1. If $g(x - 1) = x^2 + 1$, find $g(2)$.

A. 1 B. 2 C. 5 D. 9 E. 10

2. Airport runways are labeled by two numbers giving the nonnegative clockwise angles less than 360° of the runway's direction measured from north to the nearest 10°, divided by 10. Thus a runway with heading 223° is labeled 22. What is the other number on this runway?

A. 4 B. 14 C. 16 D. 32 E. 40

3. The equation $a^3 + b^3 + c^3 = 2008$ has a solution in which $a$, $b$, and $c$ are distinct even positive integers. Find $a + b + c$.

A. 20 B. 22 C. 24 D. 26 E. 28

4. For how many different integers $b$ is the polynomial $x^2 + bx + 16$ factorable over the integers?

A. 2 B. 3 C. 4 D. 5 E. 6

5. Let $f(x) = x^2 - 2x + 4$. Which of the following is a factor of $f(x) - f(2y)$?

A. $x + 2y$ B. $x + 2y + 2$ C. $x - 2y + 2$ D. $x + 2y - 2$ E. none of these

6. In square *MATH*, *M* and *A* lie on a circle of radius 20, and the circle is tangent to side $\overline{TH}$ at the midpoint of $\overline{TH}$. Find the lengths of the sides of the square.

A. 24 B. 26 C. 28 D. 30 E. 32

7. A fair coin is labeled A on one side and M on the other; a fair die has two sides labeled T, two labeled Y, and two labeled C. The coin and die are each tossed three times. Find the probability that the six letters can be arranged to spell AMATYC.

A. $\frac{1}{60}$ B. $\frac{1}{48}$ C. $\frac{1}{36}$ D. $\frac{1}{24}$ E. $\frac{1}{12}$

8. What is the value of $(\log_{624} 625)(\log_{623} 624)(\log_{622} 623)\ldots(\log_6 7)(\log_5 6)$?

A. 2 B. 2.5 C. 4 D. 5 E. 6

9. The letters AMATYC are written in order, one letter to a square of graph paper, to fill 100 squares. If three squares are chosen at random without replacement, find the probability to the nearest 1/10 of a percent of getting three A's.

A. 3.3% B. 3.7% C. 4.0% D. 7.3% E. 11.1%

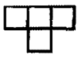
10. A student committee must consist of two seniors and three juniors. Five seniors are able to serve on the committee. What is the least number of junior volunteers needed if the selectors want at least 600 different possible ways to pick the committee?

A. 6 B. 7 C. 8 D. 9 E. 10

11. Ed drives to work at a constant speed S. One day he is halfway to work when he immediately turns around, speeds up by 8 mph, and drives home. As soon as he is home, he turns around and drives to work at 6 mph faster than he drove home. His total driving time is exactly 67% greater than usual. Find S in mph and write the answer in the corresponding blank on the answer sheet.

12. Each bag to be loaded onto a plane weighs either 12, 18, or 22 lb. If the plane is carrying exactly 1000 lb of luggage, what is the largest number of bags it could be carrying?

A. 80 B. 81 C. 82 D. 83 E. 84

13. An 8x8 checkerboard is exactly covered by 16  shaped tiles. What is the least possible number of tiles for which the  is horizontal?

A. 0 B. 2 C. 4 D. 6 E. 8

14. Call a positive integer *biprime* if it is the product of exactly two distinct primes (thus 6 and 15 are biprime, but 9 and 12 are not). If N is the smallest number such that N, N + 1, and N + 2 are all biprime, find the largest prime factor of N(N + 1)(N + 2).

A. 13 B. 17 C. 29 D. 43 E. 47

15. You have 8 identical red counters and n identical green counters. You find that you can line them up in a single row in such a way that the number of counters whose right-hand neighbor is the same color equals the number of counters whose right-hand neighbor is the other color. What is the largest possible value of n?

A. 17 B. 19 C. 21 D. 25 E. 27

16. If $b$ and $c$ are positive integers such that $b/11$, $c/b$, and $c/15$ all lie in the interval (1.5, 1.8), find $b + c$.

A. 43 B. 44 C. 45 D. 46 E. 47

17. Let $r$, $s$, and $t$ be nonnegative integers. For how many such triples $(r, s, t)$ satisfying the system $\begin{cases} rs + t = 24 \\ r + st = 24 \end{cases}$ is it true that $r + s + t = 25$?

A. 23 B. 24 C. 25 D. 26 E. 27

18. In $\Delta ABC$, AB = AC = 25 and BC = 14. The perpendicular distances from a point P in the interior of $\Delta ABC$ to each of the three sides are equal. Find this distance.

A. $\frac{9}{2}$ B. $\frac{19}{4}$ C. 5 D. $\frac{21}{4}$ E. $\frac{11}{2}$

19. The digits 1 to 9 can be separated into 3 disjoint sets of 3 digits each so that the digits in each set can be arranged to form a 3-digit perfect square. Find the last two digits of the sum of these three perfect squares.

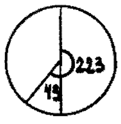
A. 26 B. 29 C. 34 D. 46 E. 74

20. The sequence $\{a_n\}$ is defined by $a_0 = a_1 = a_2 = 1$, and $a_{n-3}a_n - a_{n-2}a_{n-1} = (n-3)!$ for $n \geq 3$. If $5^k$ is the largest power of 5 that is a factor of $a_{100}a_{101}$, find $k$.

A. 20 B. 22 C. 24 D. 25 E. 26

# AMATYC Contest, Round 2 Feb–March 2008

① $g(2)=g(3-1)=3^2+1=10$ (E)

② 

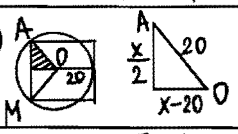
$223-180=43\approx 40$

$40\div 10=4$ (A)

③ $4^3+6^3+12^3=2008$, $a+b+c=22$ (B)

④ $b=\pm 17, \pm 10, \pm 8$ (E)

⑤ $f(x)-f(2y)=x^2-2x-4y^2+4y=(x-2y)(x+2y-2)$ (D)

⑥ 

$\left(\frac{x}{2}\right)^2+(x-20)^2=20^2$, $x=32$ (E)

⑦ $p(\text{coin: MAA or AMA or AAM})=\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot 3$

$p(\text{die: 1T, 1Y, 1C})=\frac{6}{6}\cdot\frac{4}{6}\cdot\frac{2}{6}=\frac{2}{9}$; $\frac{3}{8}\cdot\frac{2}{9}=\frac{1}{12}$ (E)

⑧ $\frac{\log_5 625}{\log_5 624}\cdot\frac{\log_5 624}{\log_5 623}\cdot\ldots\cdot\frac{\log_5 7}{\log_5 6}\cdot\frac{\log_5 6}{\log_5 5}=\frac{\log_5 5^4}{1}=4$ (C)

⑨ $\underbrace{\text{AMATYC, AMATYC}, \ldots, \text{AMATYC}}_{16}$, AMAT; letter A occures $16\cdot 2+2$

$p(AAA)=\frac{34}{100}\cdot\frac{33}{99}\cdot\frac{32}{98}\approx 0.037=3.7\%$ (B)

⑩ $(2s \,\&\, 3j)$ from $(5s \,\&\, xj)$: $C(5,2)\times C(x,3)=10\cdot\frac{x(x-1)(x-2)}{3!}\geq 600$,

$x(x-1)(x-2)\geq 360$, but $8\cdot 7\cdot 6=336<360$, thus min $x=9$ (D)

⑪ the total time is $\frac{d/2}{s}+\frac{d/2}{s+8}+\frac{d}{s+14}=1.67\cdot\frac{d}{s}\Longrightarrow s=$ (42) mph

⑫ $x+y+z\longrightarrow \max$ and $12x+18y+22z=1000\,|\div 6$, $2x+3y=\frac{500-11z}{3}$

$500-11z$ is divisible by 3 if $z=1+3n$; if $z=1$, then $2x+3y=163$ and

$x_{max}=(163-3\cdot 1)\div 2=80$, so $x=80$, $y=z=1$ and $x+y+z=82$ (C)

⑬ Consider the tiles that touch the horizontal boundaries of the checkerboard (upper and lower). At least 2 upper and at least 2 lower tiles must be horizontal, the total minimum $=4$ (C)

⑭ $33=3\cdot 11$, $34=2\cdot 17$, $35=5\cdot 7$ are biprime, the largest factor $=17$ (B)

⑮ starting from g=green, we submit the string (ggrg) eight times to get 8r and $(1+3\cdot 8)g$, which is 25 green counters (D)

⑯ $\frac{b}{11}\in(1.5, 1.8)\longrightarrow b=17, 18, 19$; $\frac{c}{15}\in(1.5, 1.8)\longrightarrow c=23, 24, 25, 26$

$\frac{c}{b}\in(1.5, 1.8)\longrightarrow c=26$, $b=17$, $b+c=43$ (A)

⑰ $(r-t)(s-1)=0$, if $s\neq 1$, $r=t=1$, $s=23$; if $s=1$, $(r,t)=(0,24), (1,23), \ldots (24,0)$ (D)

⑱ By Heron's f-la, $\frac{P}{2}=32$, $A_\Delta=\sqrt{37\cdot 7\cdot 7\cdot 18}=168$ or $\frac{P}{2}\cdot x=32x$, $x=\frac{21}{4}$ (D)

⑲ $361+529+784=1674$ (E)

⑳ If $a_0=a_1=a_2=1$ and $a_{n+1}\cdot a_n=(n-1)!$ $(n=1,2,3,\ldots)$ then

$a_n=\frac{(n-1)!}{a_{n-1}}=\frac{(n-1)!\,a_{n-2}}{(n-2)!}=\frac{(n-1)!\,(n-3)!}{(n-2)!\,a_{n-3}}$ and

$a_{n-3}a_n-a_{n-2}a_{n-1}=\frac{(n-1)!\,(n-3)!}{(n-2)!}-(n-2)!=(n-2)!\left[\frac{n-1}{n-2}-1\right]=(n-3)!$, so

$a_{100}a_{101}=100!$ has a factors $5\cdot 1, \ldots, 5\cdot 20$, but $5^2|25, 50, 75, 100$, total $20+4$ (C)