1. If $f(x) = 4x - 2$ and $g(x) = x^2$, find $f(g(f(1)))$.

A. 2 B. 6 C. 14 D. 18 E. 34

2. Define the operation $\lozenge$ by $a \lozenge b = a^2 + b$. Find $(1 \lozenge 3) \lozenge (3 \lozenge 1)$.

A. 20 B. 26 C. 35 D. 104 E. 110

3. The equation $x^5 + y^3 + z^2 = 2007$ has a unique solution (a, b, c), for positive integers a, b, and c. Find a + b + c.

A. 18 B. 21 C. 24 D. 27 E. 30

4. The ratio of the width to the length of a rectangle is 0.6. If the length is increased by 20% and the width is increased by 10%, the ratio of the width to the length of the new rectangle is

A. 0.5 B. 0.52 C. 0.54 D. 0.55 E. 0.56

5. Replace each letter of AMATYC with a digit 0 through 9 to form a six-digit number (identical letters are replaced by identical digits, different letters are replaced by different digits). If the resulting number is the smallest such number which is a perfect square, find the sum of its digits (that is, A + M + A + T + Y + C).

A. 25 B. 26 C. 27 D. 28 E. 29

6. For $0 < t < \pi/2$, $\sec t \tan t + \csc t =$

A. $\sec t \cot^2 t$ B. $\sec^2 t \cot t$ C. $\tan t \csc^2 t$ D. $\sec t \csc^2 t$ E. $\sec^2 t \csc t$

7. If $\ln s = \frac{2}{3}$ and $\ln t = \frac{1}{6}$, find $\log_{st} e^5$.

A. 3.6 B. 5 C. 5.4 D. 6 E. 10.8

8. A seven-digit number contains each of the digits 1 through 7. The first digit is divisible by 7, the first two digits (viewed as a two-digit number) are divisible by 6, the first three digits are divisible by 5, and so forth. What are the last two digits of the number?

A. 43 B. 41 C. 63 D. 61 E. 21

9. In how many different ways can the expression pqrpqrpqrr be written using the Commutative Property of Multiplication without using any exponents?

A. 1024 B. 864 C. 4200 D. 16800 E. 3628800

10. A point is chosen at random from the interior of square *ABCD*. Find the probability that the point is closer to one of the points *A* or *C* than to the midpoint of diagonal *AC*.

A. $\frac{1}{4}$ B. $\frac{1}{3}$ C. $\frac{3}{8}$ D. $\frac{1}{2}$ E. $\frac{3}{4}$

11. The set of points whose distance from (-5, -6) is three times their distance from (3, 2) forms a circle. Find its radius.

A. $2\sqrt{3}$ B. $3\sqrt{2}$ C. $2\sqrt{5}$ D. $4\sqrt{2}$ E. $3\sqrt{3}$

12. Knaves always lie and knights always tell the truth. Al says, "Either Bo or Ed is a knight," Bo says, "Cy is a knight," Cy says, "Ed is a knave," and Ed says, "Al is a knave." How many of them are knights?

A. 0 B. 1 C. 2 D. 3 E. 4

13. A triangle has sides of length 10, 17, and 21. Find the sum of the reciprocals of the lengths of its three altitudes.

A. 1/4 B. 3/11 C. 2/7 D. 1/3 E. 2/5

14. How many different 3-letter strings can be formed from the letters of FACULTY MATH LEAGUE (no letter can be used in a given string more times than it appears in the phrase)? Place your answer in the corresponding blank on the answer sheet.

15. For the system $\begin{cases} x+2y-2z=5 \\ 2x+ky+3z=1 \\ x+3y+kz=2 \end{cases}$, there are exactly two values of k for which the system is inconsistent. Find the absolute value of their difference.

A. 1 B. 2 C. 4 D. 6 E. 8

16. The letters of the alphabet are assigned consecutive positive integers starting with n, so that A = n, B = n+1, C = n+2, . . . , , Z = n+25. If n is chosen to be the least value for which, of the letters AMATYC, all but one are prime, which of the following is true?

A. Y is not prime B. A + C = M + 1 C. A*C = M D. A + C is a square E. A + M + C = Y

17. If $2a - 4b = 128b^3 - 16a^3$ and $a \neq 2b$, find the value of $a^2 + 2ab + 4b^2$.

A. -1/8 B. -1/2 C. 1/4 D. 1/2 E. 1

18. Six letters addressed to different recipients are randomly placed into their six addressed envelopes. Find the probability that exactly two letters are placed into the proper envelopes.

A. $\frac{1}{8}$ B. $\frac{3}{16}$ C. $\frac{4}{15}$ D. $\frac{5}{16}$ E. $\frac{3}{8}$

19. When $P(x) = x^{50} - 2x^{49} - 2x^{47} + 8x^{45} + 2x^3 - 4x + 2$ is divided by $D(x) = x^3 - 2x^2 - x + 2$, the remainder is

A. $x^2 + 2x + 2$ B. $x^2 + 3x + 1$ C. $x^2 + 3x + 2$ D. $x^2 + 2x + 1$ E. $x^2 + x + 1$

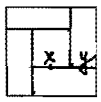
20. There are exactly three ordered pairs (a, b), (c, d), and (e, f) which satisfy the equation $2x^3 + 12xy + 2y^3 - 16 = 0$ and form the vertices of an equilateral triangle. Find the area of the triangle. (Hint: $2x^3 + 12xy + 2y^3 - 16$ has a factor $x + y + b$ for some integer b).

A. $6\sqrt{3}$ B. $8\sqrt{2}$ C. $5\sqrt{5}$ D. $4\sqrt{6}$ E. $9\sqrt{2}$

AMATYC Contest Round 1 Oct-Nov 2007

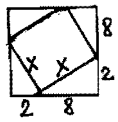
① $x + 4\frac{1}{3}x = 64,\ \frac{16}{3}x = 64,\ x = 12$ (C)

② $3\Delta 2 = 3\cdot 2 + 2 = 8,\ 2\Delta 3 = 2\cdot 3 + 3 = 9,$
$8\Delta 9 = 8\cdot 9 + 9 = 81$ (D)

③ 
$x^2 = \frac{4}{9}(x+2y)^2 \,\Big|\, \div x^2$
$9 = 4\left(1 + 2\frac{y}{x}\right)^2,$
$\frac{y}{x} = \frac{1}{4},\ \frac{x+y}{x} = \frac{5}{4}$ (A)

④ $6\cdot 24\cdot 7 = 1008$, difference $= 8$ (A)

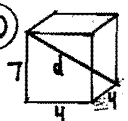
⑤ if all coins are nickels, they worth $5\cdot 24 = 120$¢; to increase up to 175¢, we need to replace $(175-120)\div 5 = 11$ coins (B)

⑥ 
$A = x^2 = 8^2 + 2^2$
$A = 64 + 4 = 68$ (E)

⑦ $\frac{s\cdot 12}{60} = s - 16,\ s = 5s - 80,\ s = 20$ (D)

⑧ $\left.\begin{array}{l}\frac{A}{2} + B = 7\\ \frac{A}{2} + C = 8\end{array}\right\} \Rightarrow \frac{A+B+C}{3} = \frac{7+8}{3} = 5$ (C)

⑨ $948^2 = 898{,}704$
$8+9+8+7+0+4 = 36$ (E)

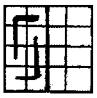
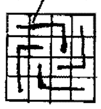
⑩ 
$d^2 = 4^2 + 4^2 + 7^2$
$d^2 = 81,\ d = 9$ (B)

⑪ 3 people $= 4\%$
difference $= x$ people $= 60\% - 40\% = 20\%$
$x = \frac{3\cdot 20}{4} = 15$ (C)

⑫ $2! \times 3! \times 3!$
$= 2\times 6\times 6 = 72$ (B)

⑬ $N + (2N)^2$
$= N\cdot(1+4N) =$ prime
if $N = 1, -1$ (C)

⑭ 16 should be paired with 9; if 15 paired with 1, then 8 is without pair, therefore, 15 is with 10; and 8 paired with 1; then 7 with 2, 6 with 3; next 14 with 11, 13 with 12, and 5 with 4. The largest difference is $16 - 9 = 8 - 1 = 7$ (B)

⑮  If you divide the square by the vertical line, each part (left and right) could be covered by 2 ways, total $= 2\times 2 = 4$ ways; dividing the square by the horizontal line, we have again 4 ways to cover. Plus 2 ways as shown. Totally we have 10 ways.  (E)

⑯ $\theta = \frac{360}{n}$, $1,2,3,4,5,6 \mid 360$, but $7 \nmid 360$ (E)

⑰ $\frac{m}{15} + \frac{n}{21} = \frac{7m+5n}{3\cdot 5\cdot 7},\ 3 \mid 7m+5n \Rightarrow 3 \mid m+2n$
for each of $m = 1,2,4,7,8,11,13,14$ we have 6 cases, total $= 8\times 6 = 48$ cases (B)

⑱ $r(s^2-1) = 14s - 13$
$s = 0, 2$ (only!)
$(r,s,t) = (5,2,4)$
or $(13,0,14)$ (A)

⑲ $\frac{K^2 + (K+1)^2 + \cdots + (K+16)^2}{17} = \frac{17K^2 + K(2+4+\cdots+32) + (1^2+2^2+\cdots+16^2)}{17}$
$= K^2 + 16K + 88 = (K+8)^2 + 24$
$24 = 2^3\cdot 3 = 2^r\cdot m,\ r = 3$ (D)

⑳ $\frac{x}{\sin 2\theta} = \frac{9}{\sin\theta} \Rightarrow x = 18\cdot\cos\theta$, (C)
$9^2 = 7^2 + x^2 - 14x\cdot\frac{x}{18},\ x^2 = 144,\ x = 12$