(100 - P) = 430 \]

Solve this equation to get

\[ P = 60 \text{ pounds. Then } C = 40 \text{ pounds. So, the answer is a) 60 pounds of peanuts and 40 pounds of cashews.} \]
4) Suppose two burgers and a shake cost $9.30 and a burger and two shakes cost $7.80. Which of the following is the cost of three burgers and four shakes? a) $17.50 b) $21.30 c) $18.90 d) $22.50 e) $19.20 [Problem submitted by Vin Lee, LACC Professor of Mathematics, Source: Vin Lee]

Solution: Let B be the cost of a burger and S the cost of a shake.

\[2B + S = 9.3 \quad \rightarrow \quad S = 9.3 - 2B\]
\[B + 2S = 7.8 \quad \rightarrow \quad B + 2(9.3 - 2B) = 7.8 \quad \rightarrow \quad B = 3.6\]
\[S = 9.3 - 2B \quad \rightarrow \quad S = 9.3 - 2(3.6) \quad \rightarrow \quad S = 2.1\]

Therefore, \[3B + 4S = 3(3.6) + 4(2.1) = 19.2\]. So, the answer is e) $19.20.
5) Which of the following a solution of \((x + 3)^3 + (x + 5)^3 = 8\)?

a) \(x = -4\)  b) \(x = -3\)  c) \(x = -2\)  d) \(x = -1\)  e) \(x = 0\) [Problem submitted by Kian Kaviani, LACC Mathematics Department Chairman, Source: Kian Kaviani]

Solution: The fastest and easiest way to answer this question is substitute the choices into the given equation to see that the only solution listed is b) \(x = -3\).

Another approach is to let \(t = x + 4\). Then \(x + 3 = t - 1\) and \(x + 5 = t + 1\). Substitute these values into the given equation to get \((t - 1)^3 + (t + 1)^3 = 8\). Then factor the left side as the sum of two cubes.

\[
\begin{align*}
[(t - 1) + (t + 1)]((t - 1)^2 - (t - 1)(t + 1) + (t + 1)^2) &= 8 \\
[2t][t^2 + 3] &= 8 \\
[2t][t^2 + 3] &= 2 \cdot 4 \quad \rightarrow \quad 2t = 2 \quad \rightarrow \quad t = 1 \\
[2t][t^2 + 3] &= 2 \cdot 4 \quad \rightarrow \quad t^2 + 3 = 4 \quad \rightarrow \quad t = 1 \\
t = 1 \quad \rightarrow \quad 1 = x + 4 \quad \rightarrow \quad x = -3
\end{align*}
\]
6) Suppose Julie can do a job in 4 hours and Janice in 5 hours. If they work together for 2 hours and then Julie quits, how long will it take Janice to finish the job? 

a) 12 minutes  b) 18 minutes  c) 20 minutes  d) 25 minutes  e) 30 minutes [Problem submitted by Vin Lee, LACC Professor of Mathematics, Source: Vin Lee]

Solution: Julie can do the job in four hours; so, her rate is \( \frac{1}{4} \) of the job per hour. Janice’s rate is \( \frac{1}{5} \) of the job per hour. Working together their rate is \( \frac{1}{4} + \frac{1}{5} = \frac{9}{20} \) of the job per hour. Using the equation “rate x time = work done” we get \( \frac{9}{20} \cdot 2 = \frac{9}{10} \) of the job done in the first two hours leaving \( \frac{1}{10} \) of the job for Janice to do. \( \frac{1}{5} t = \frac{1}{10} \rightarrow t = \frac{1}{2} \) hour for Janice to finish. So, the answer is e) 30 minutes.
7) When rolling two fair dice, what is the probability of one die landing on a number less than three and the other die on an odd number?

a) $\frac{5}{18}$  b) $\frac{11}{36}$  c) $\frac{1}{3}$  d) $\frac{13}{36}$  e) $\frac{1}{4}$  [Problem submitted by Vin Lee, LACC Professor of Mathematics, Source: Vin Lee]

Solution: There are 36 possible outcomes when rolling two dice. There are 11 outcomes satisfying the given conditions: (1, 1), (1, 2), (1, 3), (1, 5), (2, 1), (2, 3), (2, 5), (3, 1), (3, 2), (5, 1), (5, 2). So the probability is $\frac{11}{36}$, which is b).
8) If \(x, x + 2,\) and \(x + 3\) are the first three terms of a geometric series, what is the sum of the first five terms of the series?

a) \(-\frac{31}{4}\)  b) \(\frac{31}{4}\)  c) \(-\frac{29}{4}\)  d) \(\frac{33}{4}\)  e) \(\frac{33}{4}\)

[Problem submitted by Vin Lee, LACC Professor of Mathematics, Source: Vin Lee]

Solution: A geometric series has the form \(a + ar + ar^2 + ar^3 + \cdots\). For any geometric the second term divided by the first, the third term divided by the second, the fourth term divided by the third, and so on are each of these is equal to \(r\), called the common ratio. Therefore,

\[
\frac{x + 2}{x} = \frac{x + 3}{x + 2},
\]

which can be solved to get \(x = -4\). So, the first three terms of the series are -4, -2, and -1. The common ratio is \(\frac{1}{2}\). The answer to the question is

\[
(-4) + (-2) + (-1) + (-\frac{1}{2}) + (-\frac{1}{4}) = -\frac{31}{4},
\]

which is a).
9) There are 6 men and 7 women in a ballroom dancing class. If 4 men and 4 women are chosen, how many man/woman pairings are possible?

a) $2 \cdot 3^2 \cdot 6 \cdot 7$  
b) $2 \cdot 5^2 \cdot 6^2 \cdot 7$  
c) $2 \cdot 5^2 \cdot 6^2 \cdot 7^2$  
d) $2^2 \cdot 5^2 \cdot 6^2 \cdot 7$  
e) $2 \cdot 5 \cdot 6 \cdot 7$

[Problem submitted by Megan Khatoonabadi, LACC Professor of Mathematics, Source: Megan Khatoonabadi]

Solution: The number of ways to choose 4 men out of 7 is $\binom{7}{4}$. The number of ways to choose 4 women out of 6 women is $\binom{6}{4}$. Once the 4 men and 4 women are chosen, then the number of ways to pair them is $4 \cdot 3 \cdot 2 \cdot 1 = 4!$. The answer to the question is

$$\binom{7}{4} \cdot \binom{6}{4} \cdot 4! = \frac{7!}{(4!) \cdot (3!)} \cdot \frac{6!}{(4!) \cdot (2!)} \cdot 4! = \frac{(7!) \cdot (6!)}{(3!) \cdot (2!) \cdot (4!)} = 2 \cdot 5^2 \cdot 6^2 \cdot 7,$$

which is b).
10) Suppose an undergraduate class has a total of 10 freshmen, 20 sophomores and 15 juniors. If these students are randomly lined up in a single line, which of the following is the probability that all the freshmen line up next to each other?

a) \( \frac{(35!)(10!)}{45!} \)  b) \( \frac{(36!)}{45!} \)  c) \( \frac{(36!)(10!)}{45!} \)  d) \( \frac{36!}{(10!)(45!)} \)  e) \( \frac{36 \cdot (35!)}{45!} \)  [Problem submitted by Megan Khatoonabadi, LACC Professor of Mathematics, Source: Megan Khatoonabadi]

Solution: The total number of ways the 45 students can line up is \( \frac{45!}{(10!)(20!)(15!)} \). The number of ways the freshmen can line up together is \( 36 \cdot \frac{35!}{(20!)(15!)} \). Now divide to get the probability: \( \frac{(36!)(10!)}{45!} \), which is c).
11) If $s_1 = 1!$, $s_2 = 1! + 2!$, $s_3 = 1! + 2! + 3!$, $s_4 = 1! + 2! + 3! + 4!$, and so on; then what is the one’s digit of $s_{100}$?

a) 5  b) 0  c) 9  d) 3  e) 1  [Problem submitted by Kian Kaviani, LACC Mathematics Department Chairman, Source: Kian Kaviani]

Solution: 

$s_1 = 1! = 1$
$s_2 = 1! + 2! = 3$
$s_3 = 1! + 2! + 3! = 9$
$s_4 = 1! + 2! + 3! + 4! = 33$
$s_5 = 1! + 2! + 3! + 4! + 5! = 153$

Note that for $n \geq 5$ the one’s digit of $n!$ is zero. Therefore, if $n \geq 4$, then the one’s digit of $s_n$ is 3. So, the answer is d) 3.
12) How many real number solutions does the equation \( x^2 = 2^x \) have?  

a) 0  b) 1  c) 2  d) 3  e) more than 3  

[Problem submitted by Vin Lee, LACC Professor of Mathematics, Source: Vin Lee]

Solution: Let \( y_1 = x^2 \), \( y_2 = 2^x \), and graph the two functions in the same coordinate system using the integers from -2 to 6 for the x-coordinates of each function. Note that the graph of \( y_1 \) is an upward opening parabola whose vertex is \((0,0)\), decreasing on the interval \((-\infty, 0)\), and increasing on the interval \((0, \infty)\).

<table>
<thead>
<tr>
<th>x</th>
<th>( y_1 = x^2 )</th>
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<tbody>
<tr>
<td>-2</td>
<td>4</td>
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<tr>
<td>6</td>
<td>36</td>
<td>64</td>
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</table>

Solutions to the given equation \( x^2 = 2^x \) are the \( x \) values for which \( y_1 = y_2 \). Two of these can be seen above are \( x = 2 \) and \( x = 4 \). There will not be any solutions greater than 4 because the exponential function is above the quadratic function and is increasing at a greater rate than the quadratic function for \( x > 4 \).

Note also that \( y_1 > y_2 \) when \( x = -1 \) and \( y_1 < y_2 \) when \( x = 0 \); so, the curves intersect between \( x = -1 \) and \( x = 0 \) [at approximately \( x = -0.72 \)]. So the answer to the question is d) 3 solutions.
13) Suppose an equilateral triangle is inscribed in a circle. Which of the following is closest the area of the triangle divided by the area of the circle?

a) .30  b) .35  c) .40  d) .45  e) .50  [Problem submitted by Vin Lee, LACC Professor of Mathematics, Source: Vin Lee]

Solution: Let $R$ be the radius of the circle and $L$ the length of each side of the triangle. Situate the triangle with one of its sides horizontal and the opposite vertex above. Draw a line segment from the center of the circle to the lower right vertex of the triangle. Now draw a vertical height of the triangle. Note that the segment of the height from the base of the triangle to the center of the circle, the segment from the center of the circle to the lower right vertex of the triangle, and the right half of the base of the triangle form a 30, 60, 90 degree triangle whose hypotenuse is $R$ and one of whose legs is $\frac{1}{2}L$. So, $\frac{R}{\frac{1}{2}L} = \frac{2}{\sqrt{3}}$ and $R = \frac{L}{\sqrt{3}}$. Now let the height of the inscribed triangle be $h$. Then $\frac{h}{\frac{1}{2}L} = \frac{\sqrt{3}}{1}$ and $h = \frac{\sqrt{3}}{2}L$. The area of the circle is $A = \pi R^2 = \pi \left(\frac{L}{\sqrt{3}}\right)^2 = \frac{\pi L^2}{3}$. The area of the triangle is $A = \frac{1}{2} \text{base} \cdot \text{height} = \frac{1}{2} \cdot \frac{\sqrt{3}}{2}L \cdot \frac{\sqrt{3}}{2}L = \frac{\sqrt{3}}{4}L^2$. The area of the triangle divided by the area of the circle is $\frac{3\sqrt{3}}{4\pi}$. Approximating $\pi$ with 3.14 and $\sqrt{3}$ with 1.7 gives $\frac{\pi}{3\sqrt{3}} \approx .3911$. A closer approximation is .4135. Either of these is closest to c) .40.
14) Suppose an arithmetic sequence [a sequence of the form $a, a+d, a+2d, a+3d, \ldots$] has 3 consecutive terms which are $3, -7x - 6, \ldots$. What is the sum of the next two terms?

a) -11 only  

b) $-\frac{183}{2}$ only  
c) $-\frac{179}{2}$ only  
d) either -11 or $-\frac{183}{2}$  
e) either $-\frac{183}{2}$ or $-\frac{179}{2}$  

[Problem submitted by Vin Lee, LACC Professor of Mathematics, Source: Vin Lee]

Solution: In an arithmetic sequence, the difference between consecutive terms is $d$. So, $3 - 10x^2 = d$ and $(-7x - 6) - 3 = d$, which means

$$3 - 10x^2 = (-7x - 6) - 3$$
$$10x^2 - 7x - 12 = 0$$
$$(2x - 3)(5x + 4) = 0 \quad \rightarrow \quad x = -\frac{4}{5}, \frac{3}{2}$$

$$x = -\frac{4}{5}, \frac{3}{2}$$

If $x = -\frac{4}{5}$, then $d = 3 - 10 \left(-\frac{4}{5}\right)^2 = -\frac{17}{5}$. Now let $a_k = -7x - 6 = -\frac{2}{5}$. Then the sum of the next two terms is $a_{k+1} + a_{k+2} = -\frac{19}{5} - \frac{36}{5} = -11$

If $x = \frac{3}{2}$, then $d = 3 - 10 \left(\frac{3}{2}\right)^2 = -\frac{39}{2}$. Now let $a_k = -7x - 6 = -\frac{33}{2}$. Then the sum of the next two terms is $a_{k+1} + a_{k+2} = -36 - \frac{111}{2} = -\frac{183}{2}$.

Therefore, the answer is d) either -11 or $-\frac{183}{2}$. 
15) Suppose a circle is inscribed in an equilateral triangle. Which of the following is closest to the area of the circle divided by the area of the triangle?
a) .57  b) .61  c) .65  d) .69  e) .73  [Problem submitted by Kian Kaviani, LACC Mathematics Department Chairman, Source: Kian Kaviani]

Solution: Let $R$ be the radius of the circle and $L$ the length of each side of the triangle. Situate the triangle with one of its sides horizontal and the opposite vertex above. Draw a line segment from the center of the circle to the lower right vertex of the triangle. Now draw a vertical radius to the point on the base of the triangle which is tangential to the circle. These two line segments together with the right half of the base form a 30, 60, 90 degree triangle the length of whose legs are $R$ and $\frac{L}{2}$. Then $\frac{R}{L} = \frac{1}{\sqrt{3}}$. So, $R = \frac{L}{2\sqrt{3}}$ and the area of the circle is

$$A = \pi R^2 = \pi \left( \frac{L}{2\sqrt{3}} \right)^2 = \frac{\pi L^2}{12}$$

Let $h$ be the height of the triangle. Then $\frac{h}{L} = \frac{\sqrt{3}}{1}$ So, $h = \frac{\sqrt{3}}{2} L$ and the area of the triangle is

$$A = \frac{1}{2} \text{base} \times \text{height} = \frac{1}{2} L \cdot \frac{\sqrt{3}}{2} L = \frac{\sqrt{3}}{4} L^2$$

Now divide the area of the circle by the area of triangle to get $\frac{\pi}{3\sqrt{3}}$. Approximating $\pi$ as 3.14 and $\sqrt{3}$ as 1.7 gives $\frac{\pi}{3\sqrt{3}} \approx .6157$. A closer approximation is .6046. Either of these is closest to b) .61.
16) Let $s_n = 1! + 2! + 3! + \cdots + n!$ For how many integers $n$ is $\sqrt{s_n}$ an integer?

a) 1  b) 2  c) 3  d) 4  e) more than 4 [Problem submitted by Kian Kaviani, LACC Mathematics Department Chairman, Source: Kian Kaviani]

Solution: $s_1 = 1! = 1 \quad \rightarrow \quad n = 1$ is a solution

$s_2 = 1! + 2! = 3$

$s_3 = 1! + 2! + 3! = 9 \rightarrow \quad n = 3$ is a solution

$s_4 = 1! + 2! + 3! + 4! = 33$

$s_5 = 1! + 2! + 3! + 4! + 5! = 153$

Note that for $n \geq 4$ the ones digit of $s_n$ is 3; so, $\sqrt{s_n}$ is not an integer. Therefore, the answer to the question is b) 2
17) How many real number solutions does this equation have? \( |x^2 - 4| + |x| + 2x = 2 \)

a) 1  b) 2  c) 3  d) 4  e) more than 4 [Problem submitted by Kian Kaviani, LACC Mathematics Department Chairman, Source: Kian Kaviani]

Solution: Divide the possible x values into four cases: A. \( x < -2 \)  B. \( -2 \leq x < 0 \)  C. \( 0 \leq x < 2 \)  D. \( x \geq 2 \)

Case A: \( |x^2 - 4| + |x| + 2x = x^2 - 4 - x + 2x = 2 \)
\[ x^2 + x - 6 = 0 \quad \rightarrow \quad (x + 3)(x - 2) = 0 \quad \rightarrow \quad x = -3 \]

Case B: \( |x^2 - 4| + |x| + 2x = 4 - x^2 - x + 2x = 2 \)
\[ x^2 - x - 2 = 0 \quad \rightarrow \quad (x + 1)(x - 2) = 0 \quad \rightarrow \quad x = -1 \]

Case C: \( |x^2 - 4| + |x| + 2x = 4 - x^2 + x + 2x = 2 \)
\[ x^2 - 3x - 2 = 0 \quad \rightarrow \quad x = \frac{3+\sqrt{17}}{2}, \text{ but neither of these is in the interval } [0,2). \]

Case D: \( |x^2 - 4| + |x| + 2x = x^2 - 4 + x + 2x = 2 \)
\[ x^2 + 3x - 6 = 0 \quad \rightarrow \quad x = \frac{-3+\sqrt{33}}{2}, \text{ but neither of these is in the interval } [2,\infty). \]

Therefore the answer to the question is 2 real solutions, which is b).
18) Which of the following is the sum of $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + 100 \cdot 101$?

a) 343,040  b) 346,400  c) 322,400  d) 348,400  e) none of these [Problem submitted by Kian Kaviani, LACC Mathematics Department Chairman, Source: Kian Kaviani]

Solution:

\[
\begin{align*}
1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + 100 \cdot 101 &= \sum_{n=1}^{100} n(n + 1) \\
&= \sum_{n=1}^{100} (n^2 + n) \\
&= \sum_{n=1}^{100} n^2 + \sum_{n=1}^{100} n \\
&= \frac{100(100+1)(2\cdot100+1)}{6} + \frac{100(100+1)}{2} \\
&= \frac{100(100+1)(2\cdot100+1)}{6} + \frac{3\cdot100(100+1)}{6} \\
&= \frac{100(100+1)(2\cdot100+3)}{6} \\
&= \frac{100(100+1)(2\cdot100+4)}{6} \\
&= \frac{100(100+1)(100+2)}{3} \\
&= 100 \cdot 101 \cdot 34 \\
&= 343,400
\end{align*}
\]

Therefore, the answer is e) none of these.
19) Suppose two bicycle riders are headed toward each other, the first at 10 mph and the second at 15 mph. At the instant the bicyclists are 30 miles apart, a bird flying at 20 mph and headed in the same direction as the first rider, passes the first rider. When the bird reaches the second rider, it instantaneously changes direction and heads back toward the first rider. When the bird reaches the first rider it again instantaneously changes direction and heads toward the second rider. If the bird continues to go back and forth at 20 mph, how far will it travel from the time the bikers are 30 miles apart until they reach each other?

a) 24 miles  b) 26 miles  c) 27 miles  d) 32 miles  e) none of these  [Problem submitted by Vin Lee, LACC Professor of Mathematics, Source: Vin Lee]

Solution: Since one rider is going 10 mph and the other 15 mph, the gap between them is being closed at a rate of 25 mph.

\[\text{time} = \frac{\text{distance}}{\text{speed}} = \frac{30 \text{ miles}}{25 \text{ mph}} = \frac{6}{5} \text{ hours}\]

So, the bird is flying 20 mph for \(\frac{6}{5}\) hours.

Therefore, the distance the bird flies is \(\text{speed} \cdot \text{time} = 20 \text{ mph} \cdot \frac{6}{5} \text{ hours} = 24 \text{ miles}\), which is a).
20) Solve for \( x \): \( 3 \cdot 16^x + 36^x - 2 \cdot 81^x = 0 \).

a) \( x = 2 \)  b) \( x = -\frac{1}{2} \)  c) \( x = \frac{1}{4} \)  d) \( x = \frac{3}{2} \)  e) \( x = \frac{1}{2} \)  [Problem submitted by Kian Kaviani, LACC Mathematics Department Chairman, Source: Kian Kaviani]

Solution: Divide both sides of the equation by \( 36^x \) to get

\[
3 \cdot \left( \frac{16}{36} \right)^x + 1 - 2 \cdot \left( \frac{81}{36} \right)^x = 0
\]

\[
3 \cdot \left( \frac{4}{9} \right)^x + 1 - 2 \cdot \left( \frac{9}{4} \right)^x = 0,
\]

Now let \( u = \left( \frac{4}{9} \right)^x \) to get

\[
3u + 1 - \frac{1}{u} = 0
\]

\[
3u^2 + u - 1 = 0
\]

\[
(3u - 2)(u + 1) = 0 \quad \rightarrow \quad u = -1, \frac{2}{3}
\]

However, \( u = \left( \frac{4}{9} \right)^x > 0 \) \( \rightarrow \) \( u \neq -1 \) \( \rightarrow \) \( u = \frac{2}{3} \)

So, \( u = \left( \frac{4}{9} \right)^x \) \( \rightarrow \) \( \frac{2}{3} = \left( \frac{4}{9} \right)^x = \left( \frac{2}{3} \right)^{2x} \) \( \rightarrow \) \( x = \frac{1}{2} \) \( \rightarrow \) e) \( \frac{1}{2} \) is the answer.
21) There are two quadratic polynomials with real number coefficients, $x^2 + ax + b$ and $x^2 + cx + d$, each of which will divide into $x^4 + 9$ with no remainder. For these two quadratics, what is $a + b + c + d$?

a) 0  b) $2\sqrt{3}$  c) $2\sqrt{6}$  d) 3  e) 6  [Problem submitted by Kian Kaviani, LACC Mathematics Department Chairman, Source: Kian Kaviani]

Solution: $x^4 + 9 = x^4 + 6x^2 + 9 - 6x^2$

$x^4 + 9 = (x^2 + 3)^2 - 6x^2$

$= (x^2 + 3)^2 - (\sqrt{6}x)^2$

$= (x^2 + 3 - \sqrt{6}x)(x^2 + 3 + \sqrt{6}x)$

$= (x^2 - \sqrt{6}x + 3)(x^2 + \sqrt{6}x + 3)$  →  The answer is e) 6.
22) Which of the following is the sum of \( 3 + 33 + 333 + 3,333 + \cdots + 3,333,333,333 \)?

a) 3,703,703,613 b) 3,703,703,616 c) 3,703,703,700 d) 3,703,703,703 e) none of these

[Problem submitted by Kian Kaviani, LACC Mathematics Department Chairman, Source: Kian Kavianil]

Solution:

\[
3 + 33 + 333 + 3,333 + \cdots + 3,333,333,333 = \frac{1}{3} [9 + 99 + 999 + \cdots + 9,999,999,999] \\
= \frac{1}{3} [(10 - 1) + (10^2 - 1) + (10^3 - 1) + \cdots + (10^{10} - 1)] \\
= \frac{1}{3} [10 + 10^2 + 10^3 \cdots + 10^{10} - 10] \\
= \frac{1}{3} [11,111,111,110 - 10] \\
= \frac{1}{3} \cdot 11,111,111,100 \\
= 3,703,703,700, \text{ which is c).}
\]
23) Which of the following is equal to \( \frac{1}{\sqrt[3]{2} + \sqrt[4]{4} + \sqrt{2} + \sqrt[6]{2} + \sqrt[6]{2} + 1} \)?

a) \( \frac{1}{2^3 - 1} \)  

b) \( \frac{1}{\frac{1}{2^6 - 1}} \)  

c) \( \frac{1}{2^5 - 1} \)  

d) \( \frac{1}{\frac{1}{2^5} - 1} \)  

e) \( \frac{1}{\frac{1}{6^6 - 1}} \)  

[Problem submitted by Kian Kaviani, LACC Mathematics Department Chairman, Source: Kian Kaviani]

Solution:

\[
\sqrt[3]{32} + \sqrt[4]{4} + \sqrt{2} + \sqrt[6]{2} + 1 = 2^\frac{1}{3} + 4^\frac{1}{4} + 2^\frac{1}{2} + 2^\frac{1}{6} + 1 \\
= 32^\frac{1}{6} + 4^\frac{1}{4} + 2^\frac{1}{2} + 2^\frac{1}{6} + 1 \\
= 1 + 2^\frac{1}{6} + 2^\frac{2}{6} + 2^\frac{3}{6} + 4^\frac{1}{6} + 2^\frac{5}{6} \\
= 1 + 1 \cdot (2^\frac{1}{6}) + 1 \cdot (2^\frac{1}{6})^2 + 1 \cdot (2^\frac{1}{6})^3 + 1 \cdot (2^\frac{1}{6})^4 + 1 \cdot (2^\frac{1}{6})^5 \\
= \sum_{n=1}^{6} 1 \cdot (2^\frac{1}{6})^{n-1} \\
= \frac{1 \cdot [1 - (2^\frac{1}{6})^6]}{1 - 2^\frac{1}{6}} \\
= \frac{1 \cdot [1 - 2]}{1 - 2^\frac{1}{6}} \\
= \frac{1}{2^\frac{1}{6} - 1} \\

Therefore, \( \frac{1}{\sqrt[3]{32} + \sqrt[4]{4} + \sqrt{2} + \sqrt[6]{2} + \sqrt[6]{2} + 1} = 2^\frac{1}{6} - 1 \), which is c).
Suppose a circle is inscribed in a square each of whose sides is one unit in length. Let \( A_1 \) be the area of the square minus the area of the circle. Next a second square is inscribed inside the first circle and a second circle is inscribed inside the second square. Let \( A_2 \) be the area of the second square minus the area of the second circle. Then a third square is inscribed inside the second circle and a third circle inside the third square. Let \( A_3 \) be the area of the third square minus the area of the third circle. If this process is repeated until \( A_{10} \) is found, what is \( A_1 + A_2 + A_3 + \cdots + A_{10} \)?

a) \( \frac{1023}{512} \left(1 - \frac{\pi}{4}\right) \)  
b) \( \frac{1023}{1024} \left(1 - \frac{\pi}{4}\right) \)  
c) \( \frac{511}{512} \left(1 - \frac{\pi}{4}\right) \)  
d) \( \frac{255}{256} \left(1 - \frac{\pi}{4}\right) \)  
e) none of these  

[Problem submitted by Kian Kaviani, LACC Mathematics Department Chairman, Source: Kian Kaviani]

Solution: Since the radius of the first circle is \( \frac{1}{2} \), its area is \( \frac{\pi}{4} \); so, \( A_1 = 1 - \frac{\pi}{4} \). The diagonal of the second square is 1; so, the side of the second square is \( \frac{1}{\sqrt{2}} \) and the radius of the second circle is \( \frac{1}{2\sqrt{2}} \). Therefore, \( A_2 = \left(\frac{1}{\sqrt{2}}\right)^2 - \pi \left(\frac{1}{2\sqrt{2}}\right)^2 = \frac{1}{2} - \frac{\pi}{8} = \frac{1}{2} \left(1 - \frac{\pi}{4}\right) \). The diagonal of the third square is twice the radius of the second circle, \( 2 \cdot \frac{1}{2\sqrt{2}} = \frac{1}{\sqrt{2}} \); so, the length of each of its sides is \( \frac{1}{2} \) and the radius of the third circle is \( \frac{1}{4} \). Therefore, \( A_3 = \left(\frac{1}{2}\right)^2 - \pi \left(\frac{1}{4}\right)^2 = \frac{1}{4} - \frac{\pi}{16} = \frac{1}{4} \left(1 - \frac{\pi}{4}\right) \).

Continuing with this pattern, we get

\[
A_1 + A_2 + A_3 + \cdots + A_{10} = \left(1 - \frac{\pi}{4}\right) + \frac{1}{2} \left(1 - \frac{\pi}{4}\right) + \frac{1}{4} \left(1 - \frac{\pi}{4}\right) + \cdots + \frac{1}{512} \left(1 - \frac{\pi}{4}\right)
\]

\[
= \left(1 - \frac{\pi}{4}\right) \left(1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots + \frac{1}{2^n}\right)
\]

\[
= \left(1 - \frac{\pi}{4}\right) \cdot \frac{1 - \left(\frac{1}{2}\right)^{10}}{1 - \frac{1}{2}}
\]

\[
= \left(1 - \frac{\pi}{4}\right) \cdot \frac{1 - \left(\frac{1}{2}\right)^{10}}{\frac{1}{2}}
\]

\[
= 2 \cdot \frac{1023 - 1}{1024} \left(1 - \frac{\pi}{4}\right)
\]

\[
= \frac{1023}{512} \left(1 - \frac{\pi}{4}\right), \text{ which is a}.\]
25) In the drawing below ABC is a sector of a circle whose central angle is 90\(^\circ\). Angle ADB is also 90\(^\circ\). Angle EBD is 30\(^\circ\). The length of segment AD is 20 more than the length of segment BD. Which of the following is the radius of the circle?

a) 20 \hspace{1cm} b) \frac{30(\sqrt{3}-1)}{2} \hspace{1cm} c) \frac{(\sqrt{3}-1)^2+30\sqrt{3}}{3} \hspace{1cm} d) 25(\sqrt{3}-1) \hspace{1cm} e) none of these

[Problem submitted by Angela Wayne, LACC Professor of Mathematics, Source: global-math.com]

Solution: Note that \(\triangle BDE\) is a \(30^\circ, 60^\circ, 90^\circ\) implying that the ratio of the lengths of its sides is \(\sqrt{3}:1:2\). Let \(x\) be the length of segment BD. Then the length of segment ED is \(\frac{1}{\sqrt{3}}x\) and the length of segment BE is \(\frac{2}{\sqrt{3}}x\).

Now let \(r\) be the radius of the circle, \(y\) the length of segment AE, and \(z\) the length of segment CE to get the drawing below.
Since angle BED is $60^\circ$, so is angle AEC. Therefore, $\triangle ACE$ is $30^\circ, 60^\circ, 90^\circ$, and one of its sides is $r$. This implies $z = \frac{1}{\sqrt{3}}r$ and $y = \frac{2}{\sqrt{3}}r$.

Note that $z + \frac{2}{\sqrt{3}}x = r$ $\rightarrow$ $\frac{1}{\sqrt{3}}r + \frac{2}{\sqrt{3}}x = r$ $\rightarrow$ $x = \frac{(\sqrt{3}-1)}{2}r$.

We are given that the length of AD = the length of BD + 20 cm, which may be written as

$y + \frac{1}{\sqrt{3}}x = x + 20$ $\rightarrow$ $\frac{2}{\sqrt{3}}r + \frac{1}{\sqrt{3}} \cdot \frac{(\sqrt{3}-1)}{2}r = \frac{(\sqrt{3}-1)}{2}r + 20$, which can be solved for $r$ to get $r = 20$. So, the answer is a) 20.